Provincial Department of Education - NWP

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Third Term Test - Grade 12 - 2023

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Index No. : ....

Combined Mathematics - I

Time : 03 hours

Additional time for reading 10 minutes

Use additional reading time to go through the question paper, select the questions you will answer decide which of them you will prioritise.

Index number | | | | | | | |

Instructions:

* *This question paper consists of two parts*
  Part A (Questions 1 - 10) and **Part B** (Questions 11 - 17)
* **Part A:**
  *Answer **all** questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.*
* **Part B:**
  *Answer **five** questions only. Write your answers on the sheets provided*
* *At the end of the time allotted, tie the answer scripts of the two parts together so that* **Part A** *is on the top of* **Part B** *and hand them over to the supervisor.*
* *You are permitted to remove **only** **Part B** of the question paper from the Examination Hall*

For Examiners' Use only

(10) Combined Mathematics I		
Part	Question No.	Marks
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	

Total

In numbers	
In words	

Part A

(01) Sketch the rough graphs of $y = |x - 1|$ and $y = 1 - x^2$ in the same diagram. Hence or otherwise, find all real values of x satisfying the inequality $2|x - 2| < 4 - x^2$.

(02) Solve the equation $\text{Log}_{1/2}(2x - 1) + \log_{2x-1} 4 = 1 \ (x > ½)$.

(03) $$\lim_{x \to 2} \frac{\sqrt{2x-1} - \sqrt{x+1}}{\sin \frac{\pi x}{2}} = -\frac{1}{\sqrt{3}\,\pi}$$

(04) The remainders, when the polynomial $f(x) = x^3 + px^2 + qx - 6$ is divided by $(x-1)$ and $(x+2)$ are equal. Show that $p - q = 3$. Also if it is given that $(x-2)$ is a factor of $f(x)$, obtain the value of P.

(05) Determine the values of A, B and C such that $4x^2 - x - 1 = A(x^2 - 3x + 2) + B(x - 2) + C(x - 1)$

Hence or otherwise, find the partial fractions of $\dfrac{4x^2 - x - 1}{(x - 2)(x - 1)}$.

(06) The point P divides the line BC in the ratio 1:2 internally. If $B \equiv (-1, 1)$ and $C \equiv (1, 4)$, write the equation of the straight line passing through P and the point $(2,1)$.

(07) A curve C is parametrically given by $x = (t+1)^3$ and $y = t^3 + 1$. Write the equation of the tangent and the normal line drawn to the curve C at the point relevant to $t = 2$.

(let the gradient of the tangent x gradient of the normal = -1)

(08) Write the set of values of λ such that the roots of the quadratic equation

$(1+2\lambda)x^2 - 10x + \lambda - 2 = 0, \quad \lambda \in \mathbb{R} - \{-\frac{1}{2}\}$ are real.

(09) If $\frac{b+c}{5} = \frac{c+a}{6} = \frac{a+b}{7}$ according to the standard notation, using the sine rule for a triangle, show that $\frac{\text{Sin A}}{4} = \frac{\text{Sin B}}{3} = \frac{\text{Sin C}}{2}$

(10) Find the general solutions of the equation

$(\sqrt{3} - 1)\ \text{Sin}^2 x - (\sqrt{3} + 1)\ \text{Sin}\ x\ \ \text{Cos}\ x + 1 = 0$

(11) (a) The roots of the quadratic equation $px^2 + qx + r = 0$ $(p, q, r, \in \mathbb{R})$ are α and β. Write the values of $\alpha + \beta$ and $\alpha\beta$ in terms of p, q and r.

If $\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$, show that $2p^2r = r^2q + q^2p$

Also show that the quadratic equation with roots

$\sqrt{\frac{\alpha}{\beta}}$ and $\sqrt{\frac{\beta}{\alpha}}$ is $\sqrt{pr}\, x^2 + qx + p = 0$

(b) Let $\frac{x^2 - bx}{ax + c} = \frac{k - 1}{k + 1}$ where $a > b$ and $a, b, c \in \mathbb{R}$.

Express the above quadratic equation in the from $AX^2 + BX + C = 0$. If the roots of this equation are equal in magnitude and opposite in direction show that $k = \frac{a - b}{a + b}$.

Also if the roots of that quadratic equation are real, find the range of values of K.

(c) The remainder, when the polynomial function $f(x) = x^3 - 3x^2 + kx + 8$ where $k \in \mathbb{R}$ is divided by $x - 2$, is 2. Find the value of k. For that value of k, if $g(x) = x^2 - x - 4$, find a function $p(x)$, such that $p(x) = f(x) + g(x)$. Show that $(x - 2)$ is a factor of $p(x)$.

Find λ, μ such that $p(x) = (x - 2)\,[(x - 2)^2 + \lambda\,(x-2) + \mu\,]$. Hence **deduce that** the remainder, when $p(x)$ is divided by $(x - 2)^2$

(12) (a) Sketch the rough graph of the function $f(x) = \frac{x^2 - 4}{|x - 2|}$

Hence show that $\lim\limits_{x \to 2} \frac{x^2 - 4}{|x - 2|}$ does not exist.

(b) Show that $\lim\limits_{x \to \frac{\pi}{6}} \frac{(8\,\mathrm{Sin}^3 x - 1)(2\,\mathrm{Cos}\, x - \sqrt{3})}{(x - \frac{\pi}{6})^2} = -3\sqrt{3}$

(c) Sketch the rough graph of $y = x\,(x - 4)$. Hence deduce the graph of $y = |\,x\,(x - 4)\,|$ in a seperate coordinate plane. Also sketch the rough graph of $y = 4 - 2\,|\,x - 2|$ in the same coordinate plane and **deduce that** $|\,x\,(x - 4)\,| > 4 - 2\,|\,x - 2|$ for all $x \in \mathbb{R}$ except for three values of x. Write these three special values also.

(13) (a) Prove that $\log_a b = \dfrac{\log_c b}{\log_c a}$ where $a, b, c \in \mathbb{R}^+$

Hence **deduce that** $\log_{a^k} b = \dfrac{1}{k} \log_a b$

Hence or otherwise, Show that

$$\log_2 x + \log_{2^2} x^2 + \log_{2^3} x^3 + \ldots\ldots\ldots\ldots\ldots\ldots + \log_{2^n} x^n = \log_2 x^n$$

(b) Find the domain and the range of the function $f(x) = \sqrt{x^2 - 4x - 5}$.

If there exist a function $g(x) = x - 3$, $g : \mathbb{R} - (2, 8) \longrightarrow \mathbb{R} - (-1, 5)$, write $f{\circ}g$. Hence **deduce that** domain of the function $f{\circ}g$.

(c) Find the partial fractions of the function $\dfrac{x^2}{(x+1)(x+2)}$

Find λ, μ and γ such that $x^2 = \lambda(x+1)(x+2) + \mu(x+1) + \gamma$.

Hence deduce the partial fractions of $\dfrac{1}{(x+1)(x+2)}$.

(14) (a) Using the first principles, show that the first derivative of $\sin x$ is $\cos x$.

If $y = \dfrac{\sin x - \cos x}{\sin x + \cos x}$, Show that $\dfrac{dy}{dx} = 1 + y^2$

find $\dfrac{d^2y}{dx^2}$ and show that $\dfrac{d^2y}{dx^2} = 0$ if and only of $y = 0$.

show that $\dfrac{d^2y}{dx^2} = 8(2 - \sqrt{3})^2$, when $x = \dfrac{\pi}{3}$

(b) A curve is parametrically given by $x = a\cos^2\theta$ and $y = b\sin\theta$ where $(\theta \neq n\pi,\ n \in \mathbb{Z})$.

Differentiating the parametric functions show that $\dfrac{dx}{d\theta} = \dfrac{-2ay}{b^2} \dfrac{dy}{d\theta}$

Hence show that $\dfrac{dy}{dx} = \dfrac{-b^2}{2ay}$.

Also obtain that $\dfrac{d^2y}{dx^2} = \dfrac{b^2}{2ay^2} \dfrac{dy}{dx} = \dfrac{-b^4}{4a^2y^3}$

Show that $\dfrac{d^2y}{dx^2} = \dfrac{-b}{4a^2}$ when $\theta = \dfrac{\pi}{2}$

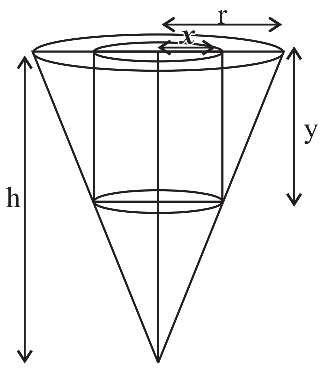
(15) (a) Let $f(x) = \dfrac{x^2}{(x-1)^2}$ for $x \neq 1$. Using the standard notation show that

$f'(x) = \dfrac{-2x}{(x-1)^3}$ for $x \neq 1$

Also it is given that $f''(x) = \dfrac{2(2x+1)}{(x-1)^4}$, $x \neq 1$

find the turning points of the graph of $y = f(x)$ by finding the sign of $f'(x)$. Find the intervals on which $f(x)$ is increasing and the intervals on which $f(x)$ is decreasing. Finding the sign of $f''(x)$, find the coordinates of the inflection points of the graph of $y = f(x)$. Sketch the graph of $y = f(x)$ indicating the asymptotes, turning points and the point of inflection. Using the graph or otherwise find the intervals of values of k for the quadratic equation $(1-k)x^2 + 2kx - k = 0$ to have two distinct real roots. Here $k \in \mathbb{R}$.

(b) A cylinder with radius x and height y is kept inside a hollow cone of radius r and height h. so that the two axes coincide each other as shown in the figure.



Show that $y = \dfrac{h(r-x)}{r}$ and show that the volume of the cylinder $v = \dfrac{\pi h}{r} x^2(r-x)$. By examining the sign of $\dfrac{dv}{dx}$, show that the value of x such that the volume (v) of the cylinder is maximum is $\dfrac{2r}{3}$. Also show that the ratio between the volume of the cone and the volume of the cylinder is 9:4.

(16) (a) For a triangle ABC, using the standard notation, state the cosine rule.

In a triangle ABC, the lengths of the sides BC, CA and AB are $x+y$, x and $x-y$ respectively. Show that $\cos A = \dfrac{x-4y}{2(x-y)}$. If the magnitude of the angle A is 120°, show that $5y = 2x$. Using the sine rule **deduce** that $\dfrac{5\sin A}{7} = \dfrac{\sin B}{1} = \dfrac{5\sin C}{3}$

(b) Prove that $\sin(A+B) - \sin(A-B) = 2\cos A \sin B$

Hence deduce that $\cos(A+B) - \cos(A-B) = -2\sin A \sin B$

Show that $\sin(60-x)\sin(60+x) = \dfrac{1}{2}\cos 2x + \dfrac{1}{4}$ and hence find the value of λ such that $\sin x \sin(60-x)\sin(60+x) = \lambda \sin 3x$. **Hence** find the general solutions of the equation $\sin x \sin(60-x)\sin(60+x) = \dfrac{1}{8}$

(c) Solve the equation $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}\left(\dfrac{8}{31}\right)$

(17) (a) Let $f(x) = 4(\sin^4 x + \cos^4 x)$.

Find the values of A and B such that $f(x) = A + B\cos 4x$, $A, B \in \mathbb{R}$

Hence **deduce that** $2 \leqslant f(x) \leqslant 4$. Also sketch a rough graph of $y = f(x)$ in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. **Hence or otherwise** find the values of k or interval of values of k for the equation Cos $4x$ = k to have

i) two real roots

ii) three real roots

iii) four real roots within the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(b) Prove that the identity $2\cos^2 x - 2\cos^2 2x = \cos 2x - \cos 4x$.

Hence show that $\cos\frac{\pi}{5} - \cos\frac{2\pi}{5} = \frac{1}{2}$

Hence obtain that $\cos\frac{\pi}{5} = \frac{\sqrt{5}+1}{4}$.

GRADE 12 THIRD TERM TEST PAPER

(FRONT PAGE SHOULD BE ADDED HERE)

PART A

1) A particle is projected from the level ground inclined to the horizontal. The velocity of the particle in the maximum height is $\sqrt{\frac{2}{3}}$ times the velocity at the half of the maximum height. Show that the angle of projection is $\frac{\pi}{4}$.

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2) Two vehicles A and B moving in the same direction along a straight road pass a certain point on the road at $t = 0$ with velocities $\boldsymbol{u}$, $\boldsymbol{2u}$ and uniform accelerations $\boldsymbol{2a}$, $\boldsymbol{a}$ respectively. Using velocity time graphs drawn in the same diagram for the motions of both vehicles, find the time taken for the maximum gap between A and B and also show that the maximum gap is given by $\boldsymbol{\frac{u^2}{2a}}$.

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3) The position vectors of two points A and B with respect to a fixed origin O are $\mathbf{3\underline{a}}$ and $\mathbf{6\underline{b}}$ respectively, where $\underline{a}$ and $\underline{b}$ are **non-parallel unit** vectors. The point C lies on AB such that $AC:CB = 1:2$.Find the position vector of the point C. If $OC = 2\sqrt{3}$, show that $A\hat{O}B = \frac{\pi}{3}$.

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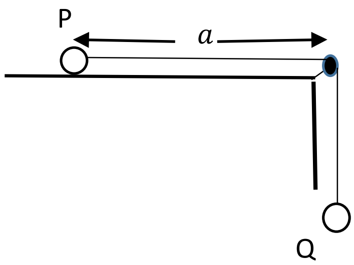
4) Two particles P and Q of masses m and $2m$, respectively are attached to the ends of a light inextensible string. The particle P is held at rest on a horizontal table with the string passing over a small smooth pulley fixed at the edge of the table. The particle Q hangs vertically below the pulley. The system is released from rest with particle P at a distance $\boldsymbol{a}$ from the pulley. In the subsequent motion, a constant frictional force of magnitude $\frac{1}{3}\boldsymbol{mg}$ acts on P . Find the acceleration of P. Also, find the speed of P at the instant when P reaches the pulley.



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5) $\underline{a}$ and $\underline{b}$ are two vectors such that $|\underline{a}| = 2$ and $|\underline{b}| = 3$ .The angle between $\underline{a}$ and $\underline{b}$ is $\frac{2\pi}{3}$,find $\underline{a}.\underline{b}$
Calculate $|\underline{a} + 2\underline{b}|$.Hence find the angle between $\underline{a}$ and $\underline{a} + 2\underline{b}$.

6) A body is placed on a rough plane inclined $\frac{\pi}{4}$ to the horizontal. The coefficient of friction between the plane and the body is $\frac{1}{3}$. A horizontal force of $6N$ is necessary to prevent the body sliding down the plane. Show that the weight of the body is $12N$.
If the motion of the body up the plane starts when the horizontal force is increased gradually, what is the minimum force required to move the particle up the plane.

7) A man rides a bicycle to due west at a velocity $5kmh^{-1}$ and a vehicle moves to the south west at a velocity $20\sqrt{2}kmh^{-1}$. Using a velocity triangle, find the magnitude and direction of the velocity of the man relative to the vehicle.

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8) ABC is an equilateral triangle of length of a side $2a$ .D is the mid-point of the side AC. The forces of magnitudes $4N, 2N, 2N$ and xN act along $\overrightarrow{BA}$, $\overrightarrow{AC}$, $\overrightarrow{BC}$ and $\overrightarrow{DB}$ respectively. If the line of action of the resultant of the system of forces cuts the line BC at a distance $\frac{2a}{3}$ from B, find the value of x.

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9) A particle of weight $50N$ is held in equilibrium in a vertical plane by means of two inextensible srings.If the tensions in the two strings are $25N$ and $25\sqrt{3}N$, find the inclinations of two strings to the vertical.

10) A particle travels on a straight path with a uniform acceleration f. The displacement in time t is a and the displacement in time $2t$ is $a + b$. Using kinematic equations, show that $f = \frac{b-a}{t^2}$.

PART B

❖ **Answer 05 questions only**

11) A balloon resting on the ground starts to ascend vertically with a uniform acceleration of $\frac{g}{12}$ .After a time t from the beginning, a body was released gently from the balloon. As a result, the acceleration of the balloon instantly increases to $\frac{g}{6}$ .After moving another time $\frac{t}{3}$ with that uniform acceleration, the balloon bursts and starts to move freely under gravity. Sketch the velocity – time graphs for the motion of the balloon and the body in the same diagram.(g is the acceleration due to the gravity)

Using the graph, find

(i) the distance, the balloon has travelled when the body was released

(ii) the time taken by the body to attain the maximum height from the beginning.

(iii) the time taken by the balloon to attain the maximum height from the beginning.

(iv) the time taken for the balloon and the body to possess equal velocities in magnitude from the beginning. and that velocity

(v) the velocity of the balloon when the body reaches the ground.

12) .A ship sails due north at a uniform velocity $2u$ along a straight path L at a distance k from a harbor P. The foot of the perpendicular drawn from P to the path L is M.The point Q lies on L, South of M such that $P\hat{Q}M = \alpha$.When the ship passes Q, a boat moving with uniform speed u start from harbor P to meet the ship. Considering the motion of the boat relative to the ship

(i) Show that the boat cannot meet the ship, if $\alpha > \frac{\pi}{6}$

(ii) If $\alpha < \frac{\pi}{6}$, using a velocity triangle, show that the boat can moves in two directions relative to earth to meet the ship and show further that the angle between these two directions is given by $2cos^{-1}(2sin\alpha)$

(iii) Let t_1 and t_2 be the times taken by the boat to meet the ship in above two directions. If $t_1 > t_2$ Show that $t_1 - t_2 = \frac{4kcot\alpha}{3u}$

13) (a) Let the position vectors of four points A, B, C and D be $\underline{a}$,$2\underline{b}$,$5\underline{a}$ and $7\underline{b}$ respectively with respect to a fixed origin O, where $\underline{a}$ and $\underline{b}$ are non-zero and non-parallel vectors.F is the point of intersection of AD and BC.Show that $\overrightarrow{OF} = \underline{a} + \lambda(7\underline{b} - \underline{a})$ for $\lambda \in$ R

Similarly, show also that $\overrightarrow{OF} = 2\underline{b} + \mu(5\underline{a} - 2\underline{b})$ for $\mu \in$ R

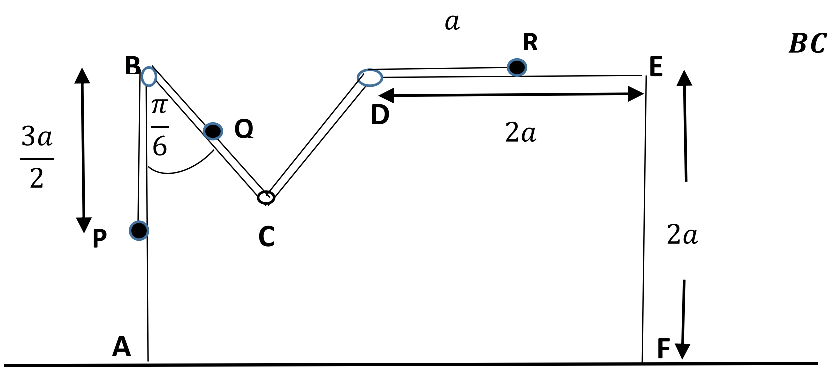
Hence, show that $\overrightarrow{OF} = \frac{1}{33}(25\underline{a} + 56\underline{b})$

(b) The length of a side of the regular hexagon $PQRSTU$ is $2m$. Forces of magnitudes $8N$, $3N$, $6N$,$2\sqrt{3}N, xN$ and yN act along $\overrightarrow{PQ}$, $\overrightarrow{PU}$, $\overrightarrow{UT}$, $\overrightarrow{QU}$, $\overrightarrow{RS}$ and $\overrightarrow{RQ}$ respectively. The resultant Of this system is a force of $10N$ parallel to $\overrightarrow{QR}$ .Find x and y.

Find the distance from P to the point, where the resultant cuts PQ.

Now, an anti-clockwise couple of magnitude $M\ Nm$ and two forces, each of magnitude FN acting along $\overrightarrow{QP}$ and $\overrightarrow{UP}$ are added to the system such that the new resultant of the system act along $\overrightarrow{PR}$ .Find the magnitudes of M and F.

14)



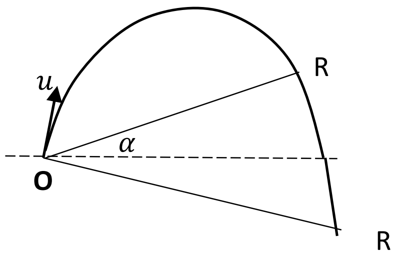
$\boldsymbol{BC = \sqrt{3}a = CD}$

The vertical cross section ABCDEF through the Centre of gravity of a smooth block of mass $\boldsymbol{5m}$ is shown in the figure. The face containing AF is placed on a smooth horizontal floor. Also, BC and DC are the lines of greatest slope of the faces containing them.$\boldsymbol{AB = 2a}$,$\boldsymbol{BC = CD = \sqrt{3}a, DE = 2a}$, $\boldsymbol{A\hat{B}C = \frac{\pi}{6}}$, and $\boldsymbol{C\hat{D}E = \frac{2\pi}{3}}$ .The particles P and Q of masses $\boldsymbol{4m}$ and $\boldsymbol{3m}$ respectively, are attached to the ends of a light inextensible string passing over a smooth light small pulley fixed to the block at B.The mass of particle R is $\boldsymbol{2m}$. The particles Q and R are attached to the ends of another light inextensible string passing through a smooth light small ring fixed to the block at C and passing over a smooth light small pulley fixed to the block at D. The particles Q and R are placed at the mid points of BC and DE and the particle P is hanging vertically. Strings are taut in the position shown in the diagram and the system is released from rest from this position. Obtain equations sufficient to determine the time taken for the particle P to reach the floor and hence find that time.

15) A particle is projected from a point O with speed u at an angle θ to the horizontal. The line drawn inclined $\alpha(< \theta)$ to the horizontal through O meets the path of the projectile at the point R.

(i) When R lies above the level of O , show that $OR = \frac{2u^2 cos\theta sin(\theta-\alpha)}{gcos^2\alpha}$

(ii) When R lies below the level of O, deduce that $OR = \frac{2u^2 cos\theta sin(\theta+\alpha)}{gcos^2\alpha}$



(iii) If it is given that u and α are constants, find the values of θ when OR becomes a maximum in above two cases

(iv) If the maximum value of OR when R lies below the level of O is three times the maximum value of OR when R lies above the level of O, show that $\alpha = \frac{\pi}{6}$

16) A smooth hemispherical bowl of radius $\sqrt{3}a$ is fixed with its edge horizontal. A thin uniform rod AB of length $4a$ and weight W rests in a vertical plane through the Centre of the bowl with its end A touching a point inside the bowl and with its other end extending outside the edge of the bowl

(i) Show that the inclination of the rod to the horizontal is $\frac{\pi}{6}$.

(ii) Find the reactions at the points, where the rod touches the bowl.

(iii) Show that $\frac{1}{4}$ of the length of the rod extends outside the edge of the bowl.

(iv) Show also that the shortest length of the rod which can be in equilibrium in this position is $2\sqrt{2}a$.

17) A ladder AB of weight W and length $2a$ is kept in equilibrium, touching end A with a rough horizontal floor and end B with a rough vertical wall. The ladder is in a vertical plane perpendicular to the wall. The coefficient of friction at both ends is μ. The Centre of gravity G divides the ladder to the ratio $AG:GB = k:1$

(i) If the inclination of the ladder to the horizontal is θ and it is in limiting equilibrium,

show that $tan\theta = \frac{k-\mu^2}{\mu(k+1)}$

(ii) If the ladder is uniform, deduce that $\theta = \frac{\pi}{2} - 2\lambda$, where λ is the angle of friction and 2λ is an acute angle.

(iii) If the inclination of the ladder to the horizontal is $\alpha\left(< \frac{\pi}{2} - 2\lambda\right)$,and the length of the ladder is $2a$ show that the moment G of the couple that should be given to the ladder in its vertical plane, just to prevent slipping is $G = Wacos(\alpha + 2\lambda)$

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