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AL/2025/10/E-I

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இலங்கைப் பரீட்சைத் திணைக்களம்
Department of Examinations, Sri Lanka

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2025
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2025
General Certificate of Education (Adv. Level) Examination, 2025

සංයුක්ත ගණිතය I
இணைந்த கணிதம் I
Combined Mathematics I

10 E I

පැය තුනයි
மூன்று மணித்தியாலம்
Three hours

අමතර කියවීම් කාලය - මිනිත්තු 10 යි
மேலதிக வாசிப்பு நேரம் - 10 நிமிடங்கள்
Additional Reading Time - 10 minutes

Use additional reading time to go through the question paper, select the questions you will answer and decide which of them you will prioritise.

Index Number							

Instructions:

- *This question paper consists of two parts;* **Part A** (Questions **1 - 10**) and **Part B** (Questions **11 - 17**).
- **Part A:** *Answer **all** questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.*
- **Part B:** *Answer **five** questions only. Write your answers on the sheets provided.*
- *At the end of the time allotted, tie the answer scripts of the two parts together so that* **Part A** *is on top of* **Part B** *and hand them over to the supervisor.*
- *You are permitted to remove **only** **Part B** of the question paper from the Examination Hall.*

For Examiners' Use only

(10) Combined Mathematics I		
Part	Question No.	Marks
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
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	16	
	17	
	Total	

Total

In Numbers	
In Words	

Code Numbers

Marking Examiner		
Checked by:	1	
	2	
Supervised by:		

[see page two

Part A

1. Using the **Principle of Mathematical Induction**, prove that $9(2^n) < (n+3)!$ for all $n \in \mathbb{Z}^+$.

2. Sketch the graphs of $y = |x-2| + 2$ and $y = 7 - |x+3|$ in the same diagram.
Hence, solve the equation $|x-2| + |x+3| = 5$.

[see page three

3. Sketch, in an Argand diagram, the locus of the points that represent the complex numbers z satisfying the equation $\left|\mathrm{Arg}(z-2i)\right|=\frac{\pi}{4}$.
Hence or otherwise, find the minimum value of $|z-1|$ for z satisfying $\left|\mathrm{Arg}(\overline{z}+2i)\right|=\frac{\pi}{4}$.

4. Let $n\in\mathbb{Z}^{+}$ be an **odd** integer. It is given that the binomial expansion of $\left(\sqrt{x}-\frac{x}{n}\right)^{n}$ contains an x^2 term. Show that $n=3$ and find the coefficient of x^2.

[see page four

5. Show that $\lim_{x \to 0} \dfrac{\tan x - \sin x}{x\left(\sqrt{3x^2+1} - \sqrt{x^2+1}\right)} = \dfrac{1}{2}$.

6. Let $k > 0$. The region enclosed by the curves $y = \left(e^{kx} - e^{-kx}\right)^2 \sqrt{e^{kx} + e^{-kx}}$, $y = 0$, $x = -\dfrac{1}{k}$ and $x = \dfrac{1}{k}$ is rotated about the x-axis through 2π radians. Show that the volume of the solid thus generated is $\dfrac{2\pi}{5k}\left(e - \dfrac{1}{e}\right)^5$.

[see page five

7. Let C be the curve given parametrically by $x = e^{t} - 2e^{-t}$ and $y = 2e^{t} + e^{-t}$ for $t \in \mathbb{R}$.

Show that $\frac{dy}{dx} = \frac{2e^{2t} - 1}{e^{2t} + 2}$.

Find the gradient of the tangent to the curve C at the point on it corresponding to $x = 1$.

8. Let $P \equiv (2, \alpha)$ and $Q \equiv (1, \beta)$, where $\alpha, \beta \in \mathbb{R}$. The point P lies on the line $l_1 : 2x - y - 1 = 0$, and the point Q lies on the straight line l_2 which is parallel to the line l_1 and such that the acute angle between PQ and l_2 is $\frac{\pi}{4}$. Find the values of α and β.

[see page six

9. Find the equations of the circles with radius 4 units and centres lying on the line $4x - 3y = 0$, and touching the circle $x^2 + y^2 = 1$ externally.

10. Express $\sin\theta - \sqrt{3}\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.
Hence, find the values of θ such that $\sin\theta - \sqrt{3}\cos\theta$ has its minimum value.

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[see page seven

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இலங்கைப் பரீட்சைத் திணைக்களம்
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අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2025
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2025
General Certificate of Education (Adv. Level) Examination, 2025

සංයුක්ත ගණිතය I
இணைந்த கணிதம் I
Combined Mathematics I

උත්තර පත්‍ර පරීක්ෂකවරුන්ගේ ප්‍රයෝජනය සඳහා පමණි

10 E I

Part B

※ Answer **five** questions only.

11. (*a*) Let $f(x) = x^2 - 2kx - k^2 - 1$, for $x \in \mathbb{R}$, where $k \in \mathbb{R}$.

Show that the equation $f(x) = 0$ has two non-zero roots that are real and distinct.

Let α and β be the roots of $f(x) = 0$, and let $r = \dfrac{1}{2\alpha}$ and $s = \dfrac{1}{2\beta}$.

Show that the quadratic equation with r and s as its roots is $4(k^2+1)x^2 + 4kx - 1 = 0$ and that $|r - s| = \dfrac{\sqrt{2k^2+1}}{k^2+1}$.

Deduce that the horizontal distance between the two points of intersection of the graphs of $y = x^3 + 9x^2 + 3x + 1$ and $y = x^3 + x^2 - x + 2$ is $\dfrac{\sqrt{3}}{2}$.

(*b*) (i) Let $a \in \mathbb{R}$. Show that if $(x - a)$ is a factor of a polynomial $p(x)$, then there exists a polynomial $s(x)$ such that $p(x) - (x-a)p'(x) = (x-a)^2 s(x)$, where $p'(x)$ is the derivative of $p(x)$.

(ii) Let $g(x) = x^3 - \lambda x^2 - 2x - (x-2)(3x^2 + \mu x - 2)$ for $x \in \mathbb{R}$, where $\lambda, \mu \in \mathbb{R}$. It is given that $(x-2)$ is a factor of $g(x)$ and that the remainder when $g(x)$ is divided by $(x-1)$ is -3. Show that $\lambda = 1$ and $\mu = -2$.
Using (i) above, **deduce** that $(x-2)^2$ is a factor of $g(x)$.

12. (*a*) Five books that are different from each other, are to be distributed among three students A, B and C.

Find the number of different ways in which

(i) A receives exactly 2 books, B receives exactly 2 books and C receives exactly 1 book,

(ii) each student receives at least 1 book.

[see page eight

(*b*) Let $U_r = \dfrac{7r+4}{r(r+1)(r+2)}$, $f(r) = \dfrac{A}{r}$ and $g(r) = \dfrac{B}{r+1}$ for $r \in \mathbb{Z}^+$, where A and B are real constants.

Find the values of A and B such that $U_r = [f(r) - f(r+2)] + [g(r) - g(r+1)]$ for $r \in \mathbb{Z}^+$.

Hence, show that $\sum_{r=1}^{n} U_r = \dfrac{9}{2} - \dfrac{2}{n+1} - \dfrac{5}{n+2}$ for $n \in \mathbb{Z}^+$.

Show further that the infinite series $\sum_{r=1}^{\infty} U_r$ is convergent and find its sum.

Find the value of the real constant m such that $\lim_{n \to \infty} \left(\sum_{r=1}^{2n} U_r + m \sum_{r=1}^{n-1} U_{n-r} \right) = 18$.

13.(*a*) Let $\mathbf{A} = \begin{pmatrix} 2 & a \\ 3 & 1 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 2 & 1 & a \\ 1 & 4 & 1 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 4+a & 7 \\ 6 & 7 \\ 3a & 4 \end{pmatrix}$, where $a \in \mathbb{R}$.

Find the values of a such that $\mathbf{A}^{-1}$ exists.

Find $\mathbf{AB}$ in terms of a.

Determine the value of a such that $\mathbf{B}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}} = \mathbf{C}$.

For this value of a, show that $\mathbf{A} - \mathbf{A}^{-1} = 3\mathbf{I}$, where $\mathbf{I}$ is the identity matrix of order 2.

(*b*) Let $w = -\dfrac{\sqrt{2}(3+i)}{(1+2i)}$. Show that $|w| = 2$ and $\operatorname{Arg} w = \dfrac{3\pi}{4}$.

Let $z \in \mathbb{C}$ be such that $|z| = 2$ and $\operatorname{Arg} z = \dfrac{\pi}{3}$.

Also, let A and B be the points representing the complex numbers z and w respectively, on an Argand diagram. Show that $AB^2 = 8 + 2\sqrt{2} - 2\sqrt{6}$.

Let C be the mid-point of AB. Using the triangle AOC, where O is the origin, **deduce** that $\sin^2\left(\dfrac{5\pi}{24}\right) = \dfrac{1}{8}\left(4 + \sqrt{2} - \sqrt{6}\right)$.

(*c*) Let $m \in \mathbb{Z}^+$. It is given that $(1 + \sqrt{3}\,i)^{3m}(1 + i)^8 = 2^{3m+4}$.

Find the least value of m.

[*see page nine*

14. (*a*) Let $f(x)=\dfrac{x^2+x+p}{(x-1)^2}$ for $x \in \mathbb{R}-\{1\}$, where $p \in \mathbb{R}$. It is given that the graph of $y=f(x)$ intersects its horizontal asymptote at a point with the x-coordinate $-\dfrac{1}{3}$. Show that $p=2$.

Show that $f'(x)$, the derivative of $f(x)$, is given by $f'(x)=-\dfrac{(3x+5)}{(x-1)^3}$ for $x \in \mathbb{R}-\{1\}$.

Hence, find the intervals on which $f(x)$ is increasing and the intervals on which $f(x)$ is decreasing.

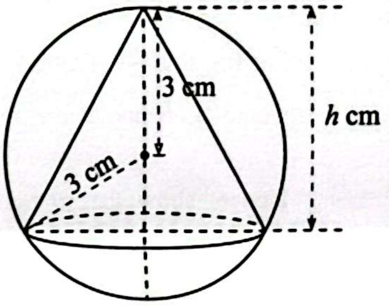
Also, find the coordinates of the turning point of the graph of $y=f(x)$.

It is given that $f''(x)$, the second derivative of $f(x)$ is given by $f''(x)=\dfrac{6(x+3)}{(x-1)^4}$ for $\mathbb{R}-\{1\}$.

Show that the graph of $y=f(x)$ has a point of inflection at $(-3, \dfrac{1}{2})$.

Sketch the graph of $y=f(x)$ indicating the turning point, the asymptotes and the point of inflection.

(*b*) A right circular cone is to be inscribed in a sphere of radius 3 cm, as shown in the figure.



Let h cm be the height of the cone. Show that the volume of the cone V cm^3 is given by $V=\dfrac{\pi}{3}\left(6h^2-h^3\right)$.

Also, show that the largest such cone that can be inscribed in the sphere is obtained when $h=4$.

15. (*a*) Let $k \in \mathbb{R}$. Find $\displaystyle\int \frac{\sqrt{x}}{(1-k^2x)}\,dx$.

(*b*) Show that $\dfrac{d}{dx}\left\{\ln\left(\dfrac{1+\sin 2x}{\cos 2x}\right)\right\}=\dfrac{2}{\cos 2x}$ for $0<x<\dfrac{\pi}{4}$.

Using integration by parts, find $\displaystyle\int (\cos 2x)\ln\left(\frac{1+\sin 2x}{\cos 2x}\right)dx$.

(*c*) Using the formula $\displaystyle\int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx$, where a is a constant, show that

$$\int_0^{\frac{\pi}{6}} \frac{\cos(x+\frac{\pi}{3})}{\sin x+\cos(x+\frac{\pi}{3})}\,dx=\int_0^{\frac{\pi}{6}} \frac{\sin x}{\sin x+\cos(x+\frac{\pi}{3})}\,dx.$$

Deduce that $\displaystyle\int_0^{\frac{\pi}{6}} \frac{\cos(x+\frac{\pi}{3})}{\sin x+\cos(x+\frac{\pi}{3})}\,dx=\frac{\pi}{12}$.

[see page ten

16. Let O be the origin, $A \equiv (1, 2)$ and $B \equiv \left(\frac{5}{2}, \frac{5}{4}\right)$. Find the equations of the angle bisectors of the angles $A\hat{O}B$ and $O\hat{A}B$, and show that these angle bisectors intersect at the point $D \equiv \left(\frac{5}{4}, \frac{5}{4}\right)$.

Find the perpendicular distance from D to the line OA.

Write down the equation of the circle C_1 that touches all three sides of the triangle OAB.

Suppose that the circle C_1 touches OA and AB at the points E and F, respectively. Show that the equation of the circle C_2 that goes through the points A, E and F is given by $4x^2 + 4y^2 - 9x - 13y + 15 = 0$.

Determine whether the intersection of the circles C_1 and C_2 is orthogonal.

17. (*a*) Write down $\sin(A + B)$ in terms of $\sin A$, $\sin B$, $\cos A$ and $\cos B$.

Deduce that $\sin 2\theta = 2\sin\theta\cos\theta$.

Let $0 < \theta < \frac{\pi}{2}$. Express the equation $\cot\theta - 2\tan\theta = \sin 2\theta$ in the form $a\cos^4\theta + b\cos^2\theta + c = 0$, where a, b and c are real constants.

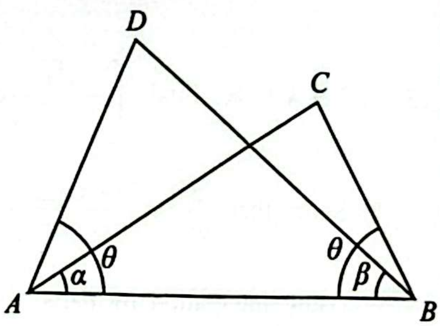
Hence, show that $\theta = \cos^{-1}\sqrt{\frac{\sqrt{17} - 1}{4}}$.

(*b*) Four distinct points A, B, C and D on a plane are such that $B\hat{A}D = A\hat{B}C = \theta$ and $3AD = 4BC$.

Let $B\hat{A}C = \alpha$ and $A\hat{B}D = \beta$. (See the figure.)

Using the Sine Rule, show that $\frac{BC}{AD} = \frac{\sin\alpha\sin(\theta + \beta)}{\sin\beta\sin(\theta + \alpha)}$.

Deduce that $\cot\theta = 3\cot\alpha - 4\cot\beta$.



(*c*) Solve the following simultaneous equations for x and y:

$\sin^{-1}\sqrt{x} = \cos^{-1}\sqrt{y}$

$\tan(\tan^{-1}3x - \tan^{-1}2y) + \tan(\tan^{-1}3y - \tan^{-1}2x) = 1$.

* * *

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AL/2025/10/E-II

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இலங்கைப் பரீட்சைத் திணைக்களம்
Department of Examinations, Sri Lanka

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2025
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2025
General Certificate of Education (Adv. Level) Examination, 2025

සංයුක්ත ගණිතය II
இணைந்த கணிதம் II
Combined Mathematics II

10 E II

පැය තුනයි
மூன்று மணித்தியாலம்
Three hours

අමතර කියවීම් කාලය - මිනිත්තු 10 යි
மேலதிக வாசிப்பு நேரம் - 10 நிமிடங்கள்
Additional Reading Time - 10 minutes

Use additional reading time to go through the question paper, select the questions you will answer and decide which of them you will prioritise.

Index Number | | | | | | | |

Instructions:

* *This question paper consists of two parts;*
  Part A (Questions 1 – 10) *and* **Part B** (Questions 11 – 17)
* **Part A:**
  Answer all questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.
* **Part B:**
  Answer five questions only. Write your answers on the sheets provided.
* *At the end of the time allotted, tie the answer scripts of the two parts together so that* **Part A** *is on top of* **Part B** *and hand them over to the supervisor.*
* *You are permitted to remove only* **Part B** *of the question paper from the Examination Hall.*

* In this question paper, g denotes the acceleration due to gravity.

For Examiners' Use only

(10) Combined Mathematics II		
Part	**Question No.**	**Marks**
A	1	
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B	11	
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	17	
	Total	

Total

In Numbers	
In Words	

Code Numbers

Marking Examiner		
Checked by:	1	
	2	
Supervised by:		

[see page two

Part A

1. A particle A of mass m is at rest on a smooth horizontal table. A particle B of mass $2m$ moving on the table with a speed u collides directly with A. The coefficient of restitution between A and B is e. Show that the speed of the particle A after the collision is $\frac{2}{3}(1+e)u$.

   It is given that the magnitude of the impulse exerted on A by B at the collision is mu. Show that $e=\frac{1}{2}$.

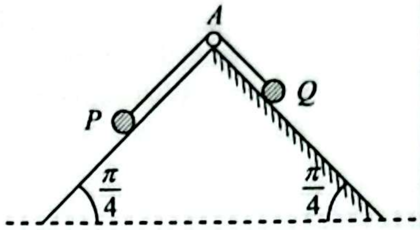
   ....

2. A particle is projected from a point O on a horizontal ground, with an initial speed u at an angle $\theta\left(0<\theta<\frac{\pi}{2}\right)$ with the horizontal. The particle moves under gravity. Find the horizontal range of the projectile and show that if it is $\frac{\sqrt{3}u^2}{2g}$, then $\theta=\frac{\pi}{6}$ or $\theta=\frac{\pi}{3}$.

   ....

[see page three

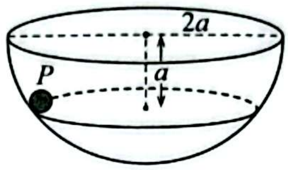
3. Two particles P and Q of masses $4m$ and m respectively are connected by a light inextensible string. The string passes over a small smooth pulley A fixed at the top of two fixed planes, each inclined at an angle $\frac{\pi}{4}$ to the horizontal, as shown in the figure. The plane on which the particle P is placed is smooth and the plane on which the particle Q is placed is rough. The coefficient of friction between Q and the rough plane is $\frac{1}{2}$. The particles are held with the string taut and the parts AP and AQ of the string lying along lines of greatest slope of the respective planes, and released from rest. It is given that P moves down the plane. Obtain equations sufficient to determine the tension of the string.



4. A car of mass 1250 kg is moving at a constant speed of 32 m s^{-1} on a straight horizontal road against a constant resistance of 1000 N. Show that the power generated by the engine of the car is 32 kW. Now, the car moves downwards on a straight road inclined at an angle $\sin^{-1}\left(\frac{1}{5}\right)$ to the horizontal against the same constant resistance. Obtain equations sufficient to determine the speed of the car at an instant when its acceleration is 4 m s^{-2} and its engine working with a power of 20 kW.

[see page four

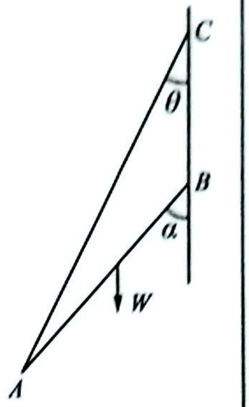
5. A thin hemispherical shell of radius $2a$ is fixed with its circular rim horizontal, as shown in the figure. A particle P of mass m is given a speed u such that it moves on the inner smooth surface of the shell in a horizontal circle whose centre is at a distance a below the centre of the circular rim of the shell. Show that $u = \sqrt{3ga}$.



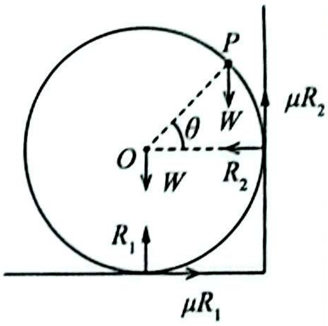
6. Let $\alpha, \beta, \gamma \in \mathbb{R} - \{0\}$. In the usual notation, let $\mathbf{a} = \alpha\,\mathbf{i} + \beta\,\mathbf{j}$ and $\mathbf{b} = \gamma\,\mathbf{i} - \beta\,\mathbf{j}$ be two vectors satisfying $|\mathbf{a}|^2 + |\mathbf{b}|^2 = 1$. Show that $\alpha^2 + 2\beta^2 + \gamma^2 = 1$.
It is given that the vectors $\mathbf{a}$ and $\mathbf{b}$ are perpendicular to each other. Show that $\beta^2 = \alpha\gamma$.
Find the value of γ if $\alpha = \frac{1}{3}$.

[see page five

7. A uniform rod AB of weight W making an angle $\alpha \left(0<\alpha<\frac{\pi}{2}\right)$ with the downward vertical is kept in equilibrium by placing the end B touching a smooth vertical wall while the other end A is suspended by a light inextensible string attached to the wall at a point C vertically above B, as shown in the figure. The string is taut and makes an angle θ with the downward vertical. The rod and the string are lying in a vertical plane perpendicular to the wall. Show that $\tan\alpha = 2\tan\theta$.



8. A particle P of weight W is attached to the rim of a uniform circular thin disc of centre O, radius a and weight W. The disc rests on a rough horizontal floor against a rough vertical wall. The coefficient of friction between the disc and the floor is μ and that between the disc and the wall is also μ. It is given that the disc is in limiting equilibrium when OP makes an angle $\theta \left(0<\theta<\frac{\pi}{2}\right)$ with the horizontal. The forces acting on the disc are marked on the figure. Show that $\cos\theta = \dfrac{2\mu(\mu+1)}{\mu^2+1}$.



[see page six

9. Let A, B and C be three events of a sample space Ω such that A and B are independent, B and C are mutually exclusive, $P(B) = \frac{1}{5}$ and $P(A \cup B) = P(B \cup C)$. Show that $P(C) = \frac{4}{5}P(A)$.
Find $P(C)$ if $P(C|A) = 2P(A \cap C) \neq 0$.

....

10. The mean of five observations arranged in ascending order as 1, p, 4, 10, q, is 6, where p and q are real numbers. These observations are transformed by the transformation $y = 3x + c$, where x denotes the original observations and c is a real number. The mean and the range of the transformed observations are 20 and 33 respectively. Find the values of p, q and c.

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[see page seven

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இலங்கைப் பரீட்சைத் திணைக்களம்
Department of Examinations, Sri Lanka

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2025
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General Certificate of Education (Adv. Level) Examination, 2025

සංයුක්ත ගණිතය II
இணைந்த கணிதம் II
Combined Mathematics II

10 E II

Part B

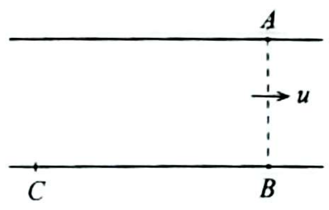
※ Answer **five** questions only.

(In this question paper, g denotes the acceleration due to gravity.)

11. (*a*) A particle P, moving with a constant deceleration of f m s^{-2} along a straight line, passes a point O with a velocity of 60 m s^{-1} at time $t = 0$ s, where $f > 0$. Just after the particle P comes to rest, it moves towards O with a constant acceleration of f m s^{-2}. At time $t = \frac{30}{f}$ s, another particle Q which was at rest at O, starts to move towards P along the same straight line with a constant acceleration of f m s^{-2} and after reaching a velocity of 30 m s^{-1} continues to move with this constant velocity. The particle Q meets the particle P, 10 seconds after the particle Q reaches the constant velocity. Sketch the velocity-time graphs for the motions of P and Q from $t = 0$ s until they meet, in the same diagram.

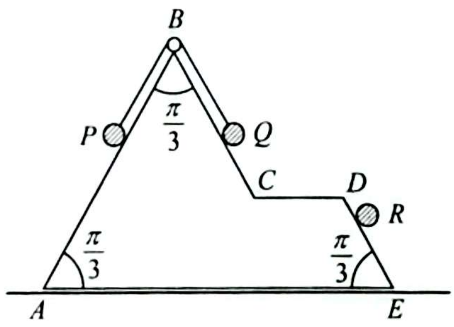
Show that $f = 3$, and that the distance from O to the point at which the particles meet is 450 m.

(*b*) A river of breadth a with parallel straight banks flows with a uniform velocity u. The points A, B and C lie on the banks such that AB is perpendicular to the banks and $BC = 2a$, as shown in the figure. Two boats P and Q begin their journeys at A and B respectively, at the same instant. The boat P travels relative to the water with a velocity of $2\sqrt{5}\,u$ in the direction of $\overrightarrow{AC}$. The boat Q travels with a speed of $\sqrt{2}\,u$ relative to the water in the direction of $\overrightarrow{BA}$ relative to earth. Sketch the velocity triangles for the motions of P and Q in the same diagram. Find the velocity of P relative to earth, and the speed of Q relative to earth.



Also, find the direction of the velocity of P relative to Q, and **hence** find the shortest distance between P and Q.

12. (*a*) The vertical cross-section $ABCDE$ through the centre of gravity of a smooth uniform block of mass $10m$ is shown in the figure. The face containing AE is placed on a smooth horizontal floor. Also, AB, BC and DE are lines of greatest slope of the faces containing them, CD is parallel to AE, and $E\hat{A}B = A\hat{B}C = D\hat{E}A = \frac{\pi}{3}$. Three particles P, Q and R of masses $5m$, m and m respectively, are held on AB, BC and DE respectively. The particles P and Q are attached to the ends of a light inextensible string passing over a smooth light small pulley fixed to the block at B. The system is released from rest with the string taut from the position shown in the figure. Obtain equations sufficient to determine the acceleration of the block and the magnitude of the reaction on P by the block.



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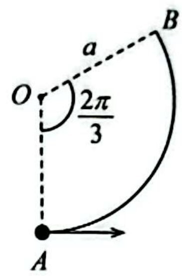
(*b*) A smooth thin rigid wire AB in the shape of a circular arc of radius a, centre O and $A\hat{O}B = \frac{2\pi}{3}$ is fixed in a vertical plane with OA vertical, as shown in the figure. A smooth small bead of mass m is kept at A and projected along the wire with a speed of $\sqrt{\frac{7ga}{2}}$. Show that the speed v of the bead when it has turned through an angle $\theta \left(0 < \theta \leq \frac{2\pi}{3}\right)$ about O is given by $v^2 = \frac{ga}{2}(3 + 4\cos\theta)$.



Also, show that after leaving the wire, the bead comes back to A, and find the speed of the bead when it reaches A.

13. Two fixed points A and B lie on a smooth horizontal table such that $AB = 16a$. One end of a light elastic string S_1 of natural length $2a$ and modulus of elasticity $3mg$ is attached to a particle P of mass m and the other end of S_1 is attached to A. Also, one end of a second light elastic string S_2 of natural length $4a$ and modulus of elasticity $4mg$, is also attached to P and the other end of S_2 is attached to B. The particle P is in equilibrium at a point C on the table, as shown in the figure.

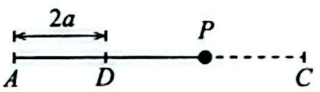
Find the lengths of AC and BC.

Now, the particle P is pulled by a distance $2a$ towards B and released from rest. Show that the equation of motion of P is given by $\ddot{x} + \omega^2(x - 6a) = 0$, where $AP = x$ and $\omega\ (> 0)$ is a constant to be determined.

By taking $X = x - 6a$, show that $\ddot{X} + \omega^2 X = 0$ and state the period of this simple harmonic motion.

Using the formula $\dot{X}^2 = \omega^2(c^2 - X^2)$, find the amplitude c and the maximum speed of P during this motion.

At the first instant when P reaches C during this motion, the string S_2 is cut.



Let D be the point on AC such that $AD = 2a$ as shown in the figure. Show that the equation of motion of P from C to D is given by $\ddot{y} + \omega_1^2 y = 0$, where $DP = y$ and $\omega_1\ (> 0)$ is a constant to be determined.

Show that the amplitude of this simple harmonic motion is $\sqrt{\frac{68}{3}}\,a$.

Find the total time elapsed from the instant when P was set in motion until it reaches the point A.

14.(*a*) In the usual notation, let $\mathbf{a} = -\mathbf{i} + \mathbf{j}$ and $\mathbf{b} = \mathbf{i} + 2\mathbf{j}$ be the position vectors of the points A and B respectively, with respect to the origin O. Let C be the point such that $\overrightarrow{OC} = 2\overrightarrow{OB}$. Also, let D be the point such that DC is parallel to AB, and AD is perpendicular to AB. Show that $\overrightarrow{OD} = -\frac{8}{5}\mathbf{i} + \frac{11}{5}\mathbf{j}$.

Let E be the point of intersection of AB and OD.

Show that $\overrightarrow{AE} = \frac{1}{10}\overrightarrow{AB}$.

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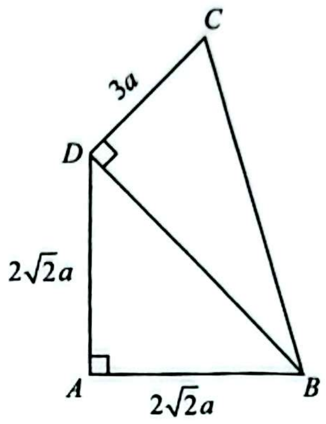
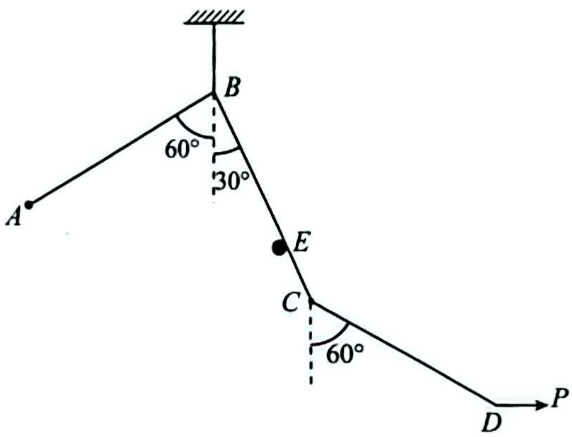
(*b*) $ABCD$ is a plane quadrilateral with $B\hat{A}D = B\hat{D}C = \frac{\pi}{2}$, $AB = AD = 2\sqrt{2}\,a$ and $CD = 3a$ as shown in the figure. Forces of magnitudes $3P$, $3P$, $2\sqrt{2}P$, $5\sqrt{2}P$ and $3\sqrt{2}P$ act along AB, AD, BD, BC and DC respectively, in the directions indicated by the order of the letters. This system of forces is equivalent to two forces of magnitudes αP and βP acting along AB and AD respectively, in the directions indicated by the order of the letters, together with a couple whose moment is M acting in the counter clockwise sense. Find the values of α, β and M.



Now, a couple acting in the plane of the quadrilateral is added to the above system of forces such that the resultant of the new system of forces goes through D. Find the moment of couple added.

15.(*a*) Three uniform rods AB, BC and CD of equal length $4a$ and weights W, $2W$ and W respectively, are smoothly jointed at the end points B and C. The end A is smoothly hinged to a fixed point. One end of a light inextensible string is attached to the joint B and other end of the string is attached to a point lying vertically above B on a horizontal ceiling. The three rods are kept in equilibrium in a vertical plane by placing the rod BC on a fixed smooth peg at E, where $BE = 3a$, and by applying a horizontal force of magnitude P at D such that the string is taut and each of the rods AB and CD is making an angle of $60°$ with the vertical and the rod BC is making an angle of $30°$ with the vertical as shown in the figure.



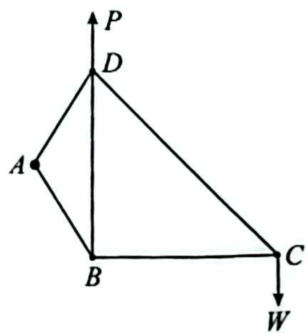
(i) Find the value of P.

(ii) Show that the magnitude of the reaction on the rod BC from the peg is $\frac{W}{3}$.

(iii) Find the tension of the string.

(*b*) The framework shown in the figure consists of five light rods AB, BC, CD, DA and DB that are smoothly jointed at their ends. It is given that $AB = AD = 2a$, $BC = BD = 2\sqrt{3}a$, and $C\hat{B}D = \frac{\pi}{2}$.

A load W is suspended at joint C and the framework is smoothly hinged to a fixed point at A and kept in equilibrium in a vertical plane with BC horizontal, by a force of magnitude P applied vertically upwards to it at the joint D. Draw a stress diagram using Bow's notation for the joints C, B and D.



Hence, find the stresses in the rods, stating whether they are tensions or thrusts.

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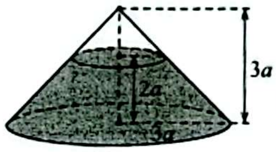
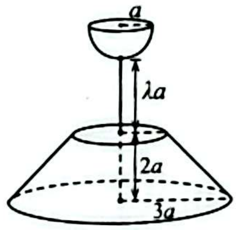
16. Show that the centre of mass of a uniform hemispherical shell of radius a is at a distance $\frac{a}{2}$ from its centre and the centre of mass of a solid uniform right-circular cone of height h is at a distance $\frac{h}{4}$ from the centre of its base.

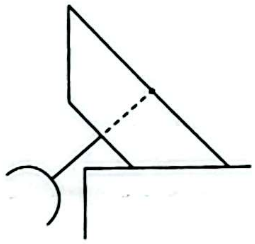
A frustum of height $2a$ is made from a solid uniform right-circular cone of base radius $3a$ and height $3a$, by cutting it through a plane parallel to its base and removing the smaller right-circular cone. (See the adjoining figure.) The mass of the right-circular cone removed to form the frustum is m. Show that the mass of the frustum is $26m$.



A uniform hemispherical shell of radius a and mass m, and the above frustum are rigidly fixed to the ends of a uniform rod of length λa and mass m to form a composite object such that the rod, the centre of the hemispherical shell and the axis of the frustum are all lying on the same straight line, as shown in the adjoining figure. Show that the centre of mass of the composite object is at a distance $\frac{3}{56}(15+\lambda)a$ from the centre of the large circular base of the frustum.



The composite object is placed on a horizontal table with the curved surface of the frustum touching the table. The adjoining figure shows the vertical cross-section through the axis of symmetry of the composite object. If the composite object is in equilibrium, show that $\lambda \leq \frac{11}{3}$.



17. (*a*) A box C contains 2 black balls and 2 white balls, and a box D contains 2 black balls and 1 white ball. These balls are identical in all aspects except for their colours. First, one ball is transferred at random from box C into box D. After that, a ball is transferred at random from box D into box C. Now, a ball is drawn at random from box D.

Find the probability that

(i) the ball drawn from box D is white,

(ii) the ball transferred first from box C into box D is black, given that the ball drawn from box D is white.

(*b*) The following table gives the data distribution of the annual savings of 100 customers of a bank.

Savings in thousands of rupees	Frequency
10 – 30	35
30 – 50	40
50 – 70	15
70 – 90	10

Find the mean, median, mode and variance of the above data distribution.

Now, an additional x number of customers are added to the data distribution and it is found that they all belong to a single class interval. The mean of the annual savings of $(100 + x)$ customers is given to be Rs. 40 000. Show that the class interval the new x customers belong to is 30 – 50.

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