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இலங்கைப் பரீட்சைத் திணைக்களம்
Department of Examinations, Sri Lanka

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2024
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2024
General Certificate of Education (Adv. Level) Examination, 2024

සංයුක්ත ගණිතය I
இணைந்த கணிதம் I
Combined Mathematics I

10 E I

පැය තුනයි
மூன்று மணித்தியாலம்
Three hours

අමතර කියවීම් කාලය - මිනිත්තු 10 යි
மேலதிக வாசிப்பு நேரம் - 10 நிமிடங்கள்
Additional Reading Time - 10 minutes

Use additional reading time to go through the question paper, select the questions you will answer and decide which of them you will prioritise.

Index Number							

Instructions:

- ❊ *This question paper consists of two parts;* **Part A** (Questions **1 - 10**) and **Part B** (Questions **11 - 17**).
- ❊ **Part A:** *Answer **all** questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.*
- ❊ **Part B:** *Answer **five** questions only. Write your answers on the sheets provided.*
- ❊ *At the end of the time allotted, tie the answer scripts of the two parts together so that* **Part A** *is on top of* **Part B** *and hand them over to the supervisor.*
- ❊ *You are permitted to remove **only** **Part B** of the question paper from the Examination Hall.*

For Examiners' Use only

(10) Combined Mathematics I		
Part	**Question No.**	**Marks**
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	

Total

In Numbers	
In Words	

Code Numbers

Marking Examiner		
Checked by:	1	
	2	
Supervised by:		

[*see page two*

Part A

1. Using the **Principle of Mathematical Induction**, prove that $7^n - 1$ is divisible by 6 for all $n \in \mathbb{Z}^+$.

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2. Sketch the graphs of $y = |x-3|+1$ and $y = 5-|x|$ in the same diagram. **Hence**, find the area of the rectangular region enclosed by these graphs.

....

[see page three

3. Shade in an Argand diagram, the region consisting of points that represent the complex numbers z satisfying the inequalities $|z-2i|\leq 2$ and $0\leq \mathrm{Arg}\left(z+2\sqrt{3}\right)\leq\frac{\pi}{6}$.
Find the greatest value of $|z|$ for the complex numbers z represented by the points in this shaded region.

4. Show that the constant term in the expansion of $\left(1+x^3\right)\left(x-\frac{1}{\sqrt{x}}\right)^9$ is 93.

[see page four

5. Show that $\lim_{x \to 4} \frac{\left(\sqrt{x-3}-1\right)}{(x-4)^2} \sin\left(\sqrt{x}-2\right) = \frac{1}{8}$.

6. The region enclosed by the curves $y = \frac{2}{x\sqrt[4]{4-x^2}}$, $y = 0$, $x = 1$ and $x = \sqrt{2}$ is rotated about the x-axis through 2π radians. Show that the volume of the solid thus generated is $\pi\left(\sqrt{3}-1\right)$.

[see page five

7. Let C be the curve given parametrically by $x = \ln t$ and $y = e^t + t\ln t$ for $t > 0$.

Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = t(e^t + \ln t + 1)$.

If the tangent drawn to the curve C at the point corresponding to $t = 1$, passes through the point $(1, a)$, show that $a = 1 + 2e$.

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8. Find the equations of the two straight lines passing through the point $A \equiv (-1, 2)$ having a perpendicular distance of 1 from the origin.

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[see page six

9. Let $A \equiv (-1, 1)$ and $B \equiv (3, 3)$. Write down the equation of the circle S with AB as a diameter. Show that the circle $x^2 + y^2 - 4x - 5y + 9 = 0$ touches the circle S internally at B.

10. Show that $\dfrac{\cot\theta}{1+\sin\theta} + \dfrac{\cot\theta}{1-\sin\theta} \equiv 4\operatorname{cosec} 2\theta$.

Hence, solve $\dfrac{\cot\theta}{1+\sin\theta} + \dfrac{\cot\theta}{1-\sin\theta} = 8\cos 2\theta$.

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[see page seven

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இலங்கைப் பரீட்சைத் திணைக்களம்
Department of Examinations, Sri Lanka

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2024
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2024
General Certificate of Education (Adv. Level) Examination, 2024

සංයුක්ත ගණිතය I
இணைந்த கணிதம் I
Combined Mathematics I

10 E I

Part B

✻ Answer **five** questions only.

11. (*a*) Let $f(x) = x^2 + 2x + c$, where $c \in \mathbb{R}$.

It is given that the equation $f(x) = 0$ has two real distinct roots. Show that $c < 1$.

Let α and β be the roots of $f(x) = 0$.

Show that $\alpha^2 + \beta^2 = 4 - 2c$.

Let $c \neq 0$ and $\lambda \in \mathbb{R}$. The quadratic equation with $\alpha + \dfrac{1}{\alpha}$ and $\beta + \dfrac{1}{\beta}$ as its roots is $2x^2 + 12x + \lambda = 0$. Find the values of c and λ.

(*b*) Let $f(x) = x^3 + px^2 + qx + p$, where $p, q \in \mathbb{R}$. The remainder when $f(x)$ is divided by $(x-2)$ is 36 more than the remainder when $f(x)$ is divided by $(x-1)$. Show that $3p + q = 29$.

It is also given that $(x+1)$ is a factor of $f(x)$. Show that $p = 6$ and $q = 11$, and factorize $f(x)$ completely.

Hence, solve $f(x) = 3(x+2)$.

12. (*a*) The parents of a family decide to invite 6 out of 15 of their close relatives for a dinner. While the father has 5 close female relatives and 3 close male relatives, the mother has 3 close female relatives and 4 close male relatives.

Find the number of different ways in which

(i) the father can invite 3 of his close female relatives and the mother can invite 3 of her close male relatives,

(ii) the father can invite 3 of his close relatives and the mother can invite 3 of her close relatives so that 3 males and 3 females are invited.

[see page eight

(*b*) Let $U_r = \dfrac{1}{r(r+2)(r+4)}$ and $f(r) = \dfrac{1}{r(r+2)}$ for $r \in \mathbb{Z}^+$.

Determine the value of the real constant A such that $f(r) - f(r+2) = AU_r$ for $r \in \mathbb{Z}^+$.

Hence, show that $\sum_{r=1}^{n} U_r = \dfrac{11}{96} - \dfrac{1}{4(n+1)(n+3)} - \dfrac{1}{4(n+2)(n+4)}$ for $n \in \mathbb{Z}^+$.

Show further that the infinite series $\sum_{r=1}^{\infty} U_r$ is convergent and find its sum.

Find the value of the real constant m such that $\lim_{n\to\infty} \sum_{r=1}^{n} (mU_r + U_{n+1-r}) = \dfrac{11}{32}$.

13. (*a*) Let $a, b \in \mathbb{R}$, $\mathbf{A} = \begin{pmatrix} 1 & 2 & -1 \\ 3 & a & 2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 0 & a & b \\ 3 & b & a \end{pmatrix}$. It is given that $2\mathbf{A} + \mathbf{B} = \begin{pmatrix} 2 & 4 & 3 \\ 9 & 5 & 4 \end{pmatrix}$.

Show that $a = 0$ and $b = 5$.

With these values for a and b, let $\mathbf{C} = \mathbf{AB}^{\mathbf{T}}$.

Find $\mathbf{C}$ and write down $\mathbf{C}^{-1}$.

Find the matrix $\mathbf{D}$ such that $\mathbf{DC} = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$.

(*b*) Let $z_1, z_2 \in \mathbb{C}$. Show that

(i) $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$

(ii) $\overline{z_1 z_2} = \overline{z_1}\,\overline{z_2}$

(iii) $z_1 \overline{z_1} = |z_1|^2$

Using the result that $\overline{\left(\dfrac{z_1}{z_2}\right)} = \dfrac{\overline{z_1}}{\overline{z_2}}$ for $z_2 \neq 0$, show that if $|z_1| = 1$ and $z_1 \neq \pm 1$, and also if $\dfrac{z_1 + z_2}{1 + z_1 z_2}$ is real, then $|z_2| = 1$.

(*c*) Express $\sqrt{3} + i$ is the form $r(\cos\theta + i\sin\theta)$, where $r > 0$ and $0 < \theta < \dfrac{\pi}{2}$.

Using De Moivre's theorem, show that $\dfrac{(\sqrt{3}+i)^{24}}{2^{23}(1+i)} = 1 - i$.

14. (*a*) Let $f(x)=\dfrac{px+q}{(x-1)(x-2)}$ for $x \in \mathbb{R} - \{1, 2\}$, where $p, q \in \mathbb{R}$. It is given that the graph of $y=f(x)$ has a stationary point at (0, 1). Show that $p=-3$ and $q=2$.

For these values of p and q, show that $f'(x)$, the derivative of $f(x)$, is given by $f'(x)=\dfrac{x(3x-4)}{(x-1)^2(x-2)^2}$ for $x \neq 1, 2$, and find the intervals on which $f(x)$ is decreasing and the intervals on which $f(x)$ is increasing.

Sketch the graph of $y=f(x)$ indicating the asymptotes and the turning points.

Hence, find the number of real solutions to the equation $x^2(x-1)(x-2) = 2 - 3x$.

(*b*) A cylinder with a top and a bottom is made to have a volume of 1024π cm^3. Let r cm be the radius of the cylinder. Show that the total surface area S cm^2 of the cylinder is given by $S=2\pi\left(\dfrac{1024}{r}+r^2\right)$ for $r > 0$.

Show that S is minimum when $r = 8$.

15. (*a*) Find the values of the real constants A and B such that $3t^2+4 = A(t^2-2t+4) + Bt(t+1)$ for all $t \in \mathbb{R}$.

Hence or otherwise, find $\displaystyle\int \frac{3t^2+4}{(t+1)(t^2-2t+4)}\,dt$.

(*b*) Using the substitution $u = x+\sqrt{x^2+3}$, show that $\displaystyle\int_0^1 \frac{1}{\sqrt{x^2+3}}\,dx = \frac{1}{2}\ln 3$.

Let J = $\displaystyle\int_0^1 \sqrt{x^2+3}\,dx$. Using integration by parts, show that 2J = 2 + $\displaystyle\int_0^1 \frac{3}{\sqrt{x^2+3}}\,dx$.

Deduce that J = $1 + \dfrac{3}{4}\ln 3$.

(*c*) Using the formula $\displaystyle\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$, where a is a constant, show that

$$\int_0^{\frac{\pi}{4}} \ln\left(\frac{\cos x}{\cos x+\sin x}\right)dx = \frac{\pi}{8}\ln\left(\frac{1}{2}\right).$$

[see page ten

16. Let $A \equiv (1, 2)$ and $B \equiv (a, b)$, where $a, b \in \mathbb{R}$. It is given that the perpendicular bisector l of the line segment AB has the equation $x + y - 4 = 0$. Find the values of a and b.

Let $C \equiv (3, 1)$. Show that the point C lies on the line l and find $A\hat{C}B$.

Let S be the circle through the points A, B and C. Show that the centre of S is given by $\left(\frac{13}{6}, \frac{11}{6}\right)$ and find the equation of S.

Hence, find the equation of the circle passing through the points A, B and the point $D \equiv (0, 3)$.

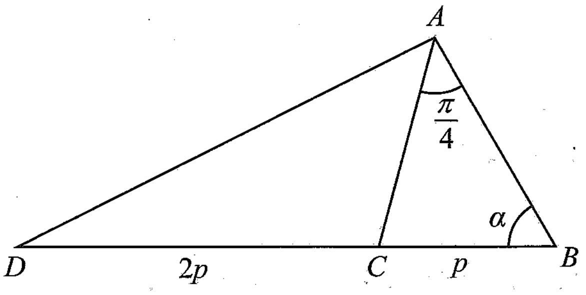
17. (*a*) Express $6\cos 2x - 8\sin 2x$ in the form $R\cos(2x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Hence, solve $6\cos 2x - 8\sin 2x = 5$.

Express $24\cos^2 x - 32\sin x\cos x$ in the form $a\cos 2x + b\sin 2x + c$, where $a, b, c\ (\in \mathbb{R})$ are constants to be determined.

Deduce the minimum value of $24\cos^2 x - 32\sin x\cos x$.

(*b*)



In the triangle ABC shown in the figure, $BC = p$, $B\hat{A}C = \frac{\pi}{4}$ and $A\hat{B}C = \alpha$. The point D lies on the extended line BC such that $CD = 2p$.

Show that $AB = p(\cos\alpha + \sin\alpha)$.

Find AD^2 in terms of p and α.

Deduce that if $AD = 3p$, then $\alpha = \tan^{-1}(5)$.

(*c*) Solve the equation $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$.

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இலங்கைப் பரீட்சைத் திணைக்களம்
Department of Examinations, Sri Lanka

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2024
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2024
General Certificate of Education (Adv. Level) Examination, 2024

සංයුක්ත ගණිතය II
இணைந்த கணிதம் II
Combined Mathematics II

10 E II

පැය තුනයි
மூன்று மணித்தியாலம்
Three hours

අමතර කියවීම් කාලය - මිනිත්තු 10 යි
மேலதிக வாசிப்பு நேரம் - 10 நிமிடங்கள்
Additional Reading Time - 10 minutes

Use additional reading time to go through the question paper, select the questions you will answer and decide which of them you will prioritise.

Index Number							

Instructions:

※ *This question paper consists of two parts;*
Part A (Questions **1 – 10**) *and* **Part B** (Questions **11 – 17**)

※ **Part A:**
*Answer **all** questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.*

※ **Part B:**
*Answer **five** questions only. Write your answers on the sheets provided.*

※ *At the end of the time allotted, tie the answer scripts of the two parts together so that* **Part A** *is on top of* **Part B** *and hand them over to the supervisor.*

※ *You are permitted to remove **only** **Part B** of the question paper from the Examination Hall.*

※ In this question paper, g denotes the acceleration due to gravity.

For Examiners' Use only

(10) Combined Mathematics II		
Part	**Question No.**	**Marks**
A	1	
	2	
	3	
	4	
	5	
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B	11	
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	17	
	Total	

Total

In Numbers	
In Words	

Code Numbers

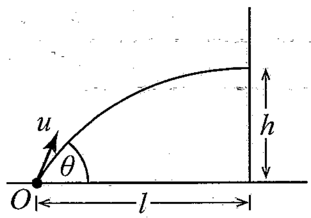
Marking Examiner		
Checked by:	1	
	2	
Supervised by:		

[*see page two*

Part A

1. A particle A of mass $2m$ and a particle B of mass m moving on a smooth horizontal table along the same straight line towards each other with speeds u and $3u$ respectively, collide directly. After the collision A and B move in opposite directions. The coefficient of restitution between A and B is e. Show that $e > \frac{1}{8}$.

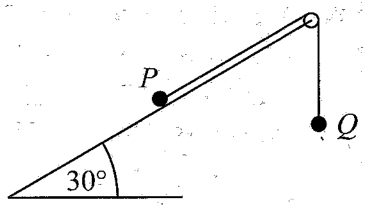
2. A particle is projected from a point O on a horizontal ground, with an initial speed u at an angle $\theta \left(0 < \theta < \frac{\pi}{2}\right)$ to the horizontal. The particle hits a vertical wall which is at a horizontal distance l from O at a height h (> 0) from the ground (see the figure).



Show that $h = l\tan\theta - \frac{gl^2}{2u^2}\sec^2\theta$ and **deduce** that $\sin 2\theta > \frac{gl}{u^2}$.

[see page three

3. A particle P of mass m lies on a rough inclined plane whose inclination to the horizontal is 30°. Particle P is connected, by a light inextensible string passing over a fixed smooth pulley at the top of the inclined plane, to a particle Q of mass $4m$ which is free to move vertically (see the figure). The part of the string on the inclined plane lies along a line of greatest slope of the plane.



The coefficient of friction between P and the plane is $\frac{1}{2}$. The system is released from rest with the string taut. It is given that P moves up the inclined plane. Obtain equations sufficient to determine the tension of the string.

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4. A car of mass M kg travels along a horizontal straight road with its engine working at a constant power of P W. There is a constant resistance of R N to the motion of the car. Find the acceleration of the car at the instant when its speed is u m s^{-1}.
Now, the car travels at a constant speed up a straight road that is inclined at an angle $\theta \left(0<\theta<\frac{\pi}{2}\right)$ to the horizontal. Find this constant speed if the car is subjected to the same resistance R N and have the same power P W.

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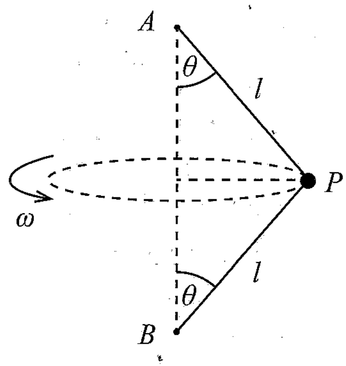
[see page four

5. A particle P of mass $2m$ is connected to two fixed points A and B lying on a vertical line by two light inextensible strings each of length l. The particle P moves in a horizontal circle with a constant angular velocity ω with both strings taut and making an angle θ $\left(0<\theta<\frac{\pi}{2}\right)$ to the vertical, as shown in the figure.

Show that the tension in the string AP is $m(l\omega^2+g\sec\theta)$.

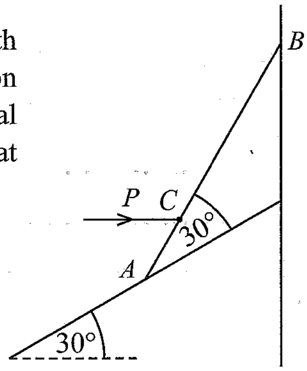


6. Let $\mathbf{u}$ and $\mathbf{v}$ be two unit vectors such that $\mathbf{u}\cdot\mathbf{v}=\frac{1}{2}$. Also, let $\mathbf{a}=\alpha\mathbf{u}+\mathbf{v}$ and $\mathbf{b}=\mathbf{u}+\beta\mathbf{v}$, where $\alpha, \beta \in \mathbb{R}$.

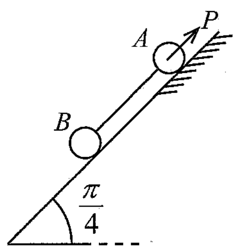
If the vectors $\mathbf{a}$ and $\mathbf{b}$ are perpendicular, and $\mathbf{a}+\mathbf{b}$ is parallel to $\mathbf{u}$, find the values of α and β.

[see page five

7. A uniform rod AB of length $4a$ and weight W is kept in equilibrium with its upper end B against a smooth vertical wall and the lower end A on a smooth plane inclined at 30° to the horizontal by applying a horizontal force P to the rod at the point C, where $AC = a$. The rod is inclined at 30° to the inclined plane, as shown in the figure. Find the value of P.



8. Two particles A and B of masses m and $2m$ respectively, are placed on a plane inclined at an angle $\frac{\pi}{4}$ to the horizontal and are connected by a light inextensible string and kept in equilibrium by a force P applied to A, as shown in the figure. The line of action of P and the string lie along a line of greatest slope of the plane. The particle A lies on the rough part of the plane and the particle B lies on the smooth part of the plane. The coefficient of friction between A and the plane is $\frac{1}{2}$.
Show that $2\left|\sqrt{2}\,P - 3mg\right| \leq mg$.



[see page six

9. Let A and B be two events of a sample space Ω. It is given that $P(A) = \frac{1}{5}$, $P(A \mid B) = \frac{1}{10}$ and $P(B \mid A) = \frac{3}{10}$. Find $P(B)$ and $P(A \cup B)$.

10. The median, the mode and the mean of the following seven observations, arranged in the ascending order, are 5, 7 and 5 respectively:

$$1,\ 3,\ 4,\ p,\ q,\ r,\ s$$

Here p, q, r and s are real numbers.

Find the values of p, q, r and s, and show that the variance of the seven observations is $\frac{38}{7}$.

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[see page seven

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கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2024
General Certificate of Education (Adv. Level) Examination, 2024

සංයුක්ත ගණිතය II
இணைந்த கணிதம் II
Combined Mathematics II

10 E II

Part B

* Answer **five** questions only.

(In this question paper, g denotes the acceleration due to gravity.)

11. (*a*) A car P that begins its journey from rest on a straight road from a point O at time $t = 0$ s, moves with a constant acceleration of f m s^{-2} for 5 seconds. It then moves with the constant speed attained at $t = 5$ s for another 5 seconds and at $t = 10$ s decelerates at a constant deceleration of f m s^{-2} and comes to rest at a point A. The car P then changes its direction instantly and returns towards O with the same constant acceleration of f m s^{-2} on the same road.

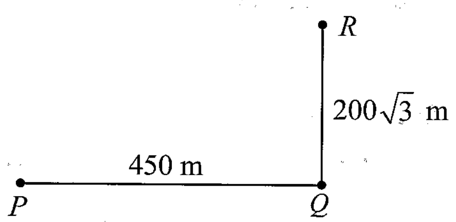
Another car Q that begins its journey with an initial speed of $10f$ m s^{-1} from the point O at $t = 10$ s, moves towards car P with a constant deceleration of f m s^{-2} along the same road. It is given that the distance between P and Q when P comes to rest at A, is 125 m. Sketch velocity-time graphs for the motions of P and Q from $t = 0$ s until they meet, in the same diagram.

Show that

(i) $f = 10$,

(ii) cars P and Q meet at $t = 17.5$ s.

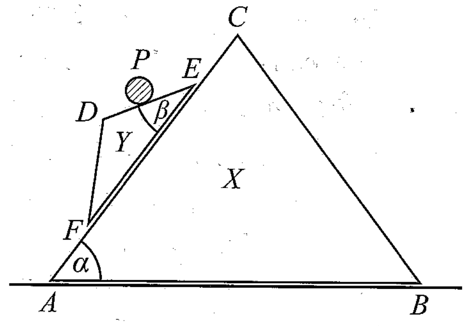
(*b*) Three boats P, Q and R are moving in straight-line paths with uniform speeds. At a certain instant, the boat Q is located 450 m east of the boat P and the boat R is located $200\sqrt{3}$ metres north of the boat Q (See the figure). The boat P sails with the intention of meeting the boat Q and the boat Q sails with the intention of meeting the boat R.



It is given that the boat P meets the boat Q in 45 seconds and the boat Q meets the boat R in 20 seconds.

Show that the speed of the boat P relative to the boat Q is 10 m s^{-1}, and find the distance between the boat P and the boat R at the instant when the boat Q meets the boat R.

12. (*a*) The vertical cross-section through the centres of mass of two smooth uniform wedges X, Y and a particle P is shown in the figure. AC, DE and EF are lines of greatest slope of the faces containing them with $B\hat{A}C = \alpha$ and $D\hat{E}F = \beta \, (< \alpha)$. The face containing AB of the wedge X of mass M_1 is placed on a smooth horizontal table. The face containing EF of the wedge Y of mass M_2 is placed on the face of X containing AC. The particle P of mass m is placed on DE. The system is released from rest. Write down equations sufficient to determine the acceleration of the wedge X, while the wedge Y moves with its face containing EF touching the face of the wedge X containing AC and the particle P moves touching DE.



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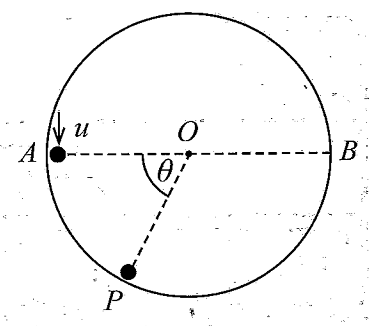
(*b*) The vertical cross-section perpendicular to the horizontal axis of a fixed hollow right-circular cylinder of radius a with a smooth inner surface is shown in the adjoining figure.



The point O is its centre, and A and B are the ends of its horizontal diameter. A particle P of mass m is projected in the vertically downward direction from A on the inner surface of the cylinder with speed u. Let v be the speed of P when OP has turned through an angle θ with P is in contact with the cylinder. Show that $v^2 = u^2 + 2ga\sin\theta$.

It is given that P leaves the inner surface of the cylinder when $\theta = \frac{7\pi}{6}$. Show that $u = \sqrt{\frac{3ga}{2}}$.

13. One end of a light elastic string of natural length a is attached to a fixed point O and the other end to a particle P of mass m, and P has been set to vertical motion. When it is moving vertically downward, it passes through the point A below O, where $OA = a$, its speed is $\sqrt{2ag}$. The particle comes to instantaneous rest at point B, $3a$ below O. Show that the modulus of elasticity of the string is $\frac{3}{2}mg$.

Also, show that the equation of motion of P is given by $\ddot{x} + \omega^2\left(x - \frac{5a}{3}\right) = 0$, where $OP = x$ for $x > a$ and ω (> 0) is a constant to be determined.

Re-write the above equation of motion by taking $X = x - \frac{5a}{3}$.

Find the centre, amplitude and the period of this simple harmonic motion of the particle.

Using the formula $\dot{X}^2 = \omega^2(C^2 - X^2)$, where C is the amplitude, find the maximum speed of P.

On its way up, show that P barely reaches O.

Show that the total time taken by P to move from B to O is $\sqrt{\frac{2a}{27g}}\left(2\pi + 3\sqrt{3}\right)$.

If the above simple harmonic motion is initiated by pulling P down and releasing, state how far the string must be pulled from its natural length.

14.(*a*) Let $OABC$ be a parallelogram with $OA = a$, $OC = 2a$ and $A\hat{O}C = \frac{\pi}{3}$. Also, let **u** and **v** be the unit vectors in the directions of $\overrightarrow{OA}$ and $\overrightarrow{OC}$ respectively.

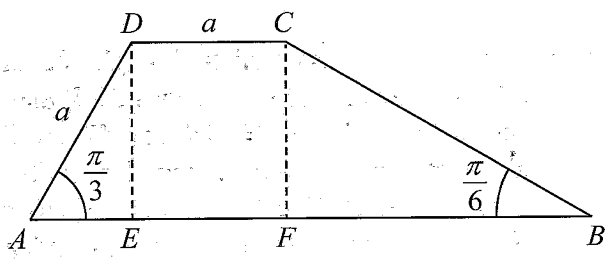
Show that $\overrightarrow{OD} = \frac{1}{2}a\mathbf{u} + 2a\mathbf{v}$, where D is the mid-point of BC.

Let E be the point on AB such that OD is perpendicular to DE.

Show that $\overrightarrow{DE} = \frac{a}{2}\mathbf{u} - \frac{a}{3}\mathbf{v}$.

Let F be the point of intersection of the extended lines OA and DE. Show that $\overrightarrow{OF} = \frac{7a}{2}\mathbf{u}$.

(*b*) Let $ABCD$ be a trapezium with AB parallel to DC, $A\hat{B}C = \frac{\pi}{6}$, $B\hat{A}D = \frac{\pi}{3}$ and $AD = DC = a$. The points E and F are on AB such that $A\hat{E}D = A\hat{F}C = \frac{\pi}{2}$ (See the figure). Forces of magnitude P, αP, βP and γP act along AB, BC, DC and AD respectively, in the directions indicated by the order of the letters. It is given that the resultant force of these is of magnitude $\sqrt{7}P$, and it passes through the points E and C in the sense from E to C. Find the values of α, β and γ.



Now, a couple is added to the system such that the line of action of the resultant of the new system passes through the point F. Find the moment of the couple added.

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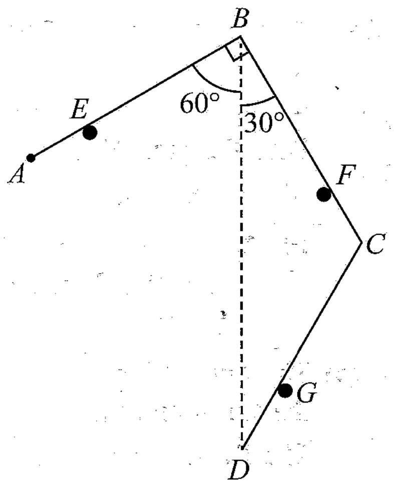
15. (*a*) Three uniform rods AB, BC and CD of equal length $4a$ and equal weight W are smoothly jointed at the end points B and C. The end A is smoothly hinged to a fixed point. The three rods are kept in equilibrium in a vertical plane by placing the rods on three smooth pegs E, F and G such that $AE = CF = DG = a$, $A\hat{B}D = 60°$, $C\hat{B}D = 30°$, and BD is vertical as shown in the figure.



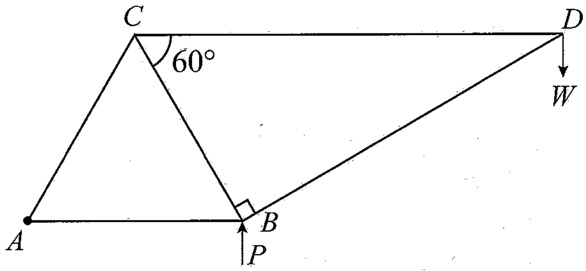
Show that

(i) the magnitude of the reaction exerted on the rod CD by the peg G is $\frac{W}{3}$ and

(ii) the magnitude of the reaction exerted on the rod BC by the peg F is $\frac{11W}{9}$.

Also, find the reaction exerted on the rod BC by the rod AB at the joint B.

(*b*) The framework shown in the figure consists of five light rods AB, BC, CA, CD and DB that are smoothly jointed at their ends. It is given that $AB = BC = CA = 2a$, $C\hat{B}D = 90°$ and $B\hat{C}D = 60°$. A load W is suspended at the joint D and the framework is smoothly hinged to a fixed point at A and kept in equilibrium in a vertical plane with AB horizontal, by a force P applied vertically upwards to it at the joint B.



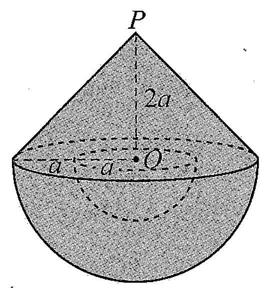
(i) Find the value of P.

(ii) Draw a stress diagram using Bow's notation for the joints D, C and B.

Hence, find the stresses in the rods, stating whether they are tensions or thrusts.

16. Show that the centre of mass of a uniform solid hemisphere of radius a is at a distance $\frac{3}{8}a$ from its centre and the centre of mass of a uniform solid right-circular cone of height h is at a distance $\frac{1}{4}h$ from the centre of its base.

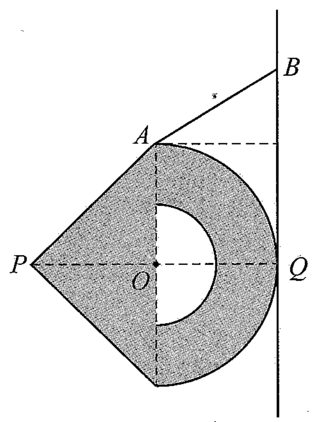
A hemispherical portion of radius a and centre O is carved out from a uniform solid hemisphere of radius $2a$, centre O and density ρ. Now, a uniform solid right circular cone of base radius $2a$ and height $2a$ with density $\lambda\rho$ is rigidly fixed to the remaining portion of the hemisphere, as shown in the adjoining figure. Show that the centre of mass of the body S thus formed, lies at a distance $\frac{(48\lambda+157)}{8(4\lambda+7)}a$ from P, where P is the vertex of the solid cone of S.



Find the value of λ such that the centre of mass of S, lies at O.

Now, suppose that λ has this value.

Let Q be the point at which the extended line PO meets the outer hemispherical surface of S. Also, let A be a point of the circular edge of S. The body S is kept in equilibrium against a rough vertical wall by means of a light inextensible string with one end attached to the point A and other end to a fixed point B on the vertical wall. In the equilibrium position, the outer hemispherical surface of S touches the wall at the point Q. The points O, A, B, P and Q lie on a vertical plane perpendicular to the wall (see the adjoining figure). Show that $\mu \geq 1$, where μ is the coefficient of friction between the outer hemispherical surface of S and the wall.



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17. (*a*) A box B_1 contains 2 white balls and 3 black balls which are identical in all aspects except for their colours. 3 balls are transferred at random from box B_1 into an empty box B_2. Then a ball is drawn at random from box B_2.

Find the probability that

(i) the ball drawn from box B_2 is white,

(ii) 2 white balls and 1 black ball are transferred from box B_1 into box B_2, given that the drawn ball from box B_2 is white.

(*b*) The times taken to solve a puzzle by 20 students were coded by subtracting 10 from each of the times and then dividing by 2.

The frequency distribution of the coded data with 2 missing frequencies is given below:

Coded times (in minutes)	**frequency**
0 – 2	2
2 – 4	f_1
4 – 6	9
6 – 8	f_2
8 – 10	1

Estimated mean for the coded times is given to be 4.4 minutes. Show that $f_1 = 6$ and $f_2 = 2$. Estimate the standard deviation and the mode of the coded times.

Now, estimate the mean, the standard deviation and the mode of the actual times taken to solve the puzzle.

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