සියලුම හිමිකම් ඇවිරිණි/மුඟුப் பதிப்புரிமையுடையது/ $All\ Rights\ Reserved$ ]

ලි ලංකා විභාග දෙපාර්තමේන්තුව ලි ලංකා විභාග දෙපාර්තමේන්තුව දි ලංකා විභාග දෙපාර්තමේන්තුව ලි ලංකා විභාග දෙපාර්තමේන්තුව இலங்கைப் பரீட்சைத் திணைக்களம் இலங்கைப் பரீட்சைத் திணைக்களம் இலங்கைப் பரீட்சைத் திணைக்களம் Department of Examinations, Sri Lanka Department o**இலங்கைப் பரீட்சைத்** பரீயி**னிக்குர் மதிணைக்களம்** 8, Sri Lanka Department of Examinations, Sri Lanka G ලංකා විභාග දෙපාර්තමේන්තුව ලි ලංකා විභාග දෙපාර්තමේන්තුව ලිකුන් ලිකුන් ලිකුන් ලිකුන් ලිකුන් ලිකුන් ලෙස ලිකුන්තුව ලිකුන් ලිකුන් ලිකුන්තුව ලිකුන් ලිකු

අධාපයන පොදු සහතික පතු (උසස් පෙළ) විභාගය, 2024 கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2024 General Certificate of Education (Adv. Level) Examination, 2024

උසස් ගණිතය II உயர் கணிதம் II Higher Mathematics II



පැය තුනයි மூன்று மணித்தியாலம் Three hours

Use additional reading time to go through the question paper, select the questions you will answer and decide which of them you will prioritise.

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| Index         | Number |  |     |     |          |  |
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### Instructions:

- \* This question paper consists of two parts.

  Part A (Questions 1 10) and Part B (Questions 11 17).
- \* Part A:

  Answer all questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.
- \* Part B:

  Answer five questions only. Write your answers on the sheets provided.
- \* At the end of the time allotted, tie the answer scripts of the two parts together so that Part A is on top of Part B and hand them over to the supervisor.
- \* You are permitted to remove only Part B of the question paper from the Examination Hall.
- \* Statistical Tables will be provided.
- \* g denotes the acceleration due to gravity.

## For Examiners' Use only

| Part | Question No. | Marks |   |  |
|------|--------------|-------|---|--|
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|     | Let $\mathbf{F} = \mathbf{i} + 2\mathbf{j} + \alpha\mathbf{k}$ , where $\alpha \in \mathbb{R}$ . A couple comprises of the force $\mathbf{F}$ acting along the line $\mathbf{r} = a\mathbf{i} + \mathbf{j} + \lambda(\mathbf{i} + 2\mathbf{j} + \alpha\mathbf{k})$ and the force $-\mathbf{F}$ acting along the line $\mathbf{r} = \mathbf{i} + b\mathbf{j} + \mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} + \alpha\mathbf{k})$ , where $a, b \in \mathbb{R}$ . If the vector moment of the couple is $\mathbf{i} - \mathbf{j} + \mathbf{k}$ , find the values of $\alpha$ , $\alpha$ and $b$ . |
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| 2.  | Relative to a fixed origin $O$ , the position vectors of three points $A$ , $B$ and $C$ are $2\mathbf{i} + \mathbf{j} + \mathbf{k}$ , $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $2\mathbf{i} - \mathbf{j} - \mathbf{k}$ , respectively. Find $\overrightarrow{AB} \times \overrightarrow{AC}$ and hence find a unit vector perpendicular to the plane containing $A$ , $B$ and $C$ .   |
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The velocity vector of a particle P at time t is given by  $\mathbf{v}(t) = (\sin t + \cos t)\mathbf{i} + (t + t^2)\mathbf{j} + e^t\mathbf{k}$ . It is also given that at t = 0, the position vector of P is equal to the acceleration vector of P. Find the position vector of P at time t.

| 5.   | A smooth sphere $P$ of mass $m$ , moving at speed $u$ , collides with an identical sphere $Q$ which is initially at rest. Immediately before the collision, the velocity of $P$ makes an angle $\theta$ with the line of centres. Immediately after the collision, the velocity of $P$ makes an angle $\phi$ with the line of centres (see the figure). The coefficient of restitution between the spheres is $e(e < 1)$ . Show that $\tan \phi = \frac{2 \tan \theta}{1 - e}$ .  |      |
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| 6.   | A uniform solid right circular cylinder of mass $M$ has radius $4a$ $6a$  |      |
| 6.   | and height $6a$ . It is given that the moment of inertia of the cylinder about an axis through the centre of mass perpendicular to the axis of the cylinder is $7Ma^2$ . The cylinder can rotate about a fixed smooth horizontal axis through a point $A$ on the circumference of one end of  |      |
| 6.   | and height $6a$ . It is given that the moment of inertia of the cylinder about an axis through the centre of mass perpendicular to the axis of the cylinder is $7Ma^2$ . The cylinder can rotate about a fixed smooth horizontal axis through a point $A$ on the circumference of one end of the cylinder. The axis of rotation is tangential to this circumference. The cylinder is held at rest with axis of the cylinder horizontal and at a height $4a$ above the point $A$ (see the figure). Show that, if the   |      |
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| 7. | A student randomly guesses answers to a choices, of which only one choice is con independent of his choice for any other Calculate the probability that  (i) answers to the first three questions a (ii) at most two of the answers to the first three questions a contract the contract three questions and the contract three questions are contract to the first three questions and the contract three questions are contract to the first three questions and the contract three questions are contract to the first three questions are contract to the contract three questions are contract three questions are contract three questions are contract to the contract three questions are contract three questions are contract to the contract three questions are contract three questions. | rrect. Assume that the question.  are all correct.  | e student's                           | r. Each q<br>choice fo | uestion ha<br>or a quest | is five           |
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| $F(x) = \begin{cases} kc(x+1) & \text{if } 0 \le x \le 3, \\ 1 & \text{if } x > 3, \\ 0 & \text{o otherwise.} \end{cases}$ Find the value of $k$ , the median of $X$ , and the mean of $X$ .  10. The probability mass function of a discrete random variable $X$ is as follows: |     |                                       | 17-7-11                                    |  |  |               |   | ariable X is g   |                                       |
|--|-----|---------------------------------------|--|--|--|---------------|---|------------------|---------------------------------------|
| Find the value of $k$ , the median of $X$ , and the mean of $X$ .  10. The probability mass function of a discrete random variable $X$ is as follows:  |     |                                       |  |  | 3,                                       |               |   |                  |                                       |
| Find the value of $k$ , the median of $X$ , and the mean of $X$ .  10. The probability mass function of a discrete random variable $X$ is as follows:  |     | F(x) =                                | $\{1$                                      | , if $x > 3$ ,   |  |               |   |                  |                                       |
| Find the value of $k$ , the median of $X$ , and the mean of $X$ .  10. The probability mass function of a discrete random variable $X$ is as follows: $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$   |     |                                       | 0  | , otherwise  | <b>).</b>                                |               |   |                  |                                       |
| 10. The probability mass function of a discrete random variable $X$ is as follows: $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$  |     | Find the value                        |  |  |  | mean of $X$ . |   |                  |                                       |
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| $ \begin{array}{c c c c c c c c c c c c c c c c c c c $  | 10. | The probability                       | y mass fun                                 | ction of a d   | iscrete rand                             | dom variable  | X is as                                 | follows:         |                                       |
| $P(X=x) \qquad 0.1 \qquad 0.2 \qquad k \qquad 0.2 \qquad 0.1$ Find the value of $k$ , the mean of $X$ and the variance of $X$ .  |     | · · · · · · · · · · · · · · · · · · · |  | 1  |  |               |   | ]                |                                       |
| Find the value of $k$ , the mean of $X$ and the variance of $X$ .  |     |                                       |  | 0.2  |  |               |   | <del>- </del>    |                                       |
|  |     |                                       | 0.1  | 1 0.2  |  | 1 0.4         |   |                  |                                       |
| If the random variable $Y$ , defined by $Y = aX - 1$ has mean 0, find the value of $a$ .   |     | ring the value                        | _ C 1. 41. :                               |  | L  |               |   |                  |                                       |
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ដ៏ជទ្ធ២ សិចិត្តថិ កុរខិបិសី /(ហ្គ $\psi$ ប់ ប្រង់ប្បព្រិស្តាយប្រាក្សា/ $All\ Rights\ Reserved$ 

ලි ලංකා විහාශ දෙපාර්තමේන්තුව ලි ලංකා විභාග දෙපාර්තමේක ලියකුලෙනුණුකු දෙප්තුර්තමේන්තුව විභාග දෙපාර්තමේන්තුව ලි ලංකා විභාග දෙපාර්තමේන්තුව මූහත්කෙන්ට பර්ධකපති නිකාක්ෂයගැට මුහත්කෙන්ට පරිධක් නිකාක්ෂයගැට මුහත්කෙන්ට පරිධක් නිකාක්ෂයගැට මුහත්කෙන්ට පරිධක් නි Department of Examinations, Sri Lanka Department o **මු බැඩාකාස්ප් S/ පාර්තමේ**න්තුව මුහත්කෙන්ට පරිධක්වේ මුහත්කෙන්ට පරිධක්වේ මුහත්කෙන්ට පරිධක්වේ පරිධක්වේ මුහත්කෙන්ට පරිධක්වේ සිට පරිධක්වේ මුහත්කෙන්ට පරිධක්වේ මුහත්කෙන්ට පරිධක්වේ සිට පරිධක්වේ පරිධක්වේ සිට ස

අධායන පොදු සහතික පතු (උසස් පෙළ) විභාගය, 2024 සහ්ඛා් பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2024 General Certificate of Education (Adv. Level) Examination, 2024

උසස් ගණිතය II உயர் கணிதம் II Higher Mathematics II



### Part B

- \* Answer five questions only.
- 11. (a) A force **F** produces a moment of  $2\mathbf{i} 2\mathbf{j} + 3\mathbf{k}$  about the origin O. The line of action of **F** lies on a plane perpendicular to the x axis and passes through the point  $A \equiv (1, 1, 0)$ .
  - (i) Determine F.
  - (ii) It is given that the force **F** has zero moment about the point  $B \equiv (p, 0, q)$ . Find the values of p and q.
  - (b) Relative to a fixed origin O, points A and B of a rigid body have position vectors  $b\mathbf{i}$  and  $3\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}$  respectively. A force  $\mathbf{F}$ , acting on the body, passes through O and its line of action is parallel to AB. Forces  $b\mathbf{i} 8\mathbf{j} + 8\mathbf{k}$  and  $p\mathbf{k}$ , also acting on the body, pass through points with position vectors  $b\mathbf{f}$  and  $3\mathbf{i} + 4\mathbf{j}$  respectively. Given that the system reduces to a couple, find  $\mathbf{F}$  and p.

Find the couple in the form  $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ .

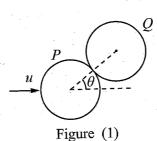
12. A lamina in the shape of a rectangle ABCD with AB = a and BC = b is immersed in a homogeneous liquid with its surface vertical and the edge AB on the free surface of the liquid. Show that the centre of pressure of the lamina is at a depth  $\frac{2b}{3}$  from the free surface of the liquid.

A cubical tank has, on one of its vertical sides, a lid PQRS in the shape of a square of side of length c which is smoothly hinged along the horizontal top edge PQ. The tank is filled with a homogeneous liquid of density  $\rho$  to the level of PQ. Find the magnitude of the minimum force that must be applied at the mid-point of RS perpendicular to the plane of PQRS in order to keep the lid closed.

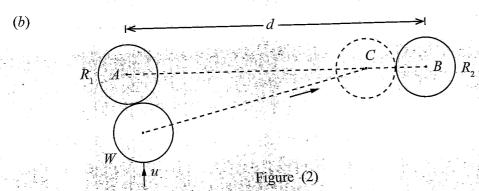
13. A particle P of mass m is projected vertically upwards under gravity with a initial velocity u. Assume that the resistance of air is  $gv^2$  per unit mass, where v is the velocity of P. Obtain the equation of motion of P and show that P comes to rest at a height  $\frac{1}{2g}\ln(1+u^2)$ .

Show that the velocity  $\omega$  with which the particle will return to the point of projection is given by  $\omega = \frac{u}{\sqrt{u^2 + 1}}$ .

**14.** (a)



A smooth sphere P moves with a speed u and makes a perfectly elastic collision with an identical sphere Q, which is at rest. Immediately before the collision, the velocity of Pmakes an angle  $\theta$  with the line of centres (see figure (1)). Find the speeds of P and Q after the collisions and show that the directions of motion of P and Q after the collisions are perpendicular.



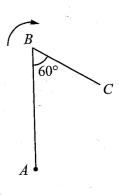
In a billiard game, all the billiard balls are identical and of radius r. Assume that all the balls are smooth and the collisions are perfectly elastic.

Two red balls  $R_1$  and  $R_2$  are initially at rest with their centres at the points A and Brespectively, where  $AB = \tilde{d}(>3r)$ . A white ball W moves with a speed u towards  $R_1$  in a direction perpendicular to AB. After hitting  $R_1$ , W changes its direction and then collides with  $R_2$ . Suppose that at the moment of hitting  $R_2$ , the centre of W lies at a point C on AB (see figure (2)). Using the result in part(a), or otherwise,

- (i) state the directions in which  $R_2$  and W will move after the  $2^{\rm nd}$  collision.
- (ii) find the speeds of  $R_2$  and W after the  $2^{\rm nd}$  collision.
- 15. A frame consists of two uniform rods AB and BC, having lengths 4a and 2a respectively, which are rigidly joined together at B in such a way that  $A\hat{B}C = 60^{\circ}$ , (see the figure). The mass per unit length of both rods is m. Show that the moment of inertia of the frame about an axis through A perpendicular to the plane of the frame is  $48ma^3$ . The frame can rotate freely about a fixed horizontal axis through A and

perpendicular to the frame. Initially AB is vertical, with B above A, and the frame is given an angular speed  $\sqrt{\frac{3g}{8a}}$ . The sense of the motion of the frame is such that, when AB is first horizontal, C is below AB. Show that angular

speed of the frame at the instant when AB is horizontal is  $\sqrt{\frac{(24+\sqrt{3})g}{24a}}$ 



- 16. A computer game consists of two levels, Level 1 and Level 2. A player starts at Level 1 and proceeds to Level 2, regardless of the outcome at Level 1. The probability of a win at Level 1 is 0.6. The probability of a win at Level 2 is 0.4 or 0.2, depending on a win at Level 1 or a loss at Level 1 respectively.
  - (i) Find the probability that a player
    - (a) wins at both levels,
    - (b) wins only one of the two levels.
  - (ii) Given that a player has won at Level 2 of a game, find the probability that he has also won at Level 1 of that game.
  - (iii) A player repeatedly plays this computer game until he wins at both levels of a game. Suppose that the outcomes of different games are independent. Find the expected number of games that a player has to play to win both levels of a game.
- 17. (a) A random variable X follows a Poisson distribution with the probability mass function  $P(X=x) = \frac{e^{-\lambda} \lambda^x}{x!} \text{ for } x = 0, 1, 2, ..., \text{ where } \lambda (>0) \text{ is a parameter.}$

Show that  $E(X) = \lambda$  and  $E(X^2) = \lambda^2 + \lambda$ .

The number of misrecognized letters per page by a character recognition system, follows a Poisson distribution with a mean of 0.5. Find the variance of the number of misrecognized letters in a page.

Given that this system has misrecognized at most two letters in a particular page, find the probability that there are no misrecognized letters in that page.

(b) Of a certain production, the copper content in a product follows a normal distribution with a mean of 59.9 grams and a standard deviation of 2.5 grams. Find the probability that the copper content of a randomly selected product from this production is more than 61 grams. Among the products with copper content less than 61 grams, find the percentage that will have copper content exceeding 60 grams.

සියලුම හිමිකම් ඇව්රිණි /ඟුඟුப් பதிப்புரிமையுடையது / $All\ Rights\ Reserved$ ]

අධායන පොදු සහතික පනු (උසස් පෙළ) විභාගය, 2024 கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2024 General Certificate of Education (Adv. Level) Examination, 2024

උසස් ගණිතය உயர் கணிதம் Higher Mathematics



පැය තුනයි மூன்று மணித்தியாலம் Three hours අමතර කියවීම් කාලය - මිනිත්තු 10 යි மேலதிக வாசிப்பு நேரம் - 10 நிமிடங்கள் Additional Reading Time - 10 minutes

Use additional reading time to go through the question paper, select the questions you will answer and decide which of them you will prioritise.

| 1 | Index Number   | * |      | ŀ   |     |        |    |    |
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### Instructions:

- \* This question paper consists of two parts.

  Part A (Questions 1 10) and Part B (Questions 11 17).
- \* Part A:

  Answer all questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.
- Answer five questions only. Write your answers on the sheets provided.
- \* At the end of the time allotted, tie the answer scripts of the two parts together so that Part A is on top of Part B and hand them over to the supervisor.
- \* You are permitted to remove only Part B of the question paper from the Examination Hall.

# For Examiners' Use only

| (11) Higher Mathematics I |              |       |  |  |  |  |
|---------------------------|--------------|-------|--|--|--|--|
| Part                      | Question No. | Marks |  |  |  |  |
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| a             | nd 5. Show that $R$ is an equivalence relation on $\mathbb{Z}$ and write down the equivalence class of   |
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| 3.         | 3. Let $f(x) = x^2 - 4x + 5$ for $x \ge 2$ . Show that $f$ is one-to-Also find $f^{-1}(x)$ .           | s one-to-one and find the range of $f$ .      |  |  |
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|            | $\begin{vmatrix} a & a^2 & 1 + a^3 \end{vmatrix}$  |   |  |  |
| ł.,        | Let $a, b, c \in \mathbb{R}$ . Show that $\begin{vmatrix} b & b^2 & 1 + b^3 \end{vmatrix} = (1 + abc)$ | (a-b)(b-c)(c-a).                              |  |  |
|            | $\begin{vmatrix} c & c^2 & 1+c^3 \end{vmatrix}$  |   |  |  |
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|                                       | =                | Show that the tangents drawn to the parabola $y^2 = 4\alpha x$ from the point $(-a, 0)$ are perpendicular  |
|                                       | 3.               |  |
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| 1                                     | 20 m<br>41<br>21 | 하게 하게 하다 사람들은 경에 집중을 가장하는 사람들이 들어가 되었다.  |
| -                                     |                  |  |
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|                                       | ii (s            | Let $a, b \in \mathbb{R}$ . Also, let $f: \mathbb{R} \to \mathbb{R}$ be the function given by $f(x) = \begin{cases} \frac{a(\sqrt{x+1}-2)}{x-3} & \text{if } x > 3, \\ x = 3 & \text{otherwise} \end{cases}$ |
|                                       | 6.               | Let $a, b \in \mathbb{R}$ . Also, let $f: \mathbb{R} \to \mathbb{R}$ be the function given by $f(x) = \begin{cases} x-3 \\ -3 \end{cases}$   |
|                                       |                  | $e^{x-3} + bx  \text{if}  x \le 3.$  |
|                                       |                  |  |
|                                       |                  | It is given that $f$ continuous and the gradient of the tangent line drawn to the graph of $f$ when $x = is e^{-3} + 1$ . Find the values of $a$ and $b$ .   |
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| 7.    | Let $f(x) = 2^x$ for $x \in \mathbb{R}$ and $g(x) = 1 + \ln(x+1)$ for $x > -1$ .   |              |  |  |  |  |  |
|-------|--|--------------|--|--|--|--|--|
|       | Find the gradient of the tangent lines drawn to the graphs of $f$ and $g$ when $x=0$ .   |              |  |  |  |  |  |
|       | Now, let $h: \mathbb{R} \to \mathbb{R}$ be the function defined by $h(x) = \begin{cases} 2^x & \text{if } x \leq 0, \\ 1 & \text{otherwise} \end{cases}$   |              |  |  |  |  |  |
|       | Now, let $n$ . If $x > 0$ . $1 + \ln(x+1)  \text{if}  x > 0.$  |              |  |  |  |  |  |
|       |  |              |  |  |  |  |  |
|       | Is h differentiable at $x = 0$ ? Justify your answer.  |              |  |  |  |  |  |
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| <br>O | Solve the differential equation $\frac{dy}{dx} = \frac{x-y}{x+y} + \frac{y}{x}$ .  |              |  |  |  |  |  |
| σ.    | dx $x+y$ $x$   |              |  |  |  |  |  |
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|---|--|--|---|---|--|
| Show that $\int \{x\}$  | $f'(x) + x^2 f''(x) \} $                       | $dx = \int f(x)  dx$   |   |   |  |
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| خُو فَاوْدَ عِنْ عِنْ كُونَ يُوْلُونِي فَوْلُونِي فِي الْمِيْزِينَ وَعَلَيْهِ عِنْ فِي فَالْمِي فِي |  | د کار در                     | ાં મુખ્યત્વે કર્યા છે.<br>મુખ્યત્વે મુખ્યત્વે | , ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; | -  |
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ដែចទួම សិចិត្តថា  $rac{1}{2}$  គ្រាប់ ប្រាប់ ប្រា

ලි ලංකා විහාග දෙපාර්තමේන්තුව ලී ලංකා විකාශ දෙපාර්තමේ**න්තුව වි**තාශ දෙපාර්තමේන්තුව විතාශ දෙපාර්තමේන්තුව ලී ලංකා විතාශ දෙපාර්තමේන්තුව இலங்கைப் பரிட்சைத் திணைக்களம் இலங்கைப் பரிட்சுத் திணைக்களும் இலங்கைப் பரிட்சைத் திணைக்களம் இலங்கைப் பரிட்சைத் திணைக்களம் Department of Examinations, Sri Lanka Department of **ඔබාස්කානය (Sri Lanka Department** of Examinations, Sri Lanka ලී ලංකා විතාහ දෙපාරතමේන්තුව ලී. ලංකා විතාහ දෙපාරතමේන්තුව ම අලංකා විතාහ දෙපාරතමේන්තුව දී ලංකා විතාශ දෙපාරතමේන්තුව இலங்கைப் பரீட்சைத் திணைக்களம் இலங்கைப் එ**ඩුවිකා වූ මාණය විතාහන් විතාහන්වේ වැඩියා වූ විතාහ**ස්සියාවේ ඉහස්

> අධායන පොදු සහතික පතු (උසස් පෙළ) විභාගය, கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2024 General Certificate of Education (Adv. Level) Examination, 2024

උසස් ගණිතය கணிகம் உயர் Higher Mathematics



# Part B

- \* Answer five questions only.
- 11. (a) Let A, B and C be subsets of a universal set S. Stating clearly the laws of Algebra of sets that you use, show that
  - (i)  $(B \cup C) \cap A' = (A' \cap B) \cup (C \cap A' \cap B')$
  - (ii)  $A \cap (B \cap C') = (A \cap B) \cap (A \cap C)'$ .
  - (b) Among a group of 400 tourists, 103 preferred to visit only Anuradhapura, 32 preferred to visit only Kandy and 71 preferred to visit only Jaffna. 19 preferred to visit all three cities. 41 do not prefer to visit any of these three cities. 235 do not prefer to visit Kandy. 200 do not prefer to visit Jaffna.

Find the number of tourists who preferred to visit only Kandy and Jaffna.

**12.** (a) Let  $a, b, c \in \mathbb{R}$  and a, b, c > 0.

Show that

(i) 
$$\frac{a+b}{2} \ge \sqrt{ab}$$
,

(ii) 
$$\frac{a^2+b^2}{c^2} + \frac{b^2+2c^2}{a^2} + \frac{2c^2+a^2}{b^2} \ge 2 + 4\sqrt{2}$$
.

(b) Find the value of  $\lambda$  such that,  $\begin{pmatrix} 3 & 2 \\ -8 & -5 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = \lambda \begin{pmatrix} u \\ v \end{pmatrix}$  for some non-zero  $u, v \in \mathbb{R}$ .

Hence or otherwise, find the equation of the line through the origin which gets mapped onto itself, under the transformation defined by  $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ -8 & -5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$ .

13. State and prove De Moivre's Theorem for a positive integral index.

Using De Moivre's theorem, show that  $\cos 5\theta = \cos^5 \theta - 10\cos^3 \theta \sin^2 \theta + 5\cos \theta \sin^4 \theta$  and  $\sin 5\theta = 5\cos^4 \theta \sin \theta - 10\cos^2 \theta \sin^3 \theta + \sin^5 \theta$  for  $\theta \in \mathbb{R}$ .

**Hence**, show that  $\tan 5\theta = \frac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{1 - 10\tan^2\theta + \tan^4\theta}$  when  $\theta \neq \frac{n\pi}{5}$  for  $n \in \mathbb{Z}$ .

Prove that  $\tan \frac{\pi}{5}$  is a root of the equation  $x^4 - 10x^2 + 5 = 0$ .

Give the other roots of the equation  $x^4 - 10x^2 + 5 = 0$  in the form  $\tan \frac{k\pi}{5}$ , stating the values of k. **Deduce** that  $\tan^2 \frac{\pi}{5} + \tan^2 \frac{2\pi}{5} = 10$ .

14. (a) Let  $C_1$  and  $C_2$  be the curves given by  $y = |x^2 - 1|$  and  $y = x^2 - x|x|$  respectively. Sketch the region S, enclosed by the curves  $C_1$  and  $C_2$ .

Show that the area of the region S is  $\frac{2}{3\sqrt{3}}(1+\sqrt{3})$ .

Also, find the volume of the solid generated by rotating S about the x-axis through  $2\pi$  radians.

- (b) Solve the differential equation  $(x^2 + 1)\frac{dy}{dx} + xy = \sqrt{x^2 + 1}\sin x$  subject to the condition y = 1 when x = 0.
- **15.** (a) Let  $I_n = \int_0^a x^n \sqrt{a^2 x^2} \, dx$  for a > 0, and  $n = 0, 1, 2, 3, \dots$ Show that  $I_n = \frac{(n-1)}{(n+2)} a^2 I_{n-2}$  for  $n \ge 2$ .

Hence, evaluate  $\int_{0}^{2} x^{4} \sqrt{4 - x^{2}} dx.$ 

(b) Find the Maclaurin series of  $\sin x$  and  $\ln(1-x)$  in ascending powers of x upto and including the term in  $x^5$ .

**Hence**, obtain the Maclaurin series of  $\sin 2x \ln(1+2x)$  in ascending powers of x upto and including the term in  $x^3$ .

Using this, find an approximate value for  $\int_{0}^{4} \sin 2x \ln \left( \frac{1+2x}{1-2x} \right) dx.$ 

- **16.** (i) Prove that the line y = mx + c is a tangent to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  if and only if  $c^2 = a^2m^2 + b^2$ .
  - (ii) Show that the locus of the point of intersection of the tangents to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  which are at right angles, is given by  $x^2 + y^2 = a^2 + b^2$ .
  - (iii) A tangent to the ellipse  $x^2 + 4y^2 = 4$  meets the ellipse  $x^2 + 2y^2 = 6$  at the points P and Q. Show that the tangents at P and Q of the ellipse  $x^2 + 2y^2 = 6$  are at right angles.
- 17. (a) Let  $f(x) = \frac{1 \cos 2x}{3 + \cos 2x}$  for  $x \in \mathbb{R}$ .
  - (i) Show that  $0 \le f(x) \le 1$  for  $x \in \mathbb{R}$ .
  - (ii) Solve the equations f(x) = 0 and f(x) = 1, and sketch the graph of y = f(x) for  $0 \le x \le 2\pi$  indicating the turning points.
  - (b) The following table gives the values of  $f(x) = e^{-x^2}$ , correct to two decimal places, for values of x between 0 and 1:

| X                 | 0.00 | 0.25 | 0.50 | 0.75 | 1.00 |
|-------------------|------|------|------|------|------|
| $f(x) = e^{-x^2}$ | 1.00 | 0.94 | 0.78 | 0.57 | 0.37 |

Applying **Simpson's Rule**, find an approximate value for  $\int_{0}^{x} e^{-x^{2}} dx$ .

An approximate value of  $\int_{0}^{1} \left(15 - e^{3-x^2}\right) dx$  is given to be 0.54. Find an approximate value for  $e^3$ .

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