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AL/2026/10/E-I

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2026
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2026
General Certificate of Education (Adv. Level) Examination, 2026

සංයුක්ත ගණිතය I
இணைந்த கணிதம் I
Combined Mathematics I

10 E I

පැය තුනයි
மூன்று மணித்தியாலம்
Three hours

අමතර කියවීම් කාලය - මිනිත්තු 10 යි
மேலதிக வாசிப்பு நேரம் - 10 நிமிடங்கள்
Additional Reading Time - 10 minutes

Use additional reading time to go through the question paper, select the questions you will answer and decide which of them you will prioritise.

Index Number							

Instructions:

- ※ *This question paper consists of two parts;*
  Part A (Questions **1 - 10**) and **Part B** (Questions **11 - 17**).
- ※ **Part A:**
  *Answer **all** questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.*
- ※ **Part B:**
  *Answer **five** questions only. Write your answers on the sheets provided.*
- ※ *At the end of the time allotted, tie the answer scripts of the two parts together so that* **Part A** *is on top of* **Part B** *and hand them over to the supervisor.*
- ※ *You are permitted to remove **only** **Part B** of the question paper from the Examination Hall.*

For Examiners' Use only

(10) Combined Mathematics I		
Part	**Question No.**	**Marks**
A	1	
	2	
	3	
	4	
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B	11	
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Total

In Numbers	
In Words	

Code Numbers

Marking Examiner		
Checked by:	1	
	2	
Supervised by:		

[*see page two*

Part A

1. Using the **Principle of Mathematical Induction**, prove that $\sum_{r=1}^{n}\left(r+\frac{1}{2}\right)=\frac{n}{2}(n+2)$ for all $n\in\mathbb{Z}^{+}$.

2. Sketch the graph of $y=|x-1|-1$ and **hence**, sketch the graph of $y=\left||x-1|-1\right|$.
Find all real values of x satisfying the inequality $\left||x-1|-1\right|\leq 1$.

[*see page three*

3. Sketch, in the same Argand diagram, the loci of the points that represent the complex numbers z satisfying

(i) $\left|\operatorname{Arg}(z-i)\right|=\dfrac{\pi}{6}$,

(ii) $\left|z-i\right|=1$ and $\operatorname{Im} z \geq 1$.

Hence, find the complex number represented by the point of intersection of these loci.

4. Let $n \in \mathbb{Z}^{+}$. Show that the constant term in the binomial expansion of $\left(\dfrac{1}{\sqrt{x}}-\sqrt{x}\right)^{2n}$ for $x>0$ is $(-1)^{n}\dfrac{(2n)!}{(n!)^{2}}$.

Hence, find the constant term in the binomial expansion of $\left(\dfrac{1}{\sqrt{2x}}-\sqrt{2x}\right)^{4}$.

[see page four

5. Show that $\lim_{x \to \frac{\pi}{3}} \dfrac{\cos x - \cos\left(\frac{\pi}{3}\right)}{\sqrt{\pi} - \sqrt{3x}} = \sqrt{\dfrac{\pi}{3}}$.

6. The region enclosed by the curves $y = \dfrac{1}{x}\sqrt{\cot\left(\dfrac{\pi}{x} - 1\right)}$, $y = 0$, $x = \dfrac{\pi}{2}$ and $x = \dfrac{2\pi}{3}$ is rotated about the x-axis through 2π radians. Show that the volume of the solid thus generated is $\ln\left[2\cos\left(\dfrac{1}{2}\right)\right]$.

[see page five

7. Let C be the curve given parametrically by $x = 2\sec\theta$ and $y = \tan\theta$ for $0 < \theta < \frac{\pi}{2}$. Show that the equation of the normal line to C at the point $(2\sec\theta,\ \tan\theta)$ is $2x\tan\theta + y\sec\theta = 5\sec\theta\tan\theta$. Also, find the value of θ such that this normal line passes through the point $(5,\ 0)$.

8. Let $A \equiv (1, 1)$ and $B \equiv (3, 5)$. Find the equations of the lines through B making an angle $\tan^{-1}\left(\frac{1}{3}\right)$ with AB.

[see page six

9. Find the equations of the circles with radii 1 that touch the circle $x^2 + y^2 = 4$ internally and whose centres lie on the line $y = \sqrt{3}\,x$.

10. It is given that $\cos\alpha = \frac{3}{5}$, where $-\frac{\pi}{2} < \alpha < 0$. Show that $\sin\left(\alpha - \frac{\pi}{4}\right) = -\frac{7}{5\sqrt{2}}$.

By taking $\cos\left(\alpha - \frac{\pi}{4}\right) = -\frac{1}{5\sqrt{2}}$, show that $\sin\left(\alpha - \frac{\pi}{12}\right) = -\frac{(7\sqrt{3}+1)}{10\sqrt{2}}$.

[see page seven

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2026
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2026
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සංයුක්ත ගණිතය I
இணைந்த கணிதம் I
Combined Mathematics I

10 E I

Part B

※ Answer **five** questions only.

11. (*a*) Let $k, \lambda \in \mathbb{R}$ and $k \neq 0$. Show that the equation $kx^2 - 2\lambda x - \frac{1}{k} = 0$ has two non-zero real roots that are distinct.

Let α and β be these roots. Show that the quadratic equation with roots $\frac{1}{\alpha} + k^2\beta^2$ and $\frac{1}{\beta} + k^2\alpha^2$ is given by $f(x) = 0$, where $f(x) = x^2 - 2(2\lambda^2 - k\lambda + 1)x + (2k\lambda - k^2 + 1)$.

It is given that the axis of symmetry of the graph of $y = f(x)$ is $x = -1$. Show that $2\lambda^2 - k\lambda + 2 = 0$.

Find the positive value of k such that the equation $2\lambda^2 - k\lambda + 2 = 0$ is satisfied by only one value of λ and also find this value of λ.

(*b*) Let $f(x) = x^4 + ax^3 + px^2 + q$ and $g(x) = x^4 + bx^3 - qx^2 + 2x - p$, where $a, b, p, q \in \mathbb{R}$. It is given that $(x+1)$ is a factor of $f(x)$ and that the remainder when $g(x)$ is divided by $(x-1)$ is 4.

Show that $a = b$.

It is also given that the equation $f(x) = f(-x)$ has a non-zero solution. Show that $a = 0$ and $p + q = -1$.

Hence, show that $(x+1)$ is a factor of $g(x)$.

12. (*a*) Numbers are to be formed using some or all of the given five digits 3, 5, 6, 7 and 8 without repetition.

Find the number of different ways in which,

(i) a number greater than 10 000 can be formed,

(ii) a number greater than 6000 can be formed.

[*see page eight*

(*b*) (i) Find the real constants A, B and C such that $\dfrac{r+5}{(r+1)(r+2)(r+3)} = \dfrac{A}{r+1} + \dfrac{B}{r+2} + \dfrac{C}{r+3}$, for $r \in \mathbb{Z}^+$.

(ii) Let $U_r = \dfrac{r+5}{(r+1)(r+2)(r+3)}$ for $r \in \mathbb{Z}^+$ and $f(r) = \dfrac{1}{r+2}$ for $r \in \mathbb{Z}^+ \cup \{0\}$.

Using part (i) above, show that

$U_r = 2f(r-1) - 3f(r) + f(r+1)$ for $r \in \mathbb{Z}^+$.

Hence, show that $\displaystyle\sum_{r=1}^{n} U_r = \frac{2}{3} - \frac{2}{n+2} + \frac{1}{n+3}$.

Show that the infinite series $\displaystyle\sum_{r=1}^{\infty} U_r$ is convergent and find its sum.

Find the value of $\displaystyle\lim_{n\to\infty}\left(\sum_{r=2}^{n+2} U_r + \sum_{r=n+1}^{2n} U_r\right)$.

13. (*a*) Let $\mathbf{A} = \begin{pmatrix} 2 & a & 1 \\ 3 & -1 & 1 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} -2 & 0 \\ b & 2 \\ 2 & 3 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} -3 & -3 \\ 5 & c \end{pmatrix}$, where $a, b, c \in \mathbb{R}$.

Find the values of a, b and c, such that $\mathbf{AB} = \mathbf{C}^{\mathrm{T}}$.

With these values of a, b and c, find the matrix $\mathbf{D}$ is such that $(\mathbf{AB})^{\mathrm{T}}\mathbf{D} = 2\mathbf{I} + \mathbf{C}$, where $\mathbf{I}$ is the identity matrix of order 2.

(*b*) Let $z_1, z_2 \in \mathbb{C}$. Show that

(i) $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$, (ii) $\overline{z_1 z_2} = \overline{z_1}\,\overline{z_2}$,

(iii) $\overline{\left(\dfrac{z_1}{z_2}\right)} = \dfrac{\overline{z_1}}{\overline{z_2}}$ for $z_2 \neq 0$, and (iv) $z_1\overline{z_1} = |z_1|^2$.

Show that if $|z_1| = |z_2| = 1$ and $z_1 \neq -z_2$, then $\left(\dfrac{1 + z_1 z_2}{z_1 + z_2}\right)$ is real.

(*c*) Show that $\dfrac{\cos\alpha + i\sin\alpha}{\cos\beta + i\sin\beta} = \cos(\alpha - \beta) + i\sin(\alpha - \beta)$, where $\alpha, \beta \in \mathbb{R}$.

Express $1 + \sqrt{3}i$ and $\sqrt{2}(1+i)$ in the form $r(\cos\theta + i\sin\theta)$, where $r > 0$ and $-\pi < \theta \leq \pi$.

Let $\omega = \dfrac{1+\sqrt{3}i}{\sqrt{2}(1+i)}$. Show that $|\omega| = 1$ and Arg $\omega = \dfrac{\pi}{12}$.

Deduce that $\omega^{18} = -i$.

[*see page nine*

14. (*a*) Let $f(x)=\alpha\left(\dfrac{x-1}{x+1}\right)^2+4$ for $x\in\mathbb{R}-\{-1\}$, where $\alpha\in\mathbb{R}$. Show that $f'(x)$, the derivative of $f(x)$, is given by $f'(x)=\dfrac{4\alpha(x-1)}{(x+1)^3}$ for $x\in\mathbb{R}-\{-1\}$.

It is given that the gradient of the tangent line to the graph of $y=f(x)$ when $x=0$ is 8. Show that $\alpha=-2$.

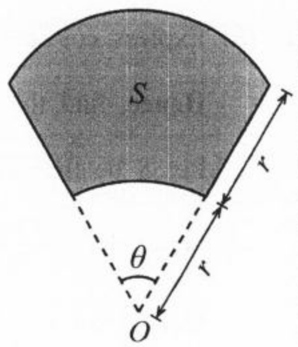
Find the interval on which $f(x)$ is increasing and the intervals on which $f(x)$ is decreasing.

Also, find the coordinates of the turning point of the graph of $y=f(x)$.

Sketch the graph of $y=f(x)$ indicating the turning point and the asymptotes.

Hence, state the range of f and the smallest value of k such that f is one-to-one on the interval $[k,\infty)$.

(*b*) As shown in the figure, the boundary of the shaded region S consists of two concentric circular arcs of radii r m and $2r$ m subtending an angle θ radians at the centre O and two straight line segments when extended pass through the centre O. The area of the region S is given to be 81 m^2. Show that the perimeter p m of S is given by $p=2\left(r+\dfrac{81}{r}\right)$ for $r>0$ and find the values of r and θ such that p is minimum.



15. (*a*) Express $\dfrac{6t^2+7t+3}{(2t-1)(4t^2+4t+5)}$ in partial fractions and **hence**, find $\displaystyle\int\frac{6t^2+7t+3}{(2t-1)(4t^2+4t+5)}\,\mathrm{d}t$.

(*b*) (i) Show that $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\tan\dfrac{x}{2}\right)=\dfrac{1}{1+\cos x}$.

(ii) Show that $\displaystyle\int_0^{\frac{\pi}{2}}\frac{x^2\sin x}{(1+\cos x)^2}\,\mathrm{d}x=\frac{\pi^2}{4}-2\int_0^{\frac{\pi}{2}}\frac{x}{1+\cos x}\,\mathrm{d}x$.

Hence, using (i), show that $\displaystyle\int_0^{\frac{\pi}{2}}\frac{x^2\sin x}{(1+\cos x)^2}\,\mathrm{d}x=\frac{\pi}{4}(\pi-4)+2\ln 2$.

(*c*) Show that $\cot\left(\dfrac{3\pi}{4}-x\right)=\dfrac{1-\cot x}{1+\cot x}$.

Using the formula $\displaystyle\int_a^b f(x)\,\mathrm{d}x=\int_a^b f(a+b-x)\,\mathrm{d}x$, where a and b are constants,

show that $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\ln(1+\cot x)\,\mathrm{d}x=\frac{\pi}{8}\ln 2$.

16. (*a*) Let $A \equiv (3, 0)$, $B \equiv (0, 2)$ and l be a line perpendicular to the line AB. Show that the equation of the line l has the form $3x - 2y = \lambda$, where $\lambda \in \mathbb{R}$. This line l intersects the x-axis at the point C and the y-axis at the point D. Show that the point of intersection of the lines AD and BC lies on the circle $x^2 + y^2 - 3x - 2y = 0$.

(*b*) Let $S_1 \equiv x^2 + y^2 - 6x - 4y + 4 = 0$ and $S_2 \equiv x^2 + y^2 + 4y - 5 = 0$. Show that the circles $S_1 = 0$ and $S_2 = 0$, have equal radii and they intersect each other.

Also, find the equations of the common tangents to these circles.

Further, let $U \equiv 2x - 4y + 3$. Find the equation of the circle that passes through the points of intersection of the circles $S_1 + U = 0$ and $S_2 = 0$, and intersects the circle $S_2 = 0$ orthogonally.

17. (*a*) Express $\cos 2x + \sin 2x$ in the form $R\sin(2x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Hence, find the general solution of the equation $1 + \cos 2x + \sin 2x = 0$.

Let S be the set consisting of these solutions.

Show that $\dfrac{1 - \cos 2x + \sin 2x}{1 + \cos 2x + \sin 2x} = \tan x$ for $x \notin S$.

Hence, solve $\dfrac{1 - \cos x + \sin x}{1 + \cos x + \sin x} = \dfrac{1}{2}\sin\dfrac{x}{2}$.

(*b*) In the usual notation, state the **Cosine Rule** for a triangle ABC.

In a triangle ABC, the point D lies on BC such that $BD : DC = 2 : 1$.

Applying the Cosine Rule for suitable triangles,

show in the usual notation that $AD^2 = \frac{1}{9}\left(6b^2 + 3c^2 - 2a^2\right)$.

Now, if $AB = 3$ cm, $AC = 2$ cm and $AD = \dfrac{\sqrt{37}}{3}$ cm, show that $B\hat{A}C = \dfrac{\pi}{3}$.

(*c*) Solve the equation $\sin\left(\tan^{-1}\left(\dfrac{1}{\sqrt{x^2+1}}\right)\right) + \tan\left(\sin^{-1}\left(\dfrac{1}{\sqrt{x^2+1}}\right)\right) = \dfrac{3}{2x}$.

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AL/2026/10/E-II

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கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2026
General Certificate of Education (Adv. Level) Examination, 2026

සංයුක්ත ගණිතය II
இணைந்த கணிதம் II
Combined Mathematics II

10 E II

පැය තුනයි
மூன்று மணித்தியாலம்
Three hours

අමතර කියවීම් කාලය - මිනිත්තු 10 යි
மேலதிக வாசிப்பு நேரம் - 10 நிமிடங்கள்
Additional Reading Time - 10 minutes

Use additional reading time to go through the question paper, select the questions you will answer and decide which of them you will prioritise.

Index Number							

Instructions:

- ※ *This question paper consists of two parts;*
  Part A (Questions **1 – 10)** *and* **Part B** (Questions **11 – 17)**
- ※ **Part A:**
  *Answer **all** questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.*
- ※ **Part B:**
  *Answer **five** questions only. Write your answers on the sheets provided.*
- ※ *At the end of the time allotted, tie the answer scripts of the two parts together so that* **Part A** *is on top of* **Part B** *and hand them over to the supervisor.*
- ※ *You are permitted to remove **only** **Part B** of the question paper from the Examination Hall.*
- ※ In this question paper, g denotes the acceleration due to gravity.

For Examiners' Use only

(10) Combined Mathematics II		
Part	**Question No.**	**Marks**
A	**1**	
	2	
	3	
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B	**11**	
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Marking Examiner		
Checked by:	1	
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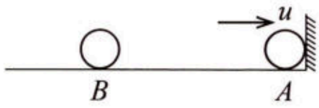
Supervised by:		

[see page two

Part A

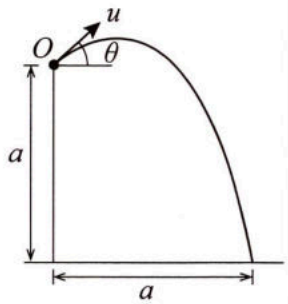
1. A particle A of mass m, moving with speed u, in a straight line on a smooth horizontal floor hits a smooth vertical wall which is perpendicular to the direction of motion of A. (See the figure.) Show that the impulse exerted on A by the wall is $m(1+e)u$, where e is the coefficient of restitution between A and the wall.

After rebounding from the wall, A collides directly with a particle B of mass $\frac{m}{e}$ which is at rest on the floor. The coefficient of restitution between A and B is also e. Show that after this collision, A comes to rest.



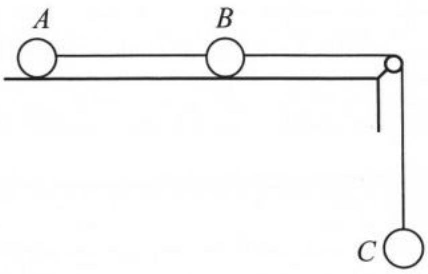
2. A particle is projected with an initial velocity u at an angle $\theta\left(0 \le \theta < \frac{\pi}{2}\right)$ to the horizontal, from a point O at a vertical distance a above a horizontal ground as shown in the figure. The particle strikes the ground at a horizontal distance a from O. Show that $a = \frac{2u^2}{g} \cdot \frac{(1+\tan\theta)}{(1+\tan^2\theta)}$.

Deduce that if $u = \sqrt{\frac{ga}{2}}$, then $\theta = 0$ or $\theta = \frac{\pi}{4}$.



[see page three

3. Three particles A, B and C are of masses m, $2m$ and $4m$, respectively. The particles A and B are attached to the ends of a light inextensible string, and are placed on a smooth horizontal table. The particles B and C are attached to the ends of another light inextensible string passing over a smooth small pulley fixed at the edge of the table. The particles A, B and C lie on the same vertical plane. The strings are taut and the particle C hangs vertically below the pulley, as shown in the figure. The system is released from rest. Find the acceleration of A.



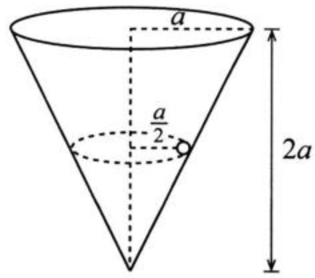
4. A car of mass 1000 kg is travelling along a straight horizontal road with its engine working at a constant power of 16 kW. The resistance to the motion of the car is 600 N. Find the acceleration of the car at an instant when its speed is 25 m s^{-1}.

Now, suppose that the car travels up a straight road that is inclined at an angle $\sin^{-1}(0.1)$ to the horizontal against a resistance of $(600 + 5v)$ N, where v m s^{-1} is the speed of the car. Obtain equations sufficient to determine the speed of the car at an instant when the acceleration of the car is 0.5 m s^{-2}, if the engine is working at the same power.

[see page four

5. A hollow right circular cone of base radius a and height $2a$ is fixed with its axis vertical and vertex downwards, as shown in the figure. A particle of mass m moves with constant speed v in a horizontal circle of radius $\frac{a}{2}$ on the smooth inner surface of the cone.

Show that $v = \sqrt{ga}$.



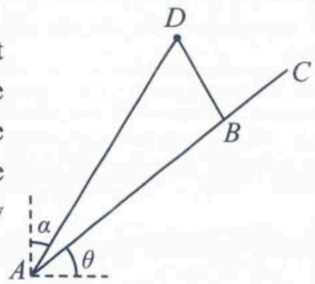
6. Let $\mathbf{u}$ and $\mathbf{v}$ be two perpendicular unit vectors, and $\mathbf{a} = t\mathbf{u} + (1-t)\mathbf{v}$, where $t \in \mathbb{R}$.

Show that $|\mathbf{a}|^2 = t^2 + (1-t)^2$.

Deduce that if $\mathbf{a}$ makes equal angles with each of $\mathbf{u}$ and $\mathbf{v}$, then $|\mathbf{a}| = \frac{1}{\sqrt{2}}$.

[see page five

7. A uniform heavy rod ABC of length $4a$ is kept in equilibrium by a light inextensible string passing over a smooth fixed peg D, attached to the end A of the rod and to the point B on the rod, where $AB = 3a$. The rod is inclined at an angle θ to the horizontal and the portion AD of the string makes an angle α with the vertical, as shown in the figure. Show that $3\tan\theta\tan\alpha = 1$.



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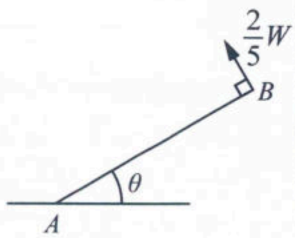
8. A uniform rod AB of weight W is kept in equilibrium by placing the end A on a rough horizontal plane and applying a force of magnitude $\frac{2}{5}W$ in a direction perpendicular to the rod at B, as shown in the figure. The rod makes an angle θ with the horizontal, and the coefficient of friction between the rod and the rough plane is μ.
Show that $\cos\theta = \frac{4}{5}$ and $\mu \geq \frac{6}{17}$.



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[see page six

9. Let A and B be two events of a sample space Ω. Show that $P(A \mid B) + P(A' \mid B) = 1$, where A' denotes the complement of the event A.

Now, it is given that $P(A \cap B) = \frac{1}{4}$ and $P(B) = \frac{1}{2}$. Find $P(A' \mid B)$.

10. For the set of data $\{x_1, x_2, \ldots, x_{20}\}$, it is given that $\sum_{i=1}^{20} (x_i - 3) = 40$ and $\sum_{i=1}^{20} (2x_i - 3)^2 = 1060$.

Show that the mean of this data set is 5 and its variance is 1.

[see page seven

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இலங்கைப் பரீட்சைத் திணைக்களம்
Department of Examinations, Sri Lanka

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2026
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2026
General Certificate of Education (Adv. Level) Examination, 2026

සංයුක්ත ගණිතය II
இணைந்த கணிதம் II
Combined Mathematics II

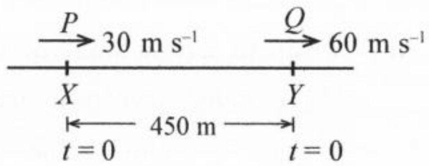
10 E II

Part B

※ Answer **five** questions only.

(In this question paper, g denotes the acceleration due to gravity.)

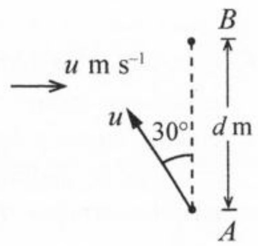
11. (*a*) X and Y are two points on a straight road that are 450 m apart, as shown in the figure. At time $t=0$, a car P passes the point X with a speed of 30 m s^{-1} in the direction of $\overrightarrow{XY}$, and moves along the road with a uniform acceleration of f m s^{-2} ($f>0$). At the same instant $t=0$, another car Q



passes the point Y with a speed of 60 m s^{-1} in the direction of $\overrightarrow{XY}$ and moves along the road with a uniform acceleration of f m s^{-2} till it reaches a speed of 90 m s^{-1} and then continues in the same direction with that constant speed. It is given that the cars P and Q meet at the instant when $t=25$ s. Sketch the velocity-time graphs for the motions of P and Q from $t=0$ to $t=25$ s, in the same diagram.

Hence, show that $f=6$ and that the distance travelled by P from X until meeting Q is 2625 m.

(*b*) A river flows due east with a uniform velocity u m s^{-1}. Points A and B are two fixed points on the surface of the river such that A is due south of B and $AB=d$ m, as shown in the figure. At a certain instant, a boat P is observed at the point A, sailing in a straight line path with a uniform velocity u m s^{-1} relative to earth in the direction making an angle 30° west of north. Find the velocity of P relative to water.



At the same instant, another boat Q is observed at the point B sailing in a straight line path with a uniform speed v m s^{-1} relative to water with the intention of intercepting P, where $\frac{3u}{2} < v < \sqrt{3}\,u$. Show that there are two possible directions in which Q can sail to intercept P.

Also, show that the difference between the times taken by Q to intercept P in these two directions is $\dfrac{d\sqrt{4v^2-9u^2}}{(3u^2-v^2)}$.

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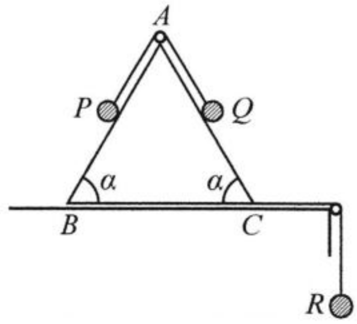
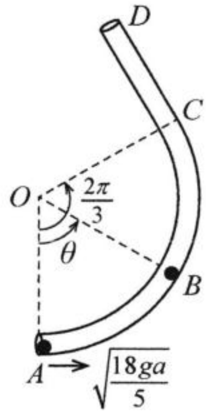
12. (*a*) The vertical cross-section ABC through the centre of gravity of a smooth uniform wedge of mass M is shown in the figure. The face containing BC is placed on a smooth horizontal table, and AB and AC are lines of greatest slope of the faces containing them. Also, $A\hat{B}C = A\hat{C}B = \alpha$. Two particles P and Q of masses $2m$ and m, respectively are attached to the two ends of a light inextensible string passing over a smooth small pulley fixed at A. The particle P is held on AB and the particle Q is held on AC with the string taut. The point C of the wedge and a particle R of mass $4m$ are attached to the ends of a light inextensible string passing over a smooth small pulley fixed to an edge of the table. The strings lie in the same vertical plane containing ABC, and the particle R hangs freely. The system is released from rest from the position shown in the figure.



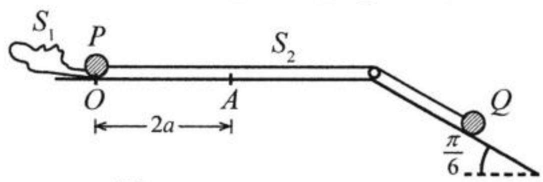
Obtain equations sufficient to determine the acceleration of the wedge.

(*b*) A smooth narrow tube $ABCD$ lying in a plane, consists of a circular portion ABC of radius a, centre O and $A\hat{O}C = \frac{2\pi}{3}$, and a straight portion CD making an angle $\frac{\pi}{2}$ with OC. The tube is fixed in a vertical plane with OA vertical, as shown in the figure. A particle of mass m is placed inside the tube at A and given a horizontal velocity of magnitude $\sqrt{\frac{18ga}{5}}$ for it to move inside the tube. Show that the speed v of the particle when it has turned through an angle $\theta \left(0 \le \theta \le \frac{2\pi}{3}\right)$ about O is given by $v^2 = \frac{2ga}{5}(4+5\cos\theta)$ and find the reaction on the particle by the tube, when it is in this position.



Assuming that the particle comes to rest before reaching D, find the distance the particle travelled along CD.

13. One end of a light elastic string S_1 of natural length $2a$ and modulus of elasticity Mg is attached to a fixed point O on a smooth horizontal table and the other end to a particle P of mass m. One end of another light inextensible string S_2 is also attached to the particle P and the other end is attached to another particle Q of mass M. The particle P is placed at O with the string S_1 slack, and the string S_2 taut and passing over a smooth small pulley fixed at the edge of the table with the particle Q lying on a smooth plane inclined at an angle $\frac{\pi}{6}$ to the horizontal, as shown in the figure. The string S_2 lies in a vertical plane with the part of it on the inclined plane lying along a line of greatest slope of the inclined plane. Let A be the point on the table such that $OA = 2a$, as marked in the figure. The system is released from rest.



Find the speed of P when it reaches A.

Show that the equation of motion of the subsequent motion of P while the string S_1 is taut is given by $\ddot{x} + \frac{Mg}{2(M+m)a}(x-a) = 0$, where $AP = x$, assuming that P does not reach the pulley.

By taking $X = x - a$, rewrite the above equation of motion in the form $\ddot{X} + \omega^2 X = 0$, where $\omega\ (>0)$ is a constant to be determined.

Using the formula $\dot{X}^2 = \omega^2(C^2 - X^2)$ find the amplitude C of this motion.

After being released from O from rest, show that the time taken by P to come to rest again for the first time is $\sqrt{\frac{2(M+m)a}{Mg}}\left[2+\pi-\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)\right]$.

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14.(*a*) In the usual notation, the position vectors of the points A and B are $\mathbf{i} + 2\mathbf{j}$ and $\alpha\mathbf{i} - \mathbf{j}$, respectively. The lines OA and OB are perpendicular, where O is the origin. Show that $\alpha = 2$.

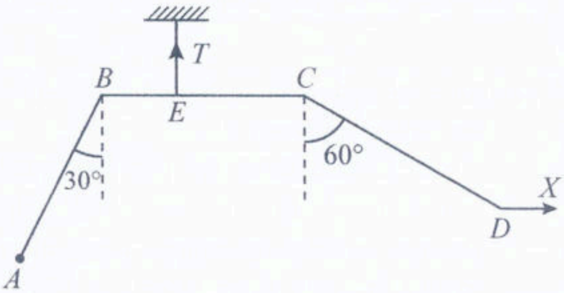
Let C be the point with the position vector $\frac{4}{3}\mathbf{i} + \mathbf{j}$. Show that the points A, B and C are collinear and find $AC : CB$.

Now, let D be the point on OC such that OD is perpendicular to AD. Show that the position vector of D is $\frac{2}{5}(4\mathbf{i} + 3\mathbf{j})$.

(*b*) A system of forces in the Oxy-plane consists of four forces $2P\mathbf{i} + 4P\mathbf{j}$, $3P\mathbf{i} - P\mathbf{j}$, $-5P\mathbf{i} + 2P\mathbf{j}$ and $2P\mathbf{i} - 3P\mathbf{j}$ acting at the points $A \equiv (2a, 0)$, $B \equiv (0, 2a)$, $C \equiv (-2a, 0)$ and $D \equiv (0, -2a)$, respectively, where P and a are positive constants. Show that this system of forces is equivalent to a single resultant force and find its magnitude, direction and the equation of the line of action.

Now, let E, F, G and H be the mid-points of AB, BC, CD and DA, respectively. Three forces of magnitudes $\sqrt{2}\alpha P$, βP and γP acting along $\overrightarrow{EG}$, $\overrightarrow{EF}$ and $\overrightarrow{GH}$, respectively, are added to the above system of forces such that the new system consisting of seven forces is equivalent to a couple of moment $8Pa$ acting in the counterclockwise sense. Find the values of α, β and γ.

15.(*a*) Three uniform rods AB, BC and CD of equal length $2a$ and of equal weight W are smoothly jointed at the end points B and C, and the end A is smoothly hinged to a fixed point. One end of a light inextensible string is attached to a point E on BC and the other end of the string is attached to a point lying vertically above E on a ceiling. The three rods are kept in equilibrium in a vertical plane by applying a horizontal force of magnitude X at D such that string is taut, rod BC is horizontal, and the rods AB and CD making angles 30° and 60°, respectively, with the vertical, as shown in the figure. Show that

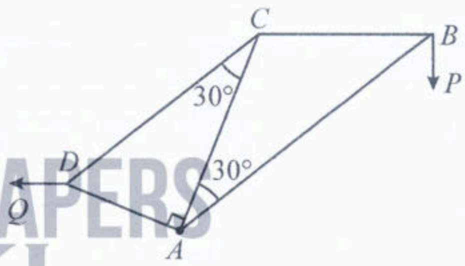


(i) $X = \frac{\sqrt{3}}{2}W$,

(ii) $T = 4W$, and

(iii) $BE = \frac{3}{4}a$.

(*b*) The framework shown in the figure consists of five light rods AB, BC, AC, AD and CD that are smoothly jointed at their ends. It is given that $AC = BC = 2a$, $B\hat{A}C = A\hat{C}D = 30^\circ$ and $C\hat{A}D = 90^\circ$.
A load P N is suspended at the joint B. The framework is smoothly hinged to a fixed point at A and kept in equilibrium in a vertical plane with BC horizontal by a horizontal force of magnitude Q N applied to it at the joint D, as shown in the figure.



If the stress of the rod BC is a tension of magnitude $10\sqrt{3}$ N, draw a stress diagram using Bow's notation for the joints B, C and D. **Hence** show that $P = 10$ and $Q = 30\sqrt{3}$, and find the stresses in the rods AB, AC, AD and DC, stating whether they are tensions or thrusts.

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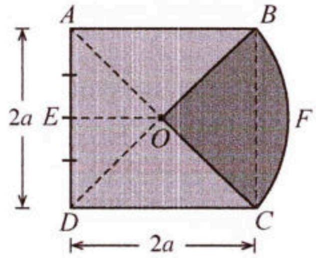
16. Show that the centre of mass of

(i) a uniform isosceles triangular lamina PQR with $PQ = PR$ and the length of the median through P is h, is at a distance $\dfrac{2h}{3}$ from P, and

(ii) a uniform lamina in the shape of a circular sector of radius a that subtends an angle 2α radians at its centre, is at a distance $\dfrac{2a}{3}\cdot\dfrac{\sin\alpha}{\alpha}$ from its centre.



The lamina L shown in the figure is formed first by removing the triangular part BOC from the uniform square lamina $ABCD$ having side-length $2a$, centre O and surface density ρ, and then by rigidly attaching the uniform lamina $OBFC$ in the shape of a circular sector having centre O and surface density $\lambda\rho$.

Show that the centre of mass of the lamina L is at a distance $\dfrac{14+\lambda(8+3\pi)}{3(6+\lambda\pi)}a$ from E, where E is the mid-point of AD.

It is given that the centre of mass of the lamina L is at O. Show that its mass M is given by $M = \dfrac{(12+\pi)}{4}a^2\rho$.

Now, the lamina L of mass M having its centre of mass at O, with a particle of mass $\dfrac{M}{2}$ attached to it at the point E, is hung freely by a string from the point A. Find the angle that AD makes with the downward vertical in the equilibrium position.

17. (*a*) An unbiased coin is tossed three times. At each toss, balls are placed in a box according to the following manner:

- If a head turns up, two black balls are placed in it.
- If a tail turns up, one white ball is placed in it.

The box is initially empty. The balls are identical in all aspects except for their colour. After tossing the coin three times, a ball is randomly drawn from the box.

(i) Find the probability that the ball drawn is black.

(ii) Given that the ball drawn is black, find the probability that after tossing the coin three times, the box contains 4 black balls and 1 white ball.

(*b*) The daily average temperature x of a city in °C for 100 days is given in the following frequency distribution:

Daily average temperature in °C x	**Number of days**
$-2 - 0$	8
$0 - 2$	13
$2 - 4$	25
$4 - 6$	34
$6 - 8$	15
$8 - 10$	5

Find the median, mode, mean and the variance of this frequency distribution.

These daily average temperatures are transformed into a new frequency distribution using the transformation $y = 12 + x$.

Find the mean and the variance of the transformed frequency distribution.

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