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தென் மாகாணக் கல்வித் திணைக்களம்
Department of Education, Southern Province

13 ශ්‍රේණිය අවසාන වාර පරීක්ෂණය - 2024 (සැප්තැම්බර්)
தரம் 13 ஆண்டிறுதிப் பரீட்சை - 2024 (செப்டம்பர்) / Grade 13 Final Term Test - 2024 (September)

Combined Maths I

10 | S | II

කාලය / நேரம் / Time } 3 hours

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Extra Reading Time - 10 Minutes

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- You are permitted to remove **only Part B** of the question paper from the Examinations Hall.
- In this paper g denotes the acceleration due to gravity.

For Examiners' Use only

(10) Combined Mathematics I		
Part	Question No.	Marks
A	1	
	2	
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	9	
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B	11	
	12	
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	Percentage	

Paper **I**	
Paper **II**	
Total	
Final Marks	

Final Marks	
In Numbers	
In Words	

Code Numbers	
Marking Examiner	
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Supervised by :	

Part A

01. If $f(n) = 3^{2n}$, using the Principle of Mathematical Induction, prove that when $f(n)$ is divided by 8 the remainder is 1, for all $n \in \mathbb{Z}^+$.

02. Sketch the graphs of $y = 1 - x^2$ and $y = 2|x - 1| - 1$ in the same diagram.
Hence or otherwise, find all real values of x satisfying the inequality, $2 - x^2 \geq 2|1 - x|$.

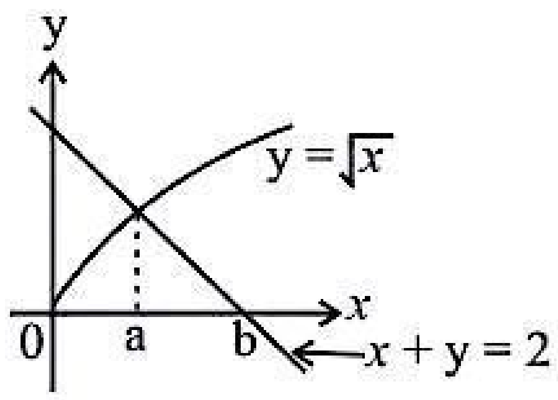
03. Find the sum of the coefficients in the binomial expansion $\left(3\sqrt{2}\,x - \frac{5}{\sqrt{2}}\right)^8$.

If the term independent of x in the given binomial expansion can be expressed as $(5^6)\mathbf{k}$, where $\mathbf{k}$ is a rational number, find the value of $\mathbf{k}$.

04. Point **P** in the Argand diagram, represents the complex number **z**. Sketch, in an Argand diagram the locus of **P** satisfying $|z - 1 - i| = 1$. Find the maximum value of $|z - 2i|$ that satisfies the above given requirement.

05. Find the value of $\lim_{x \to 0} \frac{\sin[\pi \cos^2 x]}{x^2} = \pi$.

06. Find the values of **a** and **b** shown in the figure. The region enclosed by the curves $y = \sqrt{x}, x + y = 2$ and $y = 0$ is rotated about the x - axis through 2π radians. Show that the volume of the solid thus generated is $\frac{5\pi}{6}$.



07. Show that the equation of the normal line to the curve $\frac{x^2}{3} + y^2 = 1$ at the point $P \equiv (\sqrt{3}\cos\theta, \sin\theta)$ is given by $\sqrt{3}x\sin\theta - y\cos\theta = 2\sin\theta\cos\theta$ for $0 < \theta < \frac{\pi}{3}$.

The normal line intersects x axis and y axis at A and B respectively. When **O** is the point of origin, if the area of the triangle **OAB** is $\frac{1}{2}$ square units, find the value of θ.

08. **PQR** is an isosceles triangle with a right angle at point $Q \equiv (4, 2)$. The equation of side **PR** is $x + 2y - 3 = 0$. Find the equations of sides **PQ** and **QR**.

09. Find the equations of the circles with centre of origin as its centre and touches the circle $x^2 + y^2 - 16x - 12y + 64 = 0$ internally and externally.

10. Find the general solutions for the equation $2\sin\theta = \sqrt{3}\tan\theta$.

Hence find the solutions for $2\cos\theta + \sqrt{3}\cot\theta = 0$.

Part B

*** Answer only five questions.**

11. (a) Let $f(x) = x^2 - (\lambda + 2)x + (2\lambda - 1)$, where $\lambda \in \mathbb{R}$.

Show that the equation $f(x) = 0$ has two real distinct roots. If α and β are the roots of $f(x) = 0$, write down $\alpha + \beta$ and $\alpha\beta$ in terms of λ. Find the values of λ, such that both the roots α and β are positive.

Show that the quadratic equation with roots α^2 and β^2 is given by $x^2 - (\lambda^2 + 6)x + (2\lambda - 1)^2 = 0$. Deduce that the quadratic equation with roots $(1 + \alpha^2)$ and $(1 + \beta^2)$ is $x^2 - x(\lambda^2 + 8) + 5\lambda^2 - 4\lambda + 8 = 0$.

(b) Let $f(x) = 4x^3 + 5x^2 + ax + b$ and $g(x) = x^3 + cx + 2$ where $a, b, c \in \mathbb{R}$. Here $(x - 1)$ and $(x + 2)$ are factors of $f(x)$. It is given that $g(x) = (x - 1)^2 h(x)$, where $h(x)$ is a linear polynomial. Find the values of a, b, c.

For these values of a, b and c, show that $f(x) - 4g(x) = 5x^2 + 5x - 10$.

Also show that $f(x) - 4g(x) \geq -\dfrac{45}{4}$.

Show that the remainder is $-15(x + 2)$ when $f(x) - 4g(x)$ is divided by $-15(x + 2)^2$.

12. (a) It is required to select a team of 11 players from 15 cricketers consisting of 06 batsman, 06 bowlers and 03 wicket keepers.

i) If two wicket keepers and at least four bowlers must be included in the team,

ii) If at most two wicket keepers and at least five bowlers must be included in the team,

Find the number of different ways in which a team of 11 can be selected.

(b) It is given that $U_r = \dfrac{r^2 + 3r + 1}{r^2(r + 1)^2}$ for all $r \in \mathbb{Z}^+$.

If, $f(r) = \dfrac{\lambda r + \mu}{(r + 1)^2}$ find values of λ and μ such that $f(r) - f(r - 1) = U_r$ and get $f(r)$. Hence or otherwise, find $S_n = \sum_{r=1}^{n} U_r$ for all $n \in \mathbb{Z}^+$.

Show that the infinite series $\sum_{r=1}^{\alpha} U_r$ is convergent and find its sum S.

Find the smallest integer n can take when $|S - S_n| < \dfrac{1}{10}$.

13. (a) Three matrices A, B and C are given by

$$A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}_{2\times 2}, \quad B = \begin{pmatrix} a & -1 \\ -1 & 0 \\ 1 & b \end{pmatrix}_{3\times 2}, \quad C = \begin{pmatrix} 2 & -3 \\ -3 & 1 \\ 5 & 3 \end{pmatrix}_{3\times 2}$$

Find values a and b such that $BA^T = C$.

Here $a, b \in \mathbb{R}$.

If $f(x) = x^2 - 5x + 7$, show that $f(A) = 0$.

(Here 0 is a 2×2 zero matrix)

Hence find A^{-1}.

(b) **Z** and Z_0 are two complex numbers and $\overline{Z}_0$ is the complex conjugate of Z_0.

If, $|Z - \text{Re}(Z_0)|^2 + |\text{Im}(Z_0)|^2 - 1 = \frac{1}{2}|Z - \overline{Z}_0|^2$ show that $|Z - Z_0|^2 = 2$.

Hence, draw the locus of the point representing **Z** complex number that satisfies,

$2|Z - 2|^2 + 16 = |Z - 2 + 3i|^2$ on an Argand diagram.

(c) Let $p, q \in \mathbb{Z}^+$,

If $\dfrac{\left(\cos\frac{\pi}{8} + i\sin\frac{\pi}{8}\right)^p}{\left(\cos\frac{\pi}{12} - i\sin\frac{\pi}{12}\right)^q} = i$ find the least possible positive integer value **p** and **q** can take

14. (a) Let $f(x) = \dfrac{x-2}{(x-1)^2}$ for $x \neq 1$.

Show that $f'(x)$, the derivative of $f(x)$, is given by $f'(x) = \dfrac{3-x}{(x-1)^3}$ for $x \neq 1$.

Hence find the intervals on which $f(x)$ is increasing and decreasing.

Also, find the coordinates of the turning point of $f(x)$.

It is given that $f''(x) = \dfrac{2(x-4)}{(x-1)^4}$ for $x \neq 1$.

Find the coordinates of the point of inflection of the graph $y = f(x)$.

Sketch the graph of $y = f(x)$ indicating the asymptotes, the turning point and the point of inflection

Hence, draw the graph $y = |f(x)| + 1$ in the above same diagram.

(b) **A** is a fixed point on the circle of radius **r**. **P** is a variable point on the circle.

A normal is drawn from point **A**, to the tangent of circle drawn at point **P**, and they meet at point Y.

Show that the maximum area **APY** triangle can have is $\dfrac{3\sqrt{3}\,r^2}{8}$.

15. (a) It is given that there exist constants **A, B** and **C** such that

$16x^4 + 4x^3 + 16x^2 + x + 1 \equiv A(4x^2+1)^2 + Bx(4x^2+1) + Cx^2$ for all $x \in \mathbb{R}$.

Find the values of **A, B** and **C**.

Hence, write down $\dfrac{16x^4 + 4x^3 + 16x^2 + x + 1}{x(4x^2+1)^2}$ in partial fractions.

Find $\displaystyle\int \frac{16x^4 + 4x^3 + 16x^2 + x + 1}{x(4x^2+1)^2}\,dx$

(b) Using the substitution $t = \sqrt{x}$, show that $\displaystyle\int_0^1 \frac{x^{\frac{3}{2}}}{1+x}\,dx = 2\int_0^1 \frac{t^4}{1+t^2}\,dt = \frac{1}{6}(3\pi - 8)$

Using integration by parts, show that $\displaystyle\int_0^1 x\tan^{-1}\sqrt{x}\,dx = \frac{\pi}{8} - \frac{1}{4}\int_0^1 \frac{x^{\frac{3}{2}}}{1+x}\,dx$ and find $\displaystyle\int_0^1 x\tan^{-1}\sqrt{x}\,dx$.

(c) Using the formula, $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$, where **a** and **b** are constants, show that.

$$\int_{-1}^{1} \frac{x^{2024}}{1+e^x}\,dx = \int_{-1}^{1} \frac{x^{2024}e^x}{1+e^x}\,dx$$

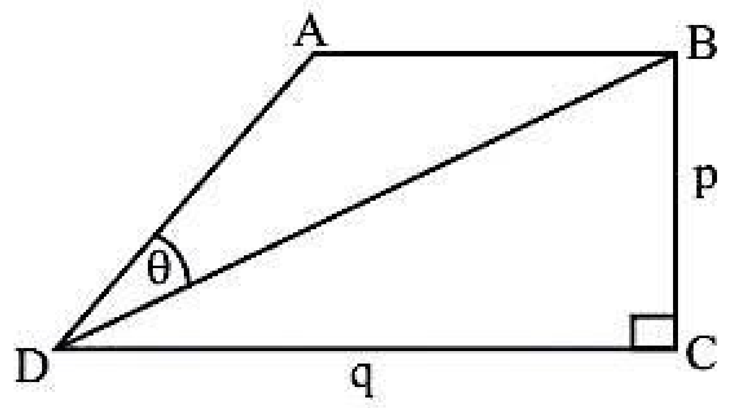
Find $\int_{-1}^{1} \frac{x^{2024}}{1+e^x}\,dx$

Deduce that $\int_{-1/2}^{1/2} \frac{x^{2024}}{1+e^{2x}}\,dx = \frac{1}{2025\,(2^{2025})}$

16. The straight line $x+y=24$ meets the x - axis and y - axis at **A** and **B** respectively.
When $N \equiv \left(\frac{a}{1+\lambda}, \frac{\lambda a}{1+\lambda}\right)$ is a point on the straight line, find the value of **a**. Here $a, \lambda \in \mathbb{R}^+$.
If **M** lies between **O** and **B** on **OB** such that $B\hat{N}M = 90^0$, find the coordinates of **M** in terms of λ. If the area of **AMN** triangle is $\frac{3}{8}$ times of **OAB** triangles area, find λ. Write down the equation of the circle S_1 whose ends of a diameter are **A** and **M**.
Show that the equation of the circle going through the intersection points of S_1 and straight line $l_1: y-x+24=0$ and orthogonal to S_1 is $x^2+y^2-44x+8y+480=0$.

17. (a) Write down **sin (A+B)** in terms of **sin A, sin B, cos A, cos B**.
Hence, deduce **sin 2A** and using $\cos 2A = 1-2\sin^2 A$, write down **sin3A** only in terms of **sin A**.
Show that, $\sin 3x \cdot \sin^3 x = -\frac{1}{8}+\frac{3}{8}\cos 2x - \frac{3}{8}\cos 4x + \frac{1}{8}\cos 6x$
Using the result find the value of $\sin^3 \frac{\pi}{12}$.

(b) The sides **AB** and **CD** of **ABCD** trapezium are parallel.
$D\hat{C}B = \frac{\pi}{2}$, $A\hat{D}B = \theta$, $BC = p$ and $CD = q$.
Show that $AB = \frac{(p^2+q^2)\sin\theta}{p\cos\theta + q\sin\theta}$



(c) Solve the equation $2\tan^{-1}x + \tan^{-1}(x+1) = \frac{\pi}{2}$.

Hence, show that $\cos\left(\frac{\pi}{4} - \frac{1}{2}\tan^{-1}\frac{4}{3}\right) = \frac{3}{\sqrt{10}}$.

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13 ශ්‍රේණිය අවසාන වාර පරීක්ෂණය - 2024 (සැප්තැම්බර්)
தரம் 13 ஆண்டிறுதிப் பரீட்சை - 2024 (செப்டம்பர்) / Grade 13 Final Term Test - 2024 (September)

Combined Maths II

10 | S | II

කාලය / நேரம் / Time } 3 hours

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Extra Reading Time - 10 Minutes

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- In this paper g denotes the acceleration due to gravity.

For Examiners' Use only

(10) Combined Mathematics II		
Part	Question No.	Marks
A	1	
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B	11	
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	Total	
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Paper **I**	
Paper **II**	
Total	
Final Marks	

Final Marks	
In Numbers	
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Code Numbers	
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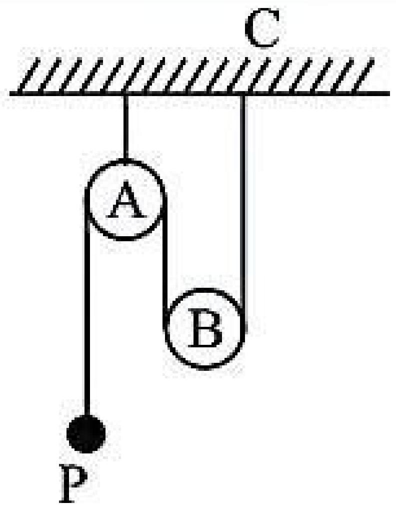
	2.
Supervised by :	

Part A

01. Two particles **A** and **B** of masses **m** and **2m**, moving in the same straight line on a smooth horizontal floor, but in opposite directions collide directly. The velocities of **A** and **B** just before collision are **u** and λ**u**, respectively. The coefficient of restitution for the collision is $\frac{1}{2}$.
Find the velocity of **B** just after collision. If $\lambda = 1$, find the kinetic energy of the system just after collision.

02. A particle is projected from a point **O** on a horizontal floor with velocity $\mathbf{u} = \sqrt{2ga}$ and at an angle θ to the horizontal. The particle just clears a vertical barrier of height $\frac{\mathbf{a}}{\mathbf{2}}$ located at a horizontal distance **a** from **0**. Show that $\tan\theta = 1$ or $\tan\theta = 3$. Here **g** is the gravitational acceleration.

03. In the figure, **A** is a fixed smooth pulley and **B** is a moving smooth pulley of mass **m**. One end of a light inextensible string is attached to a point **C** and a particle **P** of mass **2m** is attached to the other end. The system is released from rest with all strings vertical and taut. Write down equations sufficient to determine the tension of the string.



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04. A lorry of mass **5 metric ton** can travel at a maximum speed of **10 ms^{-1}** on a horizontal road. If the engine power is **25 kW**, find the road resistance on the lorry. If the lorry then moves up a straight road inclined at an angle $\sin^{-1}\frac{1}{100}$ to the horizontal with same road resistance, find its maximum speed. **($g = 10\ ms^{-2}$)**

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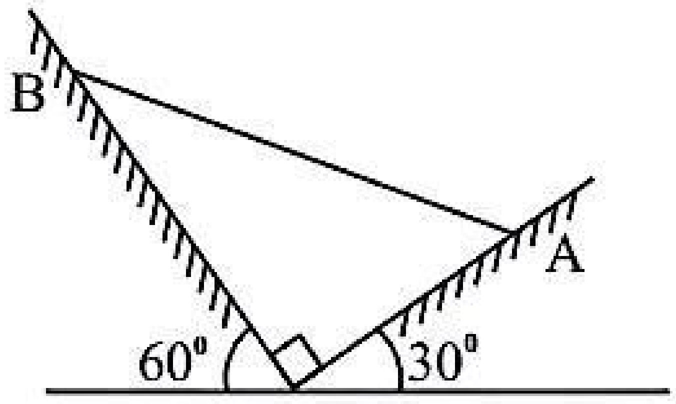
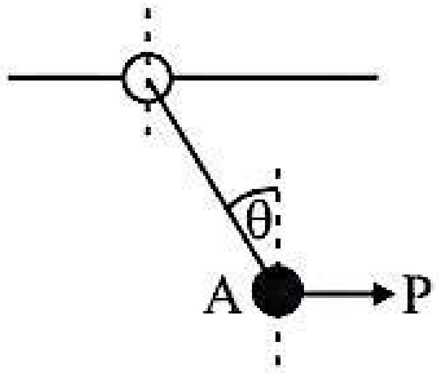
05. A light inextensible string of length $3l$ is sent through a smooth fixed ring on a ceiling and particles **A** and **B** of masses **m** and **M** are attached to its ends. Particle **B** moves in a horizontal circle with **A** as the centre, while particle **A** stays l distance vertically below the ring. Get a relationship between particle **B**'s angular velocity, **m** and **M**.

06. In the usual notation, the position vectors of two points **A** and **B**, with respect to a fixed origin **O** are $3\underline{i} + 2\underline{j}$ and $2\underline{i} + 4\underline{j}$ respectively. Let $\overrightarrow{BC} = 2\,\overrightarrow{OA}$. By considering the dot product $\overrightarrow{OC} \cdot \overrightarrow{AC}$ find $O\hat{C}A$ angle.

07. A uniform rod **AB** of weight **w** is kept at equilibrium on two smooth planes inclined at angles **30^0** and **60^0** to the horizontal (see figure). Mark the forces acting on the rod, find the angle of rod to vertical and reactions at **A** and **B** by inclined planes on rod.



08. One end of a light inextensible string is attached to a rough ring of mass **m** which is free to move along a rough horizontal wire, and the other end is attached to a particle **A** of mass **2m**. The string is kept at equilibrium at an angle θ to the vertical by a horizontal force **P** acting on particle **A**.
If the coefficient of friction between the wire and ring is $\frac{1}{2}$, show that the maximum value θ takes is $\tan^{-1}\frac{3}{4}$.



09. Let **A** and **B** be two independent events of a sample space **S**. If $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$, show that $P[(A \cup B) \cap (A' \cup B')] = \frac{5}{12}$.

10. A set of eight positive integers in ascending order is given by **1, 2, 2, 3, a, b, c, d**. They have a mode and mean of **3** and range of **5**. Find a, b, c, d integer values.

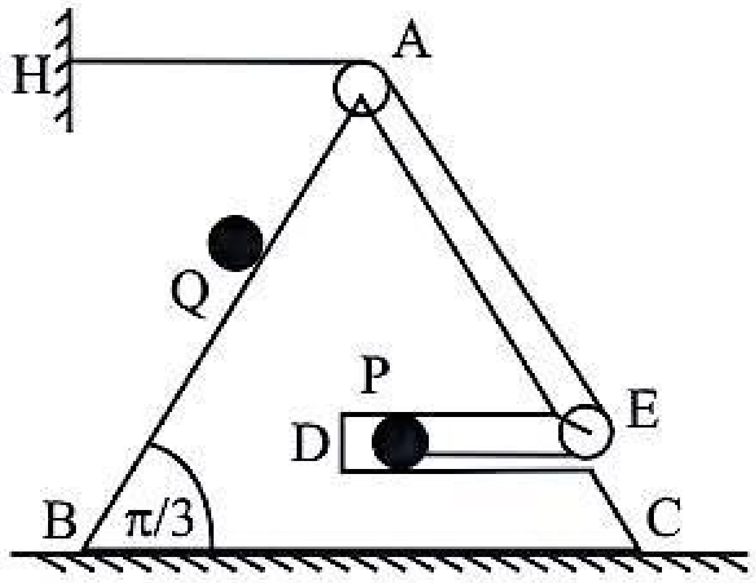
Part B

*** Answer only five questions.**

11. (a) A front open elevator mounted on a vertical tower travels upward at **u** uniform speed. At a certain instant a man in the elevator throws an object **P** vertically upward under gravity with a velocity **v (v > u)** relative to himself. At the instant when the object **P** is at rest relative to the ground, the man releases an object **Q** from rest relative to himself. The gravitational acceleration is **g** ms^{-2}. Draw the velocity time curve for the motion of **P** relative to the elevator. Hence, find the largest gap between them, and the time taken for P to be instantaneously stationary relative to the ground. By considering the velocity time curve for the motion of **Q** relative to the elevator in the same diagram, find the gap between Q and elevator when P and elevator meet again. Also find the time taken for **P** and **Q** to meet.

(b) A battle ship S is sailing due north with uniform velocity $\sqrt{3}u$ kmh^{-1}. At a certain instant, two enemy boats B_1 and B_2 start moving from a port **L**, $4\sqrt{3}$ **km** west from the point A where **S** ship is, with constant velocities **u** kmh^{-1} and $\sqrt{3}u$ kmh^{-1} art angles 60^0 and 30^0 east to north respectively at same time. Sketch the two velocity triangles for the motions of B_1 and B_2 relative to **S** in the same diagram.
Hence, if the range of the guns in S ship is **6.5 km**, what is the time B_1 boat will be in danger?
Also show that the shortest distance between **S** and B_1 and **S** and B_2 happen at the same instant, and that the gap between B_1 and B_2 at this moment is $2\sqrt{6}$ **km**.

12. (a) The triangle **ABC** in the figure is a vertical cross - section through the center of gravity of a uniform wedge of mass **M**. There is a smooth groove **DE** parallel to **BC** in the wedge. The lines **AB** and **AC** are the lines of greatest slopes of the respective faces containing them and $A\hat{B}C = \frac{\pi}{3}$.



The wedge is placed with the face containing **BC** on a fixed smooth horizontal table. A particle **P** of mass **m** is placed on **DE** and one end of a light inextensible string is attached to **P**, passing over smooth light pulleys at points **A** and **E** with the other end attached to a fixed point **H**.
Here **A** and **H** are at the same horizontal level. A particle of mass **2m** is placed at a point on **AC** and while the string is taut, the system is released from rest.
Show that the magnitude of the acceleration of the wedge, and the magnitude of **P** relative to the wedge are equal.
Show that the magnitude of the acceleration of the wedge is $\frac{\sqrt{3}\,mg}{2M + 11m}$.

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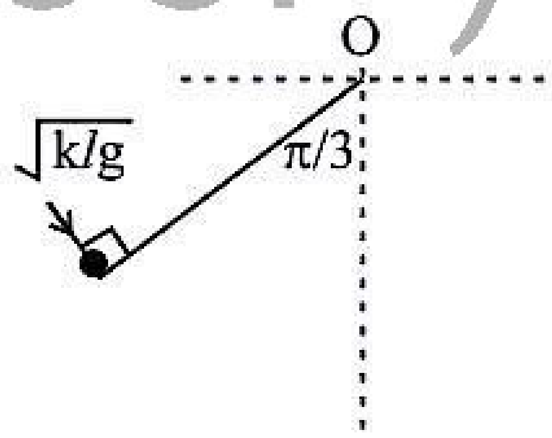
(b) A particle **P** of mass **m** is attached to one end of a light inextensible string of length l. The other end of the string is attached to a fixed point **O** and the string is held taut at a position where **OP** makes an angle $\frac{\pi}{3}$ with the lower vertical as shown in the figure and given a velocity of $\sqrt{klg}$ perpendicular to the string. Here $k > 0$.



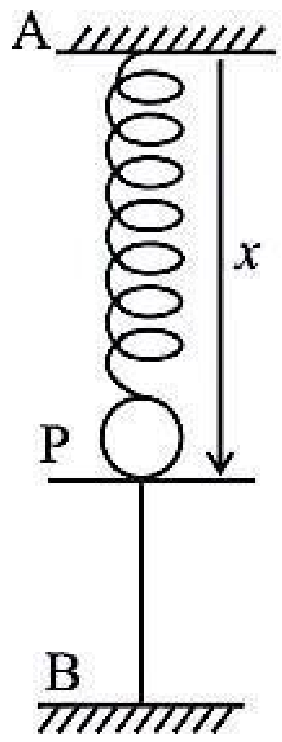
i) Show that the tension of the string when it makes an angle θ anti - clockwise to the lower vertical passing through O is $mg(k-1+3\cos\theta)$

ii) Later, if particle passes **O** horizontal level, find the range of values k can take.

iii) The particle leaves its circular motion at the moment **OP** makes an angle $\frac{\pi}{6}$ to the horizontal after crossing the horizontal line through **O**.

Find the value of **k** and find the speed of the particle at that instant.

13. A particle **P** of mass **m** is attached to two ends of an elastic bow and an inelastic string, each of natural length **2a** and **a**. The other end of the bow is attached to a fixed point **A**, and the other end of the string is attached to a point **B**, vertically below **A** such that $AB = 5a$. (See figure) The modulus of elasticity for the bow is **4 mg** and **mg** for the string.



The particle **P** is placed at a distance **4a** vertically below **A** and released from rest.

Show that when $AP = x\,(0 < x \le 4a)$.

$\ddot{x} + \frac{3g}{a}(x - 3a) = 0$

Take $X = x - 3a$ and express the above equation as $\ddot{X} + \frac{3g}{a}X = 0$.

Assuming a solution of the form $\dot{X}^2 = \frac{3g}{a}(D^2 - X^2)$ to the above equation, show that the amplitude of this simple harmonic motion $D = a$.

Show that the speed of the particle **P** at the instant $AP = \frac{5a}{2}$ is $\frac{3}{2}\sqrt{ag}$.

The string **BP** is cut at the moment when the particle **P** is at a position $AP = \frac{5a}{2}$

Show that $\ddot{x} = -\frac{2g}{a}\left(x - \frac{5a}{2}\right)$, where $AP = x\left(0 < x \le \frac{5a}{2}\right)$, for the new motion of **P**. Assuming that the solution for the above equation is of the form $x = \frac{5a}{2} + \alpha\cos\omega(t - t_0) + \beta\sin\omega(t - t_0)$, find constants α and β.

Here $\omega = \sqrt{\frac{2g}{a}}$ and t_0 is the time for motion of particle between $x = 4a$ and $x = \frac{5a}{2}$

Hence, show that the time taken for the particle **P** to reach the highest point is $t = \pi\sqrt{\frac{a}{g}}\left[\frac{1}{2\sqrt{2}} + \frac{2}{3\sqrt{3}}\right]$.

Also find **AP** distance when **P** particle reaches highest point.

14. (a) Let $\overrightarrow{OA} = \underline{a}$, $\overrightarrow{OB} = \underline{b}$ and $\overrightarrow{OC} = 2\underline{a}$. Here $\underline{a}$ and $\underline{b}$ are two non - zero, non parallel vectors. Point **D** lies on **AB** such that $AD : DB = 2 : 1$. The extended line OD meets BC at E.

Show that $3\overrightarrow{OD} = \underline{a} + 2\underline{b}$

Taking $\overrightarrow{OE} = \lambda\overrightarrow{OD}$ and $\overrightarrow{BE} = \mu\overrightarrow{BC}$ where $\lambda, \mu \in \mathbb{R}$

Show that $(\lambda - 6\mu)\underline{a} + (2\lambda + 3\mu - 3)\underline{b} = 0$.

Hence show that $5\overrightarrow{OE} = 6\overrightarrow{OD}$.

If the lines **OA** and AE are perpendicular,

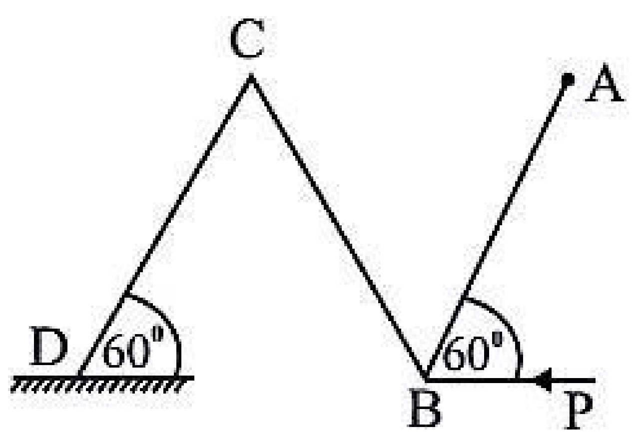
show that the angle between the vectors $\underline{a}$ and $\underline{b}$ is $\cos^{-1}\left(\frac{3\,|\underline{a}|}{4\,|\underline{b}|}\right)$

(b) The coordinates of points **A, B** and **C** are **(2, 0), (3, $\sqrt{3}$)** and **(0, 2$\sqrt{3}$)** respectively according to the rectangular cartesian axes **OX** and OY. The unit of distance here is metres. Forces of magnitudes **6$\sqrt{3}$, 4, 4$\sqrt{3}$** and λ act along **AB, BC, CA** and **CD** respectively, in the directions indicated by the order of the letters. Show that $\lambda = 2$, if the resultant force of this force system makes an anti-clockwise angle of $\frac{\pi}{6}$ with the positive x - axis. Find the magnitude of the resultant force and the coordinates of the intersection point of its line of action with x - axis.

Now a couple of magnitude **24 Nm** acting clockwise is added to the system.

Find the equation of line of action for the new force system.

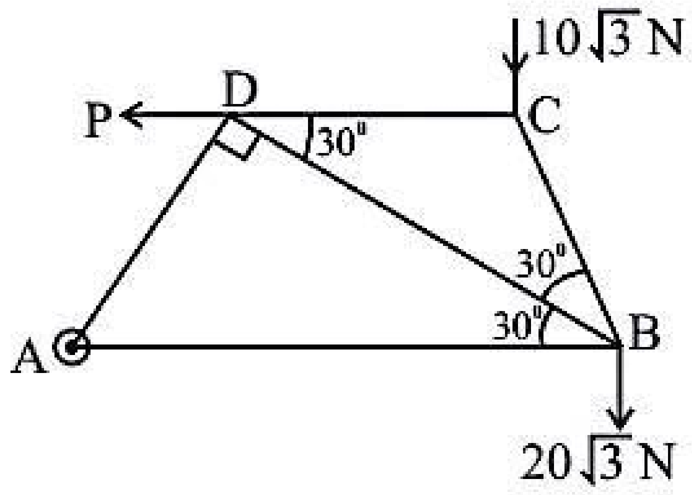
15. (a) Three uniform rods **AB, BC** and **CD**, each of length **2a**, are smoothly jointed about axes **B** and **C**. The weights of the rods **AB, BC** and **CD** are **w, w** and **2w** respectively. The end **A** is smoothly hinged to a fixed point while end **D** is on a rough horizontal plane and **B** and **D** are at the same horizontal level.



The system is kept in equilibrium with the rods making an angle $\frac{\pi}{3}$ to the horizontal by a horizontal force **P** applied at **B**. If the coefficient of friction between the surfaces at **D** is μ, show that $\mu \geq \frac{\sqrt{3}}{7}$.

Find the horizontal and vertical components of the reaction exerted by **BC** on **DC** at **C**. Also show that $P = \frac{5w}{2\sqrt{3}}$.

(b) Framework shown in the figure consists of five light rods **AB, BC, CD, DA** and **BD** smoothly jointed at their ends. Weights of **20$\sqrt{3}$ N** and **10$\sqrt{3}$ N** are suspended at the joints **B** and **C** and the framework is smoothly hinged at **A**. **AB** and **CD** are horizontal.



Find the value of **P**.

Using Bow's notation, draw stress diagram and hence find the stresses in all rods distinguishing between tensions and thrusts.

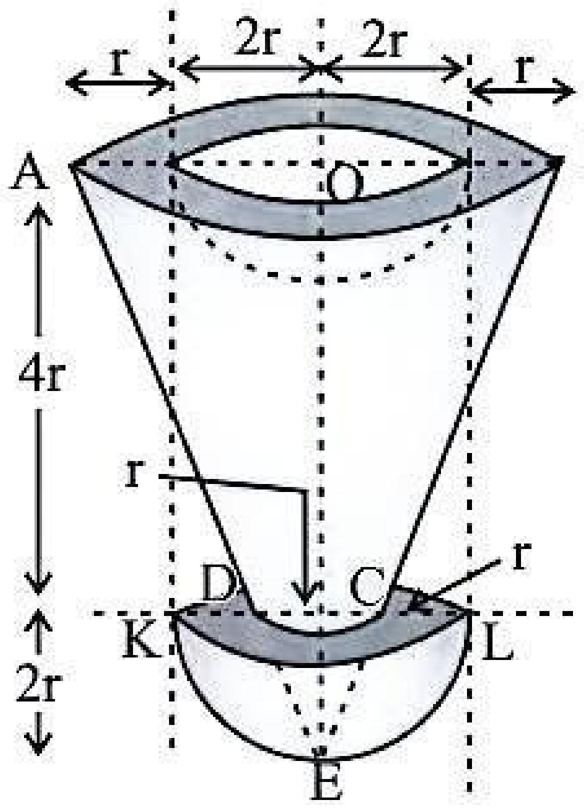
16. Show that,

i) The centre of mass of a uniform solid hemisphere of radius a is at a distance $\frac{3}{8}a$ from its centre,

ii) The centre of mass of a uniform solid right circular cone of straight height h lies on its axis of symmetry at a distance of $\frac{h}{4}$ from the centre of the base.

iii) **DEC** is removed symmetrically about the axis of a solid hemisphere **KEL** of density σ and base radius **2r** to form a hollow cone of radius r at its base.

Also, a solid hemisphere of base radius **2r** and centre **O** is removed from the base of solid cone of base radius **3r**, vertical height **6r** and density **kσ**.



It is inserted into the hollow cone of the solid hemisphere **KEL** as shown in the figure, to form a composite object. Show that the centre of mass of the composite object is at a distance αr from **E** on its axis of symmetry. Here $\alpha = \dfrac{17 + 159k}{2(7 + 19k)}$.

When this object is suspended by a string attached to point **A** and is in equilibrium, show that its axis of symmetry makes an angle $\tan^{-1}\left(\dfrac{3}{6 - \alpha}\right)$ with the vertical.

Also, any point on the curved surface of the hemisphere at the bottom of the compound object touches a smooth horizontal plane and is in stable equilibrium. Show that $k = \dfrac{11}{83}$.

17. (a) An analysis of the charts at a children's hospital revealed several probability measures of the following cases in which one of the boys treated at the hospital is randomly selected.

Case A - The child has asthma

Case B - The child has stomach ache.

Case C - The child has a cold

Given that events **A, B, C** are mutually independent, $P(A) = 0.1$, $P(A \cup B) = 0.37$ and $P(C) = 0.2$

i) Show that $P(B) = 0.3$

ii) Find $P(B'/A')$

iii) Find the probability that the child has a cold but neither stomach ache nor asthma.

iv) Given that the child suffers from only one of the above conditions, find the probability of it being asthma.

(b) Define the formulas for the mean and variance of the data set $x_1, x_2, \ldots, x_i, \ldots, x_n$.

If $y_i = ax_i + b$ for $i = 1, 2, \ldots, n$ then deduce that $\bar{y} = a\bar{x} + b$ and $\sigma_y = |a|\sigma_x$.

Get the mean and variance of the data set **1.01, 2.01, 3.01, 4.01, 5.01**.

There by find the mean and variance of the data set 2.03, 4.03, 6.03, 8.03, 10.03.

Also find the mean of the composite set of these two data sets.

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