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Provincial Department of Education - NWP

තෙවන වාර පරීක්ෂණය - 13 ශ්‍රේණිය - 2023

Third Term Test - Grade 13 - 2023

විභාග අංකය: ....

Combined Mathematics - I

Time: 03 Hours

පැය තුනයි
மூன்று மணித்தியாலம்
Three hours

අමතර කියවීම් කාලය - මිනිත්තු 10 යි
மேலதிக வாசிப்பு நேரம் - 10 நிமிடங்கள்
Additional Reading Time - 10 minutes

Use additional reading time to go through the question paper, select the questions you will answer decide which of them you will prioritise.

Index number | | | | | | | |

Instructions:

* *This question paper consists of two parts;*
  Part A (Questions 1 - 10) and **Part B** (Questions 11 - 17)
* **Part A:**
  *Answer **all** questions. Write your answers to each question in the space provided. You may use additional sheets if morespace is needed.*
* **Part B:**
  *Answer **five** questions only. Write your answers on the sheets provided*
* *At the end of the time allotted, tie the answer scripts of the two parts together so tha**Part A** is on the top of **Part B** and hand them over to the supervisor.*
* *You are permitted to remove **only** **Part B** of the question paper from the Examination Hall*

For Examiners' Use only

(10) Combined Mathematics I		
Part	Question No.	Marks
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	

Total

In numbers	
In words	

Part A

01. Using the principle of mathematical induction, prove that $n^3 - n$ is divisible by 6 for all $n \in Z^+$ and $n \geq 2$.

02. Sketch the graph of $y = |x+2|$ and $y = 7 - |x-3|$ in the same diagram. Hence or otherwise, find all real values of x satisfying the inequality $|x+1| + |x-4| < 7$.

03. On an same Argand diagram, sketch the locus representing complex numbers z satisfying $|z+i|=1$ and the locus representing complex number w satisfying $\arg(w-2)=\frac{3\pi}{4}$. Hence, find the least value of $|z-w|$ for the points on these loci.

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04 . Find the first 4 terms, in ascending powers of x, of the binomial expansion of $(2+kx)^7$ where k is a constant. Give each term in its simplest form.

Given that the coefficient of x^3 in this expansion is 1890, find the value of k.

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05. Evaluate $\lim_{x \to 0} \dfrac{x\tan 2x - 2x\tan x}{(1-\cos 2x)^2}$.

06. Show that the area of the region enclosed by the curves $y = e^x$, $y = x^2 - 1$, $x = -1$ and $x = 1$ is $\dfrac{3e^2 + 4e - 3}{3e}$.

07. A curve is given parametrically by $x = 4t - 1,\ y = \frac{5}{2t} + 10,\ \ t \in R,\ \in t \neq 0$. The curve crosses the x - axis at the point A. Find the coordinates of A. Show that an equation of the tangent to the curve at A is $y + 10x + 20 = 0$.

08. A line intersects the x - axis at $A(7, 0)$ and the y - axis at $B(0, -5)$. A variable line PQ is drawn perpendicular to AB intersecting the x - axis in P and the y - axis in Q. If AQ intersects BP at R, then find the locus of R.

09. A variable tangent line drawn to a circle whose radius is c and the centre is the origin, meets the x - axis and y - axis at A and B respectively. Find the locus of the centre of the circle passing through O, A and B.

10. Solve the equation $\sin^{-1} x - \cos^{-1} x = \sin^{-1}(3x-2)$.

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තෙවන වාර පරීක්ෂණය - 13 ශ්‍රේණිය - 2023
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Part B

❖ *Answer only five questions.*

11. *a.* The roots of the quadratic equation $x^2 - x + p = 0$ are α and β. Also the roots of $x^2 - 9x + q = 0$ are γ and δ.

If α, β, γ and δ are in a geometric progression, then find the possible values of common ratio of that progression.

Then find the possible values of p and q.

Obtain the quadratic equations whose roots are $\alpha\gamma$ and $\beta\delta$.

b. Remainder when the polynomial $g(x)$ of degree 3 is divided by $x, (x-1), (x+1)$ and $(x-2)$ are $-12, -8, -24$ and -6 respectively. Given that $Q(x) = (x+2)g(x) + 24$ show that $x, (x-1), (x+1)$ and $(x-2)$ are the factors of $Q(x)$. Hence find $Q(x)$ without finding $g(x)$.

12. *a.* A cricket team consisting of 11 players is to be formed from 16 players of whom 4 can be bowlers and 2 can keep wicket and the rest can neither be bowler nor keep wicket. In how many different ways can a team be formed so that the teams contain

i. exactly 3 bowlers and 1 wicket keeper,

ii. at least 3 bowlers and at least 1 wicket keeper?

b. U_r is the r^{th} term of the sequence $\frac{1}{2}, \frac{1\cdot 3}{2\cdot 4}, \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}, \frac{1\cdot 3\cdot 5\cdot 7}{2\cdot 4\cdot 6\cdot 8}, \ldots.$

Express $U_{(r+1)}$ in terms of U_r.

$f(r)$ is a function of r, where $f(r)=(Ar+B)U_r$; A and B are constants and $f(r+1)-f(r)=Ur$. Find the values of A and B

and **hence**, prove that $\sum_{r=1}^{n} U_r = \left[\frac{1\cdot 3\cdot 5\cdot 7.............(2n+1)}{2\cdot 4\cdot 6...................2n}-1\right]$.

13. *a.* Let $A\equiv\begin{pmatrix}2 & a\\ 3 & b\end{pmatrix}$, $B\equiv\begin{pmatrix}1\\ 1\end{pmatrix}$ and $C=\begin{pmatrix}1\\ 3\end{pmatrix}$, where $a,b\in\Re$.

It is given that $AB=C$. Show that $a=-1$ and $b=0$. Then show that $A^2=2A-3I$.

Deduce that $A^3=A-6I$

The inverse of A is denoted by A^{-1} show that $A^{-1}=\frac{1}{3}(2I-A)$. Find A^{-1}

b. i. The complex number u is defined by $u=\frac{(1+2i)^2}{2+i}$.

Express u in the form $x+iy$, where x and y are real.

Sketch an Argand diagram showing the locus of the complex number z such that $|z-u|=|u|$

ii. Find the square roots of the complex number $7-6\sqrt{2}i$. Give your answers in the form $x+yi$. Where x and y are real numbers.

iii. Find the argument and the modulus of $2\sqrt{3}-2i$.

Find all the solutions z to the equation $z^3=2\sqrt{3}-2i$.

14. *a.* Let $f(x)=\frac{(x+1)(x-2)}{(x-1)^2}$ for $x\neq 1$.

Show that $f'(x)$, the derivative of $f(x)$, is given by $f'(x)=\frac{(-x+5)}{(x-1)^3}$ for $x\neq 1$.

Hence, find the interval on which $f(x)$ is increasing and the interval on which $f(x)$ is decreasing. Also find the coordinates of the turning point of $f(x)$.

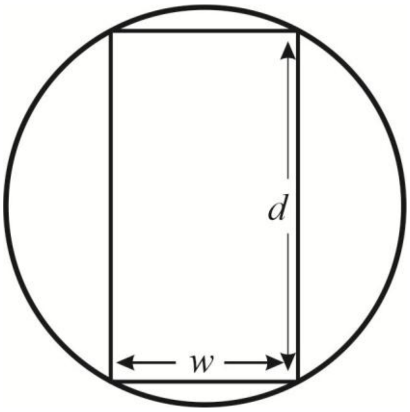
It is given that $f''(x)=\dfrac{2(x-7)}{(x-1)^4}$ for $x\neq 1$.

Find the coordinates of the points of inflection of the graph of $y=f(x)$.

Sketch the graph of for $y=f(x)$ indicating the asymptotes, the turning points and the points of inflection.

b. The strength of a beam with rectangular cross section is proportional to the product of its width w and the square of its depth d.

Find the dimensions of the strongest beam that can be cut from a cylindrical log of radius r in terms of r, considering the given figure which depicts the cross section of the log.



15. *a.* Find the value of the constants A, B and C such that

$\dfrac{1}{x(x^2+1)}=\dfrac{A}{x}+\dfrac{Bx+C}{x^2+1}$. Hence find the integrate $\int \dfrac{1}{x(x^2+1)}dx$

Use the substitution $x=\cos\theta$ show that $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{2\sin\theta}{\cos\theta+\cos^3\theta}d\theta=\ln\left(\frac{5}{3}\right)$

b. Find the value of $\int e^{-2x}\sin\pi x dx$ using the integration by parts.

c. Show that $\displaystyle\int_0^{\pi} f(x)\,dx=\int_0^{\pi} f(\pi-x)\,dx$.

If $I=\displaystyle\int_0^{\pi} x\sin^3 x dx$, using the above result, show that $I=\dfrac{1}{2}\displaystyle\int_0^{\pi}\sin^3 xdx$

Hence, find the exact value of the integral $\displaystyle\int_0^{\pi} x\sin^3 xdx$.

16. Let the equations of two circles be $x^2+y^2+2gx+2fy+c=0$ and $x^2+y^2+2g'x+2f'y+c'=0$.

If these circles intersect orthogonally, then show that $2gg'+2ff'=c+c'$.

Let u_1 and u_2 be two parallel lines passing through the points $P\equiv(1,0)$ and $Q\equiv(2,0)$ respectively. Let the line $2y-3x+7=0$ meet u_1 at A and u_2 at B.

If the length of AB is $\sqrt{13}$ units and gradient is positive, find the points $A(a,b)$ and $B(c,d)$. Where $a,b,c,d\in Z$

The vertices of a triangle are A, B and $C(6,1)$. Find the coordinates of the orthocenter. Find the equations of the circles whose diameters are AH and BC and show that those circles intersect each other orthogonally.

17. *a.* Show that $\sin^2 5^\circ+\sin^2 10^\circ+\sin^2 15^\circ+\ldots\ +\sin^2 90^\circ=9\frac{1}{2}$.

b. State and prove the **cosine rule** for any triangle ABC in the usual notation.

For any triangle ABC in the usual notation, if $\frac{b+c}{11}=\frac{c+a}{12}=\frac{a+b}{13}$, then find the ratio $\cos A:\cos B:\cos C$.

c. In a right angled triangle, the hypotenuse is $2\sqrt{2}$ times the length of perpendicular drawn from the opposite vertex of the hypotenuse. Then find the other two angles.

d. Solve the equation $81^{\sin^2 x}+81^{\cos^2 x}=30$ in the interval $[0,\pi]$.

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විභාග අංකයඃ ....

සංයුක්ත ගණිතය - II

කාලය පැය 03 යි
අමතර කියවීම් කාලය - මිනිත්තු 10 යි

උපදෙස්

- මෙම ප්‍රශ්න පත්‍රය කොටස් දෙකකින් සමන්විත වේ.
  A කොටස (ප්‍රශ්න 1-10) දක්වා B කොටස (ප්‍රශ්න 11-17)
- A කොටස
  සියලුම ප්‍රශ්නවලට පිළිතුරු සපයන්න. එක් එක් ප්‍රශ්නය සඳහා ඔබේ පිළිතුරු සපයා ඇති ඉඩෙහි ලියන්න.
  වැඩිපුර ඉඩ අවශ්‍ය වේ නම් ඔබට අමතර ලියන කඩදාසි භාවිත කළ හැකිය.
- B කොටස
  ප්‍රශ්න පහකට පමණක් පිළිතුරු සපයන්න.
- නියමිත කාලය අවසන් වූ පසු A කොටස B කොටසට උඩින් සිටින පරිදි කොටස් දෙක අමුණා විභාග ශාලාධිපතිට භාර දෙන්න.
- ප්‍රශ්න පත්‍රයෙහි B කොටස පමණක් විභාග ශාලාවෙන් පිටතට ගෙනයාමට ඔබට අවසර ඇත.

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සංයුක්ත ගණිතය I		
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B	එකතුව	
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ප්‍රතිශතය		

පත්‍රය I	
පත්‍රය II	
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අවසාන ලකුණු

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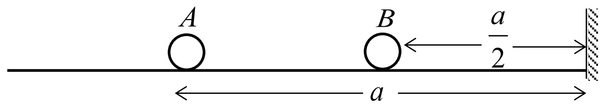
	2	
අධීක්ෂණය		

Part A

(01) Two particles A and B each of mass m, are placed in straight line on a smooth horizontal table which is placed in front of a vertical wall. The distances from the wall to the particle A and particle B are a and $a/2$ respectively.



The particle B is given an impulse towards the wall such that its velocity just after the impulse is u. The particle B hits the wall and bounces off. Then it collides with the particle A. Show that the velocity of the particle A just after the collision is $\frac{e(1+e)u}{2}$, where e is the coefficient of restitution between B and the wall and also between B and A. Also find the time taken for B to collide with A.

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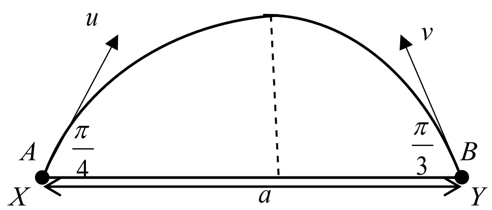
(02) X and Y are two points on a horizontal ground such that $XY = a$. Two particles A and B are projected from the points X and Y respectively at the same instant, in the vertical plane that contains XY in opposite directions such that they collide with each other after a time T at a point in space. The initial velocities of A and B are u and v respectively. Show that



$v = \sqrt{\frac{2}{3}}\, u$ and $T = \left(\frac{\sqrt{6}}{\sqrt{3}+1}\right)\frac{a}{u}$.

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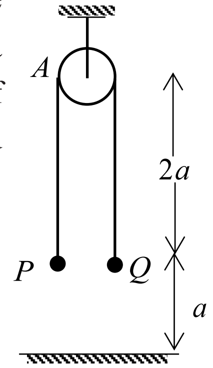
(03) Two particles P and Q of mass $2\ m$ and $3\ m$ are connected by a light inextensible string passing over a fixed smooth pulley A. When the particles P and Q are at a height of a from the ground, they are released from rest. Find the acceleration of the particle P in the subsequent motion. Show that the maximum height reached by P is $\frac{11a}{5}$.



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(04) A car of mass $1200\ kg$ moves on a horizontal road against a constant resistance of $400\ N$. The acceleration of the car is $3ms^{-2}$ when it moves with the velocity $15\ ms^{-1}$. Find the power of the car. Obtain equations sufficient to determine the acceleration of the car when it moves upward a road of inclination 30^o to the horizontal with the velocity $12\ ms^{-1}$ working with the same constant power against the same constant resistance.

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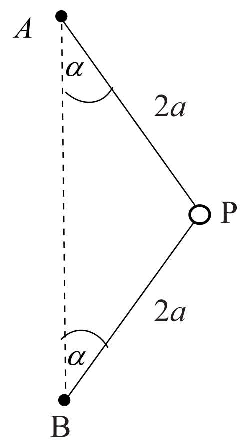
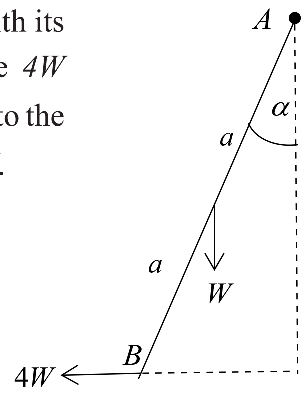
(05) A particle P of mass m is connected by two light inextensible strings each of length $2a$ attached to the points A and B in the same vertical line. The particle P moves in a horizontal circle with constant angular velocity ω. The string AP makes an angle α $(0 < \alpha < \pi/2)$ with the downward vertical and the string BP makes an angle α with the upward vertical.



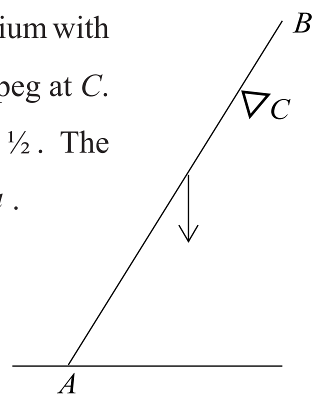
Show that $\omega > \sqrt{\dfrac{g}{2a\cos\alpha}}$.

(06) In the usual notation, the position vectors of the points A, B and C with respect to the fixed origin O are $2\mathbf{a} + 3\mathbf{b}$, $\frac{1}{3}\mathbf{a} + \frac{3}{4}\mathbf{b}$ and $k\,\mathbf{a} + 2\,\mathbf{b}$ respectively. Here $\mathbf{a}$ and $\mathbf{b}$ are two non-parallel, non-zero vectors and $k \in \mathbb{R}$. Find $\overrightarrow{AB}$ and $\overrightarrow{BC}$ in terms of $\mathbf{a}, \mathbf{b}$ and k. Find the value of k, if A, B and C are collinear.

(07) A uniform rod of length $2a$ and weight W is kept in equilibrium with its upper end A hinged to a smooth vertical wall and by a horizontal force $4W$ applied at the end B as shown in the figure. The rod makes an angle α to the vertical. Find the value of α and show that the reaction at A is $\sqrt{17}\,W$.



(08) A uniform rod AB of length $4a$ and weight W is kept in limiting equilibrium with its end A on a rough horizontal plane and the rod touching a smooth peg at C. The coefficient of friction between the rod and the horizontal plane is ½. The rod makes an angle $\tan^{-1}\left(\frac{3}{4}\right)$ with the horizontal. Show that $AC = \frac{16}{5}a$.

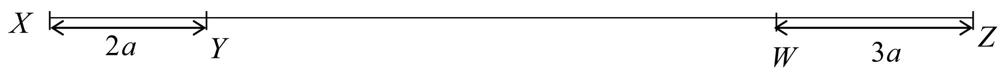


(09) Let A and B are two independent events of a sample space Ω. In the usual notation, it is given that $P(B) = 1/4$, $P(A' \cup B') = \frac{9}{10}$. Find $P(A \cap B)$, $P(A \cup B)$ and $P(B \mid A \cup B)$; where A' and B' denote complementary events of A and B, respectively.

(10) The standard deviation of marks obtained by *100* student for an examination is *10*. The z-score of a student who got *70* marks for this examination is *1.6*. Find the mean of the sample. It was later found that this mark of *70* has been entered erroneously and it should have been *60* instead. Find the correct value of the mean of the marks obtained for this examination.

* **Answer five questions only.**

(11) (a)



A motor car A starting from rest from the point X on a straight horizontal road moves with constant acceleration $3f ms^{-2}$ until it reaches the point Y. Here $XY = 2a\ m$. Then it maintains the velocity $u\ ms^{-1}$ obtained at Y, throughout the rest of its motion. When the car A reaches the point Y, another car B starts from rest from the point Z and moves in the opposite direction of the same road with constant accelerations $f\ ms^{-2}$ until it reaches W. Here $ZW = 3a\ m$. Then it maintains the velocity $v\ ms^{-1}$ obtained at W throughout its rest of the motion. Sketch the velocity - time graphs for the motions of A and B in the same diagram.

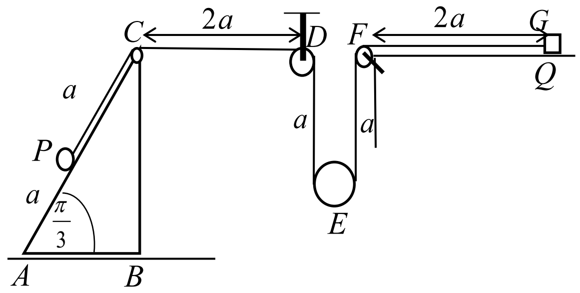
Hence, show that the time taken by car A to move from X to Y is $2\sqrt{\frac{a}{3f}}\ S$.

Also show that the time taken by car B to move from Z to W is $\sqrt{\frac{6a}{f}}\ S$.

If $XZ = 16\,a\ m$, show that the time taken by car A to meet B from Y is $\left(\frac{11 - 6\sqrt{2}}{2\sqrt{3} + \sqrt{6}}\right)\sqrt{\frac{a}{f}}\ S$.

(b) A ship A is sailing due south with uniform speed of $2u\ kmh^{-1}$ and another ship B is at a distance $a\ km$ west of A. The ship B appears to move in the direction $30°$ East of North with a velocity of $2\sqrt{3}\,u\ \ kmh^{-1}$ when it is observed from A. Show that the ship B sails in the direction $60°$ East of North with a velocity of $2u\ \ kmh^{-1}$ relative to earth. Find the shortest distance between two ships. If the shooting range of ship A is $\frac{11a}{12}$ km, show that the ship B is in danger during a time of $\frac{\sqrt{39}\ a}{36\ u}$ h.

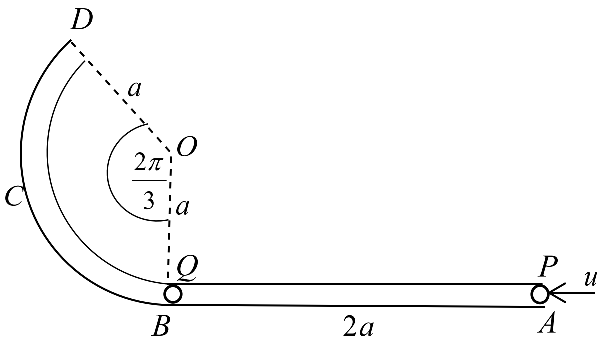
(12) (a)



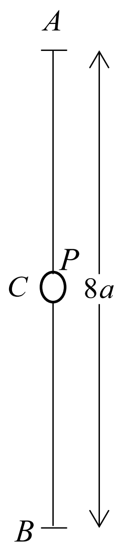
The figure shows a vertical cross-section ABC through the center of gravity of a smooth uniform block of mass $6\,m$. The face containing AB is placed on a smooth horizontal floor.

The line AC is the line of greatest slope of the face containing it. Here $B\hat{A}C = \pi/3$, $AC = CD = FG = 2a,\ DE = a$. The particle P of mass $5\,m$ is kept in the mid point of AC and the particle Q of mass $2\,m$ is kept at the point G on a smooth plane such that $FG = 2a$. The particle P and Q are attached to the ends of a light inextensible string passing over a smooth light small pulley fixed to the block at C. The string is passing over a fixed small pulley at D, underneath a movable pulley of mass m at E and passing over a fixed light smooth pulley at F. Write down the equations sufficient to determine the time taken for the particle P to reach the point A.

(b) A smooth narrow tube $ABCD$ is fixed in a vertical plane as shown in the figure. The horizontal part AB of length $2a$ is straight and BCD is an arc of radius a subtending an angle $\frac{2\pi}{3}$ at the center O. A small particle P of mass m is placed at the end A and is given a horizontal velocity u towards $\overrightarrow{AB}$. Then the particle P collides with the particle Q of mass m at B which is at rest. If the coefficient of restitution is ½ , find the velocity of Q after the collision. Show that the velocity V of particle Q is given by $V^2 = \frac{9u^2}{16} - 2ga(1 - \cos\theta)$, when $\overrightarrow{OQ}$ makes an angle, $\theta\ (0 < \theta < 2\pi/3)$ with $\overrightarrow{OB}$. Also find the reaction on the particle Q from the tube at the above position. If Q leaves the tube at the end D, show that $u > 4\sqrt{\frac{ga}{3}}$. Also find the reaction on Q from the tube at D.



(13) A and B are two fixed points in a same vertical line such that $AB = 8a$ and A is vertically below B. One end of a light elastic string of natural length a and modulus of elasticity mg is attached the point A and the other end is attached to a particle of mass m. One end of another string of length $3a$ and modulus of elasticity mg is attached to the point B and the other end is attached to the particle P. The particle P is in equilibrium at C with both strings AP and BP are vertical. Show that $AC = \frac{11a}{4}$.

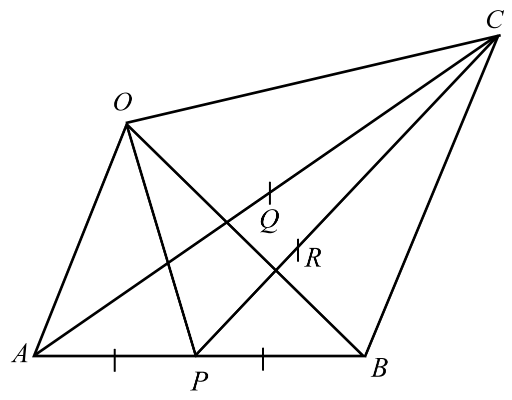


The particle P is now pulled to a point D which is at a distance $5a$ below A and released from rest. Find the tensions of strings when the particle is at a distance x from A. Write down the equation of motion of P and show that $\ddot{x} + \frac{4g}{3a}\left(x - \frac{11a}{4}\right) = 0$

By taking $y = x - \frac{11a}{4}$, show that $\ddot{y} + \frac{4g}{3a}y = 0$.

Assuming that the solution of the above equation is of the form $y = A\cos\omega t + B\sin\omega t$, find the values of A, B and ω. Show that the velocity of P, when P is at a distance a below A is $\sqrt{\frac{8ga}{3}}$ and find the time taken to reach that point.

(14) (a) As shown in the figure, the position vectors of three points A, B and C are $2\boldsymbol{a} - \boldsymbol{b}$, $4\boldsymbol{a} + 5\boldsymbol{b}$ and $-\boldsymbol{a} + 4\boldsymbol{b}$ respectively, with respect to a fixed origin O, where $\boldsymbol{a}$ and $\boldsymbol{b}$ are non - zero and non - parallel vectors. P and Q are mid points of AB and AC respectively. R is a point of PC such that $\overrightarrow{PR} = 1/3\,\overrightarrow{PC}$.



(i) Find the position vectors of Q and R in terms of **a** and **b**.

(ii) If B, R and Q are collinear, find the ratio $BR : RQ$.

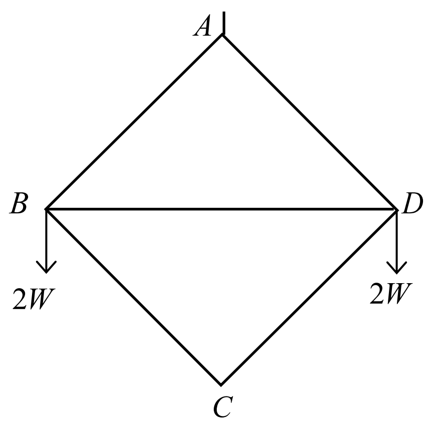
(iii) If S is a point on BQ produced such that $\overrightarrow{BS} = K\overrightarrow{BQ}$ and SC is parallel to AR, find the value of k. Here $k \in \mathbb{R}$.

(b) $ABCDEF$ is a regular hexagon of side $2a$. Forces of magnitude $2P, 1, 2, Q, 3R$ and 4 *Newton* are act along $\overrightarrow{AB}, \overrightarrow{CB}, \overrightarrow{DC}, \overrightarrow{DE}, \overrightarrow{FE}$ and $\overrightarrow{AF}$ respectively. Mark the forces in the hexagon by taking AB as the base.

(i) If the system of forces are in equilibrium, find the values of P, Q and R.

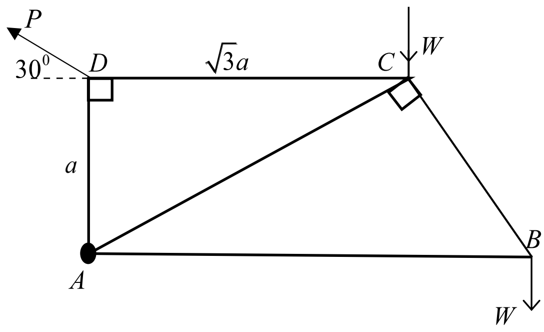
(ii) If $P = 2$, $Q = 1$ and $R = 2$, find the magnitude and the direction of the resultant of the system. Find the distance from A to the point where the resultant meet AB. If the system is reduced to a single force passing through A and a couple, find the magnitude and the direction of the single force. Also find the magnitude and the sense of the couple.

(15) (a) Four uniform rods AB, BC, CD and DA, each of length $2a$ and weight W are smoothly jointed at their ends. Two loads each of weight $2W$ are suspended at B and D. The ends B and D are connected by a light rod. $B\hat{A}D = B\hat{C}D = 120°$. The system is suspended in a vertical plane from the point A and stays in equilibrium as shown in the figure.



Find the reaction exerted on BC by CD at the joint C. Show that the thrust on the rod BD is $4\sqrt{3}W$.

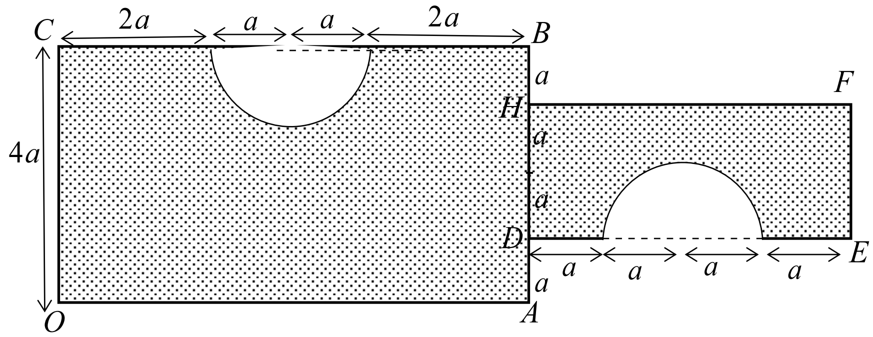
(b) The framework shown in the figure consists of five light rods AB, BC, CA, CD and AD smoothly jointed at their ends. It is given that $AD = a$, $CD = \sqrt{3}a$, $A\hat{D}C = A\hat{C}B = 90°$. At each of the joints B and C loads of W is suspended. The framework is smoothly hinged at A to a fixed point and kept in equilibrium in a vertical plane with AB and DC horizontal by a force P applied at an angle $30°$ to the horizontal at the joint D.



(i) Find the value of P.

(ii) Draw a stress diagram using Bow's notation for the joints C, B and D. Hence, find the stresses in the rods, stating whether they are tensions or thrusts.

(16) Show that the center of mass of a uniform semi - circular lamina of radius a and center O is at a distance $\frac{4a}{3\pi}$ from O on the axis of symmetry.



$OABC$ and $DEFH$ are two metal laminas made of a uniform thin metal sheet of surface density σ. A semi circle of radius a is removed from the rectangle $OABC$ and another semi circle of radius a is removed from the rectangle $DEFH$. A plane lamina is made by joining remaining parts of above rectangles as shown in the figure. Here $OA = 6a$, $OC = 4a$, $DE = 4a$ and $EF = 2a$.

The center of mass of this composited lamina lies at a distance $\bar{x}$ from OC and $\bar{y}$ from OA. Show that $\bar{x} = \frac{(272 - 11\pi)a}{2(32 - \pi)}$ and $\bar{y} = \frac{(128 - 5\pi)a}{2(32 - \pi)}$

This composite lamina is suspended freely by a light inextensible string attached to C. Find the inclination of the edge OC to the vertical in the equilibrium position.

(17) (a) Three identical boxes A, B and C, each contains 10 balls are identical in all aspects except for their colours. Box A contains 7 blue balls and 3 red balls. Box B contains 6 blue balls and 4 red balls. Box C contains 2 blue balls and 8 red balls. One of three boxes is chosen at random and 2 balls are drawn one after the another without any replacement.

Find the probability that

(i) the two balls drawn are blue.

(ii) the balls are drawn from box B, given that the two balls drawn are blue.

(b) The over-time allowances of 40 employees in a certain month are given in the following table.

Over-time allowance (in thousand rupees)	No. of employees
1 - 5	5
5 - 9	7
9 - 13	12
13 - 17	10
17 - 21	6

Estimate the mean, the mode and the standard deviation of the distribution given above. Hence, find the coefficient of skewness.

(01) let $f(n) = n^3 - n$

When $n=2$

$f(2) = 2^3 - 2$

$= 6 \times 1$

$\therefore$ the result is true for $n=1$ (5)

Take any $p \in \mathbb{Z}^+$, $p \geq 2$

Assume that the result is true for $n=p$

$f(p) = p^3 - p = 6\alpha$ —— (1) where $\alpha \in \mathbb{N}$. (5)

When $n = p+1$

$f(p+1) = (p+1)^3 - (p+1)$

$= p^3 + 3p^2 + 3p + 1 - p - 1$

$= p^3 + 3p^2 + 2p.$

from (1) (5)

$= 6\alpha + p + 3p^2 + 2p.$

$= 6\alpha + 3p^2 + 3p.$

$= 6\alpha + 3\underbrace{p(p+1)}_{2\beta}$ ($\because$ even number)

$= 6\alpha + 6\beta$

$= 6(\alpha+\beta)$

$= 6\gamma$; $\gamma \in \mathbb{N}$. (5) (5)

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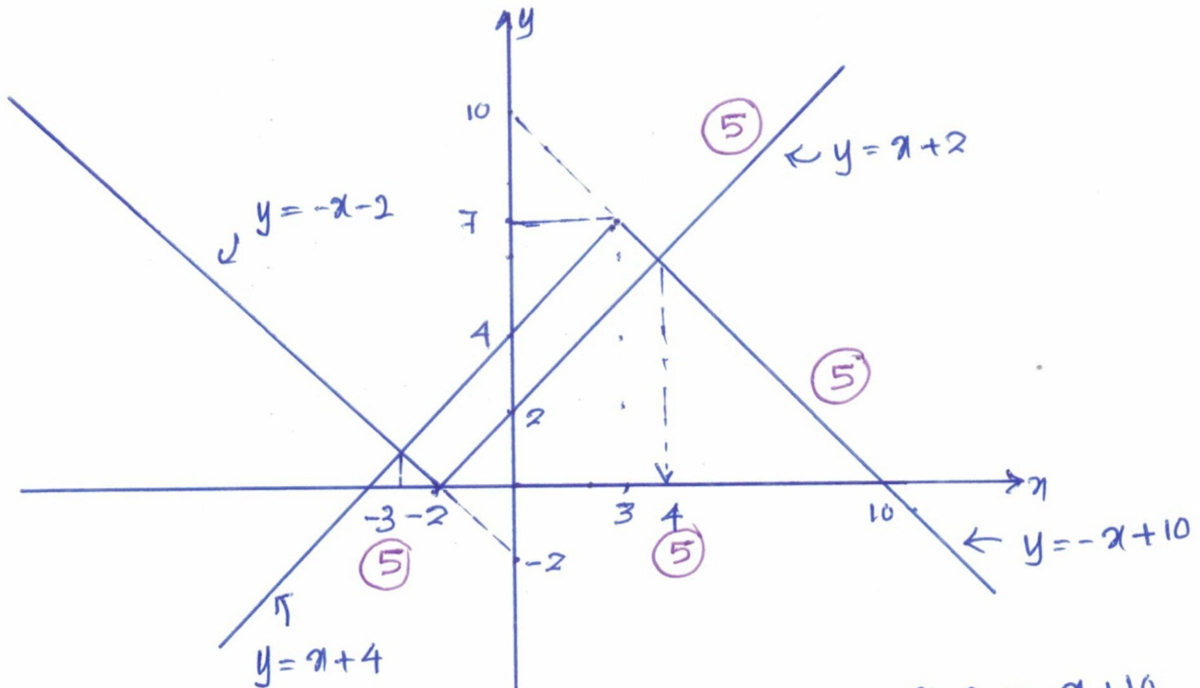
$\therefore$ If the result is true for $n = p \in \mathbb{Z}^+$. then it is also true for $n = p+1$. by using the principle of mathematical induction the result is true for all $n \in \mathbb{Z}^+$.

(02)



$x+2 = -x+10$

$2x = 8$

$x = 4$

$x+4 = -x-2$

$2x = -6$

$x = -3$

$|x+1| + |x-4| < 7$

$\downarrow x = y+1$

$|y+2| + |y-3| < 7$

$|y+2| < 7 - |y-3| \Rightarrow y \in (-3, 4)$

$|x+1| + |x-4| < 7 \Rightarrow \underline{x \in (-2, 5)}$ (5)

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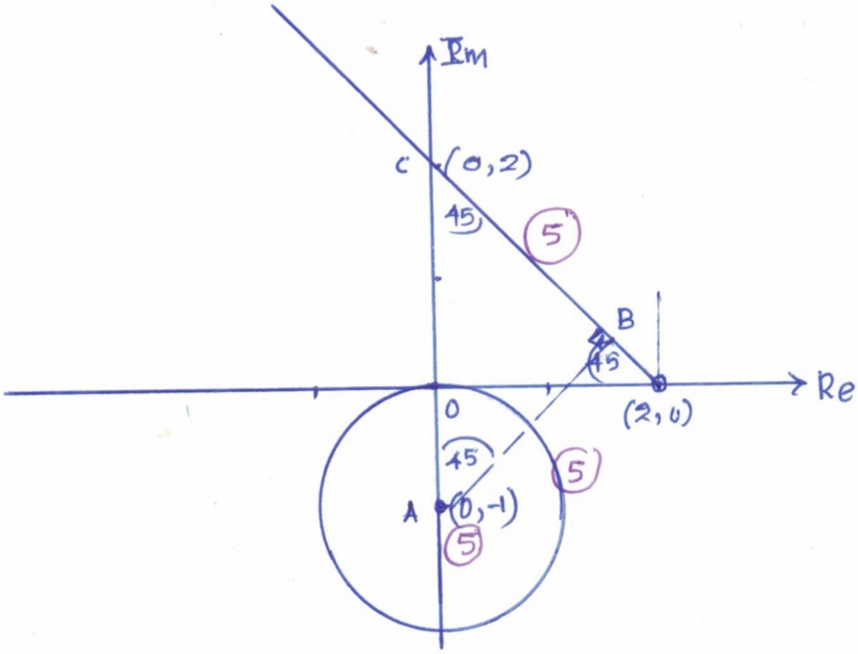
(03) $|z+i| = 1 \Rightarrow |z - (0-i)| = 1$

Centre $= (0, -1)$

Radius $= 1$

$\arg[w-2] = \frac{3\pi}{4} \Rightarrow \arg[w-(2+0i)] = \frac{3\pi}{4}$



let $AB = BC = d$.

$d^2 + d^2 = 3^2$

$2d^2 = 9$

$d = \frac{\sqrt{9}}{\sqrt{2}}$

$= \frac{3\sqrt{2}}{2}$ (5)

$|z - w|_{min} = AB - 1$

$= \frac{3\sqrt{2}}{2} - 1$ (5)

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(04) $(2+kx)^7 = {}^7C_0(2)^7 + {}^7C_1(2)^6(kx) + {}^7C_2\,2^5(kx)^2 + {}^7C_3\,2^4(kx)^3 + \dots$

$= 128 + 7 \times 64kx + 672k^2x^2 + 560k^3x^3 + \dots$ (15)

$560k^3 = 1890$ (5)

$k^3 = \frac{189}{56}$

$k^3 = \frac{27}{8}$

$k = \frac{3}{2}$ (5)

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(05) $\lim_{x \to 0} \dfrac{x\tan 2x - 2x\tan x}{(1-\cos 2x)^2}$

$= \lim_{x \to 0} \dfrac{x \cdot \dfrac{2\tan x}{1-\tan^2 x} - 2x\tan x}{(1-1+2\sin^2 x)^2}$ (5)

$= \lim_{x \to 0} \dfrac{2x\tan x\,(1-1+\tan^2 x)}{(1-\tan^2 x)\cdot 4\sin^4 x}$ (5)

$= \lim_{x \to 0} \dfrac{2x\tan^3 x}{\left(1-\dfrac{\sin^2 x}{\cos^2 x}\right)\cdot 4\sin^4 x}$

$= \lim_{x \to 0} \dfrac{2x\sin^3 x\cdot\cos^2 x}{\cos^3 x\,(\cos^2 x-\sin^2 x)\cdot 4\sin^4 x}$

$= \lim_{x \to 0} \dfrac{x}{2\cos x\cdot\cos 2x\cdot\sin x}$

$= \lim_{x \to 0} \left(\dfrac{1}{2\cos x\cos 2x}\right)\cdot\dfrac{1}{\lim_{x \to 0}\left(\dfrac{\sin x}{x}\right)}$ (5)

$= \dfrac{1}{2\times1\times1}\times\dfrac{1}{1}$ (5)

$= \dfrac{1}{2}$ (5)

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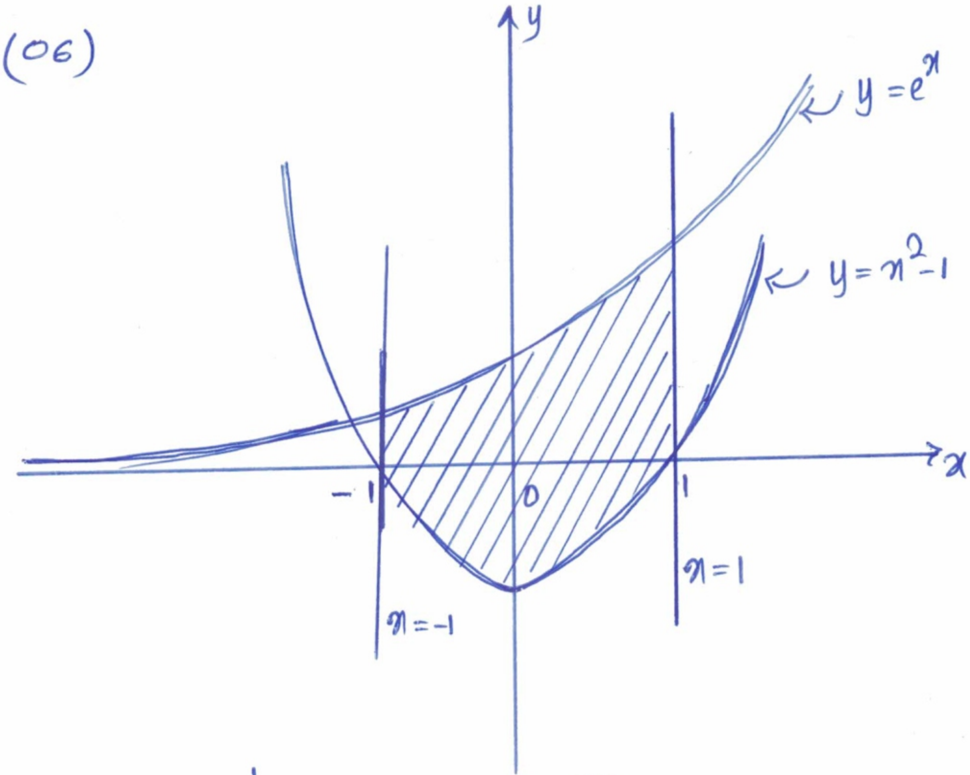
(06)



$$\text{Area} = \int_{-1}^{1} e^x - x^2 + 1 \, dx \quad (10)$$

$$= \left\{ e^x - \frac{x^3}{3} + x \right\}_{-1}^{1} \quad (5)$$

$$= \left\{ e^1 - \frac{1}{3} + 1 \right\} - \left\{ e^{-1} + \frac{1}{3} - 1 \right\} \quad (5)$$

$$= \left\{ e + \frac{2}{3} \right\} - \left\{ \frac{1}{e} - \frac{2}{3} \right\}$$

$$= e - \frac{1}{e} + \frac{4}{3}$$

$$= \frac{3e^2 + 4e - 3}{3e} \quad (5)$$

25

(07) $x = 4t - 1$

$y = \frac{5}{2t} + 10$; $t \in \mathbb{R}$, $t \neq 0$

When $y = 0$

$0 = \frac{5}{2t} + 10$

$t = -1/4$ (5)

When $t = -1/4$ then $x = 4(-\frac{1}{4}) - 1$

$= -2$

$A \equiv (-2, 0)$ (5)

$\frac{dy}{dt} = -\frac{5}{2t^2}$ $\frac{dx}{dt} = 4$

$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$

$= \frac{-5}{2t^2} \times \frac{1}{4} = \frac{-5}{8t^2}$

$\left(\frac{dy}{dx}\right) = \frac{-5}{8t^2}$ (5)

let m be the tangent of the curve at A.

$m = \left(\frac{dy}{dx}\right)_{t=-1/4} = -\frac{5 \times 16}{8} = -10$ (5)

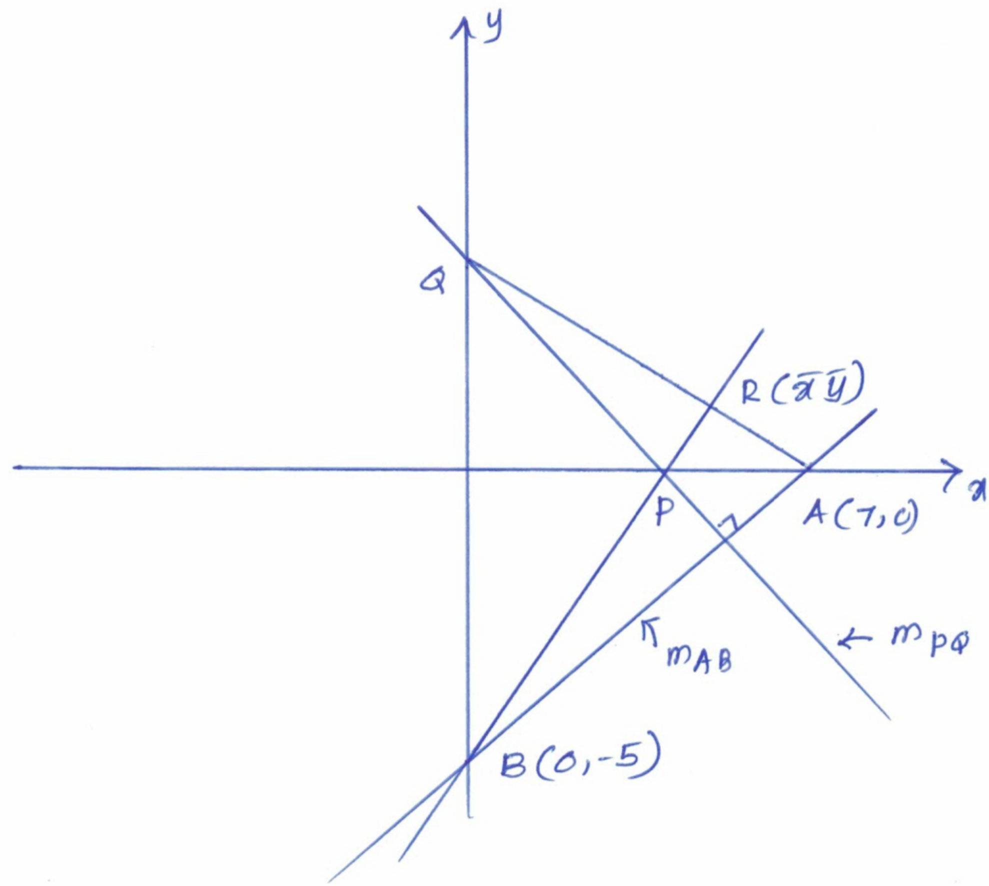
Equation of the tangent is

$\frac{y - 0}{x + 2} = -10$ (5)

$y = -10x - 20$

$y + 10x + 20 = 0$

(08)



$$m_{AB} = \frac{0+5}{7-0} = \frac{5}{7}$$

$$m_{AB} \times m_{PQ} = -1 \Rightarrow m_{PQ} = \frac{-7}{5}$$

$\therefore$ Equation of any line PQ perp. to AB is of the form $7x+5y+\lambda=0$

If $y=0$ then $x = \frac{-\lambda}{7} \Rightarrow P\left(\frac{-\lambda}{7}, 0\right)$ (5)

If $x=0$ then $y = \frac{-\lambda}{5} \Rightarrow Q\left(0, -\frac{\lambda}{5}\right)$

Let $R(\bar{x}\ \bar{y})$

AQ

$$\frac{\bar{y}-0}{\bar{x}-7} = \frac{\frac{\lambda}{5}}{7} \quad (5)$$

$$\Rightarrow \lambda = \frac{35\bar{y}}{\bar{x}-7} \quad \text{—— (1)}$$

BP

$$\frac{\bar{y}+5}{\bar{x}-0} = \frac{5}{\frac{-\lambda}{7}} \quad (5)$$

$$\lambda = \frac{-35\bar{x}}{\bar{y}+5} \quad \text{—— (2)}$$

① = ② $\dfrac{35\bar{y}}{\bar{x}-7} = \dfrac{-35\bar{x}}{\bar{y}+5}$ ⑤

$\bar{y}(\bar{y}+5) = -\bar{x}(\bar{x}-7)$

$\bar{y}^2 + 5\bar{y} - \bar{x}^2 - 7\bar{x} = 0$

$\bar{x}^2 + \bar{y}^2 - 7\bar{x} + 5\bar{y} = 0$

$\bar{x} \equiv x \quad \bar{y} \equiv y$

$x^2 + y^2 - 7x - 5y = 0$

∴ the locus of R is

$x^2 + y^2 - 7x - 5y = 0$ ⑤

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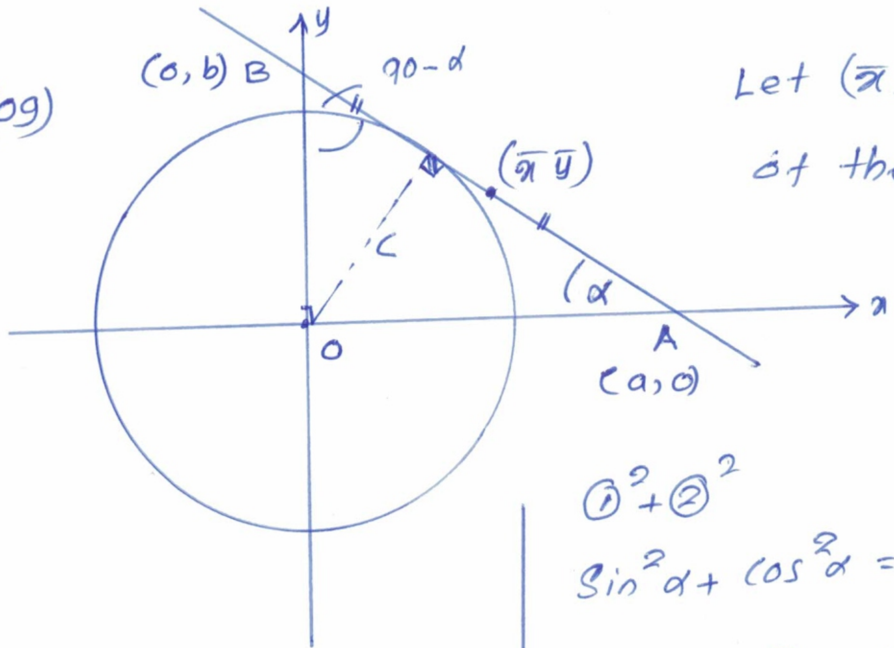
(09)



Let $(\bar{x}\ \bar{y})$ be the centre of the circle.

$\sin\alpha = \dfrac{c}{a}$ — ① ⑤

$\sin(90-\alpha) = \dfrac{c}{b}$

$\cos\alpha = \dfrac{c}{b}$ — ② ⑤

$①^2 + ②^2$

$\sin^2\alpha + \cos^2\alpha = \dfrac{c^2}{a^2} + \dfrac{c^2}{b^2}$ ⑤

$1 = \dfrac{c^2}{a^2} + \dfrac{c^2}{b^2}$ ⑤

But $\dfrac{a}{2} = \bar{x}$ and $\dfrac{b}{2} = \bar{y}$

$\Rightarrow a = 2\bar{x}$ $\Rightarrow b = 2\bar{y}$

$\therefore 1 = \dfrac{c^2}{4\bar{x}^2} + \dfrac{c^2}{4\bar{y}^2}$ ⑤

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$$\frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{c^2}$$

(10) $\sin^{-1}x - \cos^{-1}x = \sin^{-1}(3x-2)$

$\frac{\pi}{2} - \cos^{-1}x - \cos^{-1}x = \sin^{-1}(3x-2)$ (5)

$\frac{\pi}{2} - 2\cos^{-1}x = \sin^{-1}(3x-2)$

$2\cos^{-1}x = \frac{\pi}{2} - \sin^{-1}(3x-2)$

$2\cos^{-1}x = \cos^{-1}(3x-2)$ (5)

let $\cos^{-1}x = \alpha$ and $\cos^{-1}(3x-2) = \beta$

$\Rightarrow \cos\alpha = x$ $\Rightarrow \cos\beta = 3x-2$

$2\alpha = \beta$

$\cos 2\alpha = \cos\beta$

$2\cos^2\alpha - 1 = \cos\beta$

$2x^2 - 1 = 3x - 2$ (5)

$2x^2 - 3x + 1 = 0$

$2x^2 - 2x - x + 1 = 0$

$2x(x-1) - 1(x-1) = 0$

$(x-1)(2x-1) = 0$

$x = 1$ (5) or $x = \frac{1}{2}$ (5)

25

Part B

(11) a.

$x^2 - x + p = 0$ with roots α, β $\Longrightarrow$ $\alpha+\beta = 1$, $\alpha\beta = p$ (5)

$x^2 - 9x + q = 0$ with roots γ, δ $\Longrightarrow$ $\gamma+\delta = 9$, $\gamma\delta = q$ (5)

$\alpha, \beta, \gamma, \delta$ in a geometric Progression

Let $\frac{\beta}{\alpha} = \frac{\gamma}{\beta} = \frac{\delta}{\gamma} = r.$ (5)

$\Rightarrow \frac{\beta+\gamma}{\alpha+\beta} = \frac{\gamma+\delta}{\beta+\gamma} = r$ (10)

$(\beta+\gamma)^2 = (\alpha+\beta)(\gamma+\delta)$

$= 1 \times 9$

$= 9$

$(\beta+\gamma)^2 = 9$ (5)

$\beta+\gamma = \pm 3$

When $\beta+\gamma = 3$

$\frac{\beta+\gamma}{\alpha+\beta} = r$

$\frac{3}{1} = r$

$r = 3$ (5)

When $\beta+\gamma = -3$

$\frac{\beta+\gamma}{\alpha+\beta} = r$

$-\frac{3}{1} = r$

$r = -3$ (5)

$\therefore r = \pm 3$

Common ratio $= \pm 3$ (5)

45

When $r=3$

$\frac{\beta}{\alpha}=3$

$\beta=3\alpha$

$\alpha+\beta=1$

$4\alpha=1$

$\alpha=\frac{1}{4}$

$\beta=\frac{3}{4}$

$p=\frac{1}{4}\times\frac{3}{4}$

$=\frac{3}{16}$ (5)

$\frac{\delta}{\gamma}=3$

$\delta=3\gamma$

$\gamma+3\gamma=9$

$\gamma=\frac{9}{4}$

$\delta=\frac{27}{4}$

$q=\frac{9}{4}\times\frac{27}{4}$

$=\frac{243}{16}$ (5)

When $r=-3$

$\frac{\beta}{\alpha}=-3$

$\beta=-3\alpha$

$\alpha+\beta=1$

$\alpha-3\alpha=1$

$\alpha=-\frac{1}{2}$

$\beta=\frac{3}{2}$

$p=-\frac{1}{2}\times\frac{3}{2}$

$p=\frac{-3}{4}$ (5)

$\frac{\delta}{\gamma}=-3$

$\delta=-3\gamma$

$\gamma-3\gamma=9$

$\gamma=\frac{-9}{2}$

$\delta=\frac{27}{2}$

$q=\frac{-9}{2}\times\frac{27}{2}$

$q=\frac{-243}{4}$ (5)

20

$\alpha\gamma=\frac{9}{16}$ (5)

$\beta\delta=\frac{81}{16}$

$\alpha\gamma+\beta\delta=\frac{90}{16}$

$\alpha\gamma\beta\delta=\frac{9}{16}\times\frac{81}{16}$ (5)

$=\frac{724}{256}$

Equation is

$x^2-\frac{90}{16}x+\frac{724}{256}=0$ (5)

$\alpha\gamma=\frac{9}{4}$ (5)

$\beta\delta=\frac{81}{4}$

$\alpha\gamma+\beta\delta=\frac{90}{4}$ (5)

$\alpha\gamma\beta\delta=\frac{729}{16}$

Equation is

$x^2-\frac{90}{4}x+\frac{729}{16}=0$ (5)

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b) $g(0) = -12$

$g(1) = -8$

$g(-1) = -24$ (20)

$g(2) = -6$

$Q(x) = (x+2)g(x) + 24$

When $x = 0$

$Q(0) = 2g(0) + 24$

$= 2 \times (-12) + 24$

$= -24 + 24$

$= 0$

$\therefore$ x is a factor of $Q(x)$ (5)

When $x = 1$

$Q(1) = 3g(1) + 24$

$= 3 \times (-8) + 24$

$= -24 + 24$

$= 0$

$\therefore (x-1)$ is a factor of $Q(x)$ (5)

When $x = -1$

$Q(-1) = g(-1) + 24$

$= -24 + 24$

$= 0$

$\therefore (x+1)$ is a factor of $Q(x)$ (5)

When $x = 2$

$Q(2) = 4g(2) + 24$

$= 4 \times -6 + 24$

$= 0$

$\therefore (x-2)$ is a factor of $Q(x)$ (5)

$(x+2)g(x) + 24 = x(x-1)(x+1)(x-2)\lambda$ (5)

When $x = -2$

$0 + 24 = (-2)(-3)(-1)(-4)\lambda$

$24 = 24\lambda$

$\lambda = 1$ (5)

$Q(x) = x(x-1)(x+1)(x-3)$ (5)

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(12) a (i) 3 bowlers can be selected out of 4 = $^4C_3$ ways (5)

1 wicket keeper can be selected out of 2 = $^2C_1$ ways (5)

the other 7 players can be selected from the remaining 10 players } = $^{10}C_7$ (5)

Hence the total number of ways in which the cricket team can be formed

$= {}^4C_3 \times {}^2C_1 \times {}^{10}C_7$ (5)

$= \dfrac{4!}{3!(4-3)!} \times \dfrac{2!}{1!1!} \times \dfrac{10!}{7!3!}$

$= 4 \times 2 \times 120$

$= 960$ (5)

(25)

Bowlers	Wicket keeper	players	
3	1	7	$= {}^4C_3 \times {}^2C_1 \times {}^{10}C_7$
3	2	6	$= {}^4C_3 \times {}^2C_2 \times {}^{10}C_6$
4	1	6	$= {}^4C_4 \times {}^2C_1 \times {}^{10}C_6$
4	2	5	$= {}^4C_4 \times {}^2C_1 \times {}^{10}C_5$

(5)

${}^4C_3 \times {}^2C_1 \times {}^{10}C_7 = \dfrac{4!}{3!1!} \times \dfrac{2!}{1!1!} \times \dfrac{10!}{7!\times 3!} = 4 \times 2 \times 120 = 960$ ways (5) (5)

${}^4C_3 \times {}^2C_2 \times {}^{10}C_6 = \dfrac{4!}{3!1!} \times \dfrac{2!}{2!0!} \times \dfrac{10!}{6!4!} = 4 \times 1 \times 210 = 840$ way (5) (5)

${}^4C_4 \times {}^2C_1 \times {}^{10}C_6 = \dfrac{4!}{4!0!} \times \dfrac{2!}{1!1!} \times \dfrac{10!}{6!4!} = 1 \times 2 \times 210 = 420$ ways (5) (5)

(5)

$${}^4C_4 \times {}^2C_2 \times {}^{10}C_5 = \frac{4!}{0!4!} \times \frac{2}{0!2!} \times \frac{10!}{5!5!} = 1 \times 1 \times 252 = 252 \ ways$$

Total number of ways = 960+840+420+252=2472 (5)

(45)

(b) $\frac{1}{2}, \frac{1\cdot 3}{2\cdot 4}, \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}, \frac{1\cdot 3\cdot 5\cdot 7}{2\cdot 4\cdot 6\cdot 8}, \ldots$

$$U_r = \frac{1\cdot 3\cdot 5\cdot 7\cdot \ldots\ldots (2r-1)}{2\cdot 4\cdot 6\cdot 8\cdot \ldots\ldots 2r} \quad (5)$$

$$U_{r+1} = \frac{(2r+1)}{(2r+2)} U_r \quad (5)$$

(10)

$f(r) = (Ar + B) U_r$

$f(r+1) - f(r) = U_r$

$\{A(r+1) + B\} U_{r+1} - \{Ar + B\} U_r = U_r$ (5)

$\{Ar + A + B\} \cdot \frac{(2r+1)}{(2r+2)} U_r - \{Ar + B\} U_r = U_r$

$(Ar + A + B)(2r+1) - (Ar + B)(2r+2) = (2r+2)$

$2Ar^2 + 2Ar + 2Br + Ar + A + B - 2Ar^2 - 2Br - 2Ar - 2B = 2r + 2$

$Ar + A - B = 2r + 2$

$r \Rightarrow A = 2$ (5)

constant $\Rightarrow A - B = 2$

$2 - B = 2$

$B = 0$ (5)

$f(r) = 2r U_r$ (5)

(20)

$$f(r+1) - f(r) = u_r$$

$r=1$; $f(2) - f(1) = u_1$

$r=2$; $f(3) - f(2) = u_2$ (15)

$r=3$; $f(4) - f(3) = u_3$

$\vdots \qquad \vdots \qquad \vdots$

$r=n-1$; $f(n) - f(n-1) = u_{n-1}$

$r=n$; $f(n+1) - f(n) = u_n$ (10)

$$f(n+1) - f(1) = \sum_{r=1}^{n} u_r \quad (5)$$

$$\sum_{r=1}^{n} u_r = 2(n+1) \cdot u_{n+1} - 2u_1 \quad (5) \ (5)$$

$$= (2n+2)\left[\frac{1\cdot 3\cdot 5\cdot 7 \cdots (2n-1)(2n+1)}{2\cdot 4\cdot 6\cdot 8 \cdots 2n\,(2n+2)}\right] - 2 \times \frac{1}{2} \quad (5)$$

$$= \frac{1\cdot 3\cdot 5\cdot 7 \cdots (2n+1)}{2\cdot 4\cdot 6\cdot 8 \cdots 2n} - 1 \quad (5)$$

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(13) $A = \begin{pmatrix} 2 & a \\ 3 & b \end{pmatrix}$ $B = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $C = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

$$AB = \begin{pmatrix} 2 & a \\ 3 & b \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} 2+a \\ 3+b \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \quad (5)$$

$2+a=1$ $3+b=3$

$\underline{\underline{a=-1}}$ (5) $\underline{\underline{b=0}}$ (5) [15]

$$A^2 = \begin{pmatrix} 2 & -1 \\ 3 & 0 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ 3 & 0 \end{pmatrix} = \begin{pmatrix} 4-3 & 2+0 \\ 6+0 & -3+0 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 1 & -2 \\ 6 & -3 \end{pmatrix}}} \quad (5) \text{ — } (1)$$

$$2A-3I = 2\begin{pmatrix} 2 & -1 \\ 3 & 0 \end{pmatrix} - 3\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 4-3 & -2-0 \\ 6-0 & 0-3 \end{pmatrix}$$

$$= \underline{\underline{\begin{pmatrix} 1 & -2 \\ 6 & -3 \end{pmatrix}}} \quad (5) \text{ — } (2)$$

(1) = (2)

$\therefore$ $\underline{\underline{A^2 = 2A-3I}}$ (5) [15]

$A^2 \cdot A = 2AA - 3IA$ (5)

$A^3 = 2A^2 - 3A$

$= 2(2A-3I) - 3A$ (5)

$= 4A - 6I - 3A$

$= \underline{\underline{A - 6I}}$ (5) [15]

$A^2 = 2A - 3I$

$AAA^{-1} = 2AA^{-1} - 3IA^{-1}$ (5)

$A = 2I - 3A^{-1}$ (5)

$3A^{-1} = 2I - A$

$A^{-1} = \frac{1}{3}(2I - A)$ 10

$A^{-1} = \frac{1}{3}\left\{2\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} - \begin{pmatrix}2 & -1\\ 3 & 0\end{pmatrix}\right\}$ (5)

$= \frac{1}{3}\left\{\begin{pmatrix}2 & 0\\ 0 & 2\end{pmatrix} - \begin{pmatrix}2 & -1\\ 3 & 0\end{pmatrix}\right\}$

$= \frac{1}{3}\begin{pmatrix}2-2 & 0+1\\ 0-3 & 2-0\end{pmatrix}$

$= \frac{1}{3}\begin{pmatrix}0 & 1\\ -3 & 2\end{pmatrix}$ (5) 10

(b) i. $\underline{u} = \frac{(1+2i)^2}{2+i}$

$= \frac{1+4i+4i^2}{2+i}$

$= \frac{1+4i-4}{2+i}$

$= \frac{-3+4i}{2+i}$

$= \frac{(-3+4i)(2-i)}{2^2 - i^2}$

$\underline{u} = \frac{-6+3i+8i-4i^2}{4+1}$

$= \frac{-6+11i+4}{5}$

$= \frac{-2+11i}{5}$

$= \left(-\frac{2}{5}\right) + \left(\frac{11}{5}\right)i$

$= -\frac{2}{5} + \frac{11}{5}i$ 10

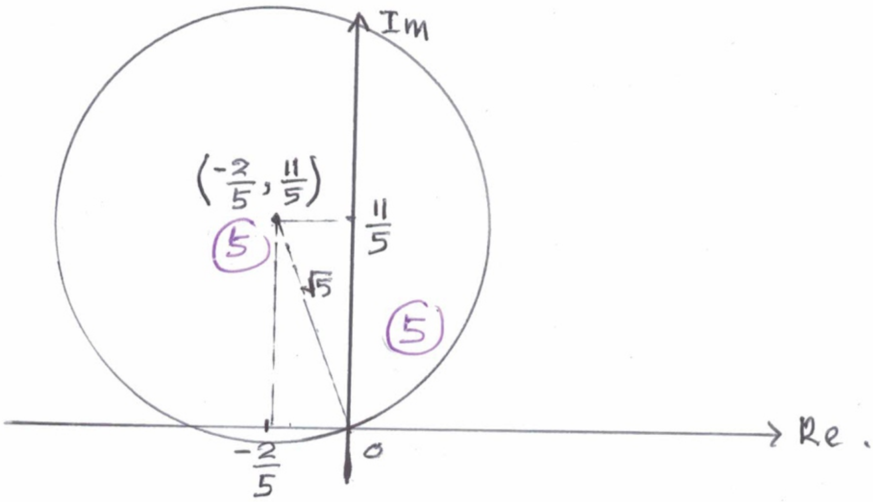
$$|z-u| = |u|$$

$$\left|z - \left(-\frac{2}{5} + \frac{11}{5}i\right)\right| = \sqrt{\left(\frac{2}{5}\right)^2 + \left(\frac{11}{5}\right)^2} \quad (5)$$

$$= \frac{1}{5}\sqrt{4+121}$$

$$= \frac{1}{5} \times 5\sqrt{5}$$

$$= \sqrt{5} \quad (5)$$



(20)

(ii) let $\sqrt{7-6\sqrt{2}i} = x + yi$ (5)

$$7 - 6\sqrt{2}i = (x+yi)^2$$

$$7 - 6\sqrt{2}i = x^2 + 2xyi + y^2i^2$$

$$7 - 6\sqrt{2}i = (x^2 - y^2) + 2xyi$$

$$x^2 - y^2 = 7 \quad (5) \text{ ——— } (1)$$

$$2xy = -6\sqrt{2} \quad (5)$$

$$xy = -3\sqrt{2}$$

$(x^2)^2 - 7x^2 - 18 = 0$

$(x^2-9)(x^2+2) = 0$

$x^2+2 \neq 0 \quad \because x, y \in \mathbb{R}$ (5)

$x^2 - 9 = 0$

$\Rightarrow x = 3$ or $x = -3$ (5)

When $x = 3$

$y = \frac{-3\sqrt{2}}{3}$

$= -\sqrt{2}$

When $x = -3$

$y = \frac{-3\sqrt{2}}{-3}$

$= \sqrt{2}$

$\therefore \sqrt{7-6\sqrt{2}\,i} = 3-\sqrt{2}\,i$ or $\sqrt{7-6\sqrt{2}\,i} = -3+\sqrt{2}\,i$ (5)

△ 30

(iii) Let $z = 2\sqrt{3} - 2i$

$|z| = \sqrt{(2\sqrt{3})^2 + 2^2}$

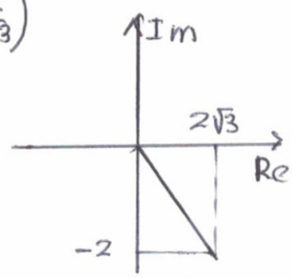
$= 2\sqrt{3+1}$

$= 4$ (5)

$\arg(z) = 2\pi - \tan^{-1}\left(\frac{2}{2\sqrt{3}}\right)$

$= 2\pi - \frac{\pi}{6}$

$= \frac{11\pi}{6}$ (5)



△ 10

$z^3 = 2\sqrt{3} - 2i$

$= 4\left[\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right]$

$z_1 = \sqrt[3]{4}\left[\cos\left(\frac{11\pi}{18}\right) + i\sin\left(\frac{11\pi}{18}\right)\right]$ (5)

$z_2 = \sqrt[3]{4}\left[\cos\left(\frac{2\pi + \frac{11\pi}{6}}{3}\right) + i\sin\left(\frac{2\pi + \frac{11\pi}{6}}{3}\right)\right]$

$z_2 = \sqrt[3]{4}\left[\cos\left(\frac{23\pi}{18}\right) + i\sin\left(\frac{23\pi}{18}\right)\right]$ (5)

(19) $z_3 = \sqrt[3]{4}\left[\cos\left(\frac{35\pi}{18}\right) + i\sin\left(\frac{35\pi}{18}\right)\right]$ (5)

△ 15   150

(14) a. $f(x) = \dfrac{(x+1)(x-2)}{(x-1)^2}$; $x \neq 1$

$$f'(x) = \frac{(x-1)^2(2x-1) - (x+1)(x-2)\cdot 2(x-1)}{(x-1)^4}$$ (20)

$$= \frac{(x-1)\{(x-1)(2x-1) - 2(x+1)(x-2)\}}{(x-1)^4}$$

$$= \frac{2x^2 - x - 2x + 1 - 2x^2 + 2x + 4}{(x-1)^3}$$

$$= \frac{-x+5}{(x-1)^3}$$ (5)

[25]

$f'(x) = 0 \Leftrightarrow x = 5$ (5)

vertical asymptote ; $x = 1$

	$-\infty < x < 1$	$1 < x < 5$	$5 < x < \infty$
Sign of $f'(x)$	$(-)$	$(+)$	$(-)$
$f(x)$ is	↘ Decreasing	↗ Increasing	↘ Decreasing
	(5)	(5)	(5)

$\therefore$ $f(x)$ is increasing on $(1, 5]$ and decreasing on $(-\infty, 1)$ and $[5, \infty)$

[20]

Turning point $\left(5, \frac{9}{8}\right)$ (5)

[5]

For $x \neq 1$; $f''(x) = \dfrac{2(x-7)}{(x-1)^4}$

$f'(x) = 0 \quad x = 7$ (5)

	$1 < x < 7$	$7 < x < \infty$
Sign of $f''(x)$	$(-)$	$(+)$
Concavity	concave down	concave up.

⑤ ⑤

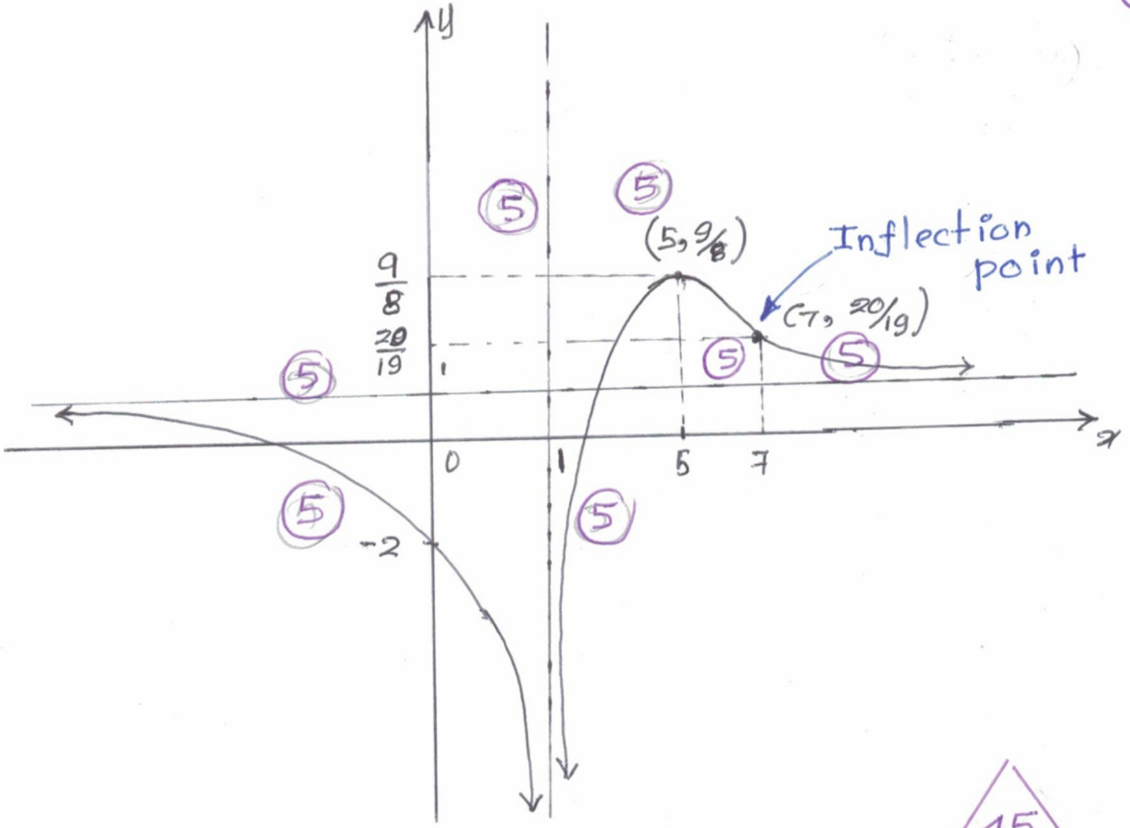
$\therefore$ Point of inflection $\left(7, \frac{20}{19}\right)$ ⑤

20

Horizontal asymptote ;

$\lim_{x \to \pm\infty}$; $y = 1$ ⑤

Vertical asymptote ; $y = 1$ ⑤



45

(b) strength $\propto wd^2$

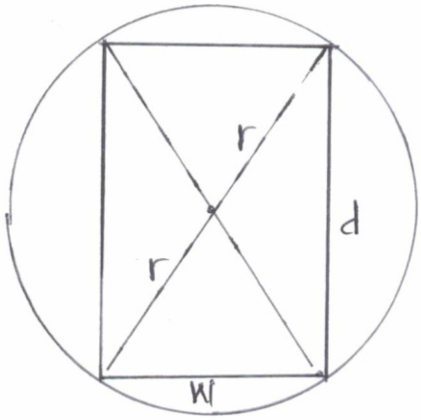
strength $(S) = kwd^2$

$S = kw(4r^2 - w^2)$

$S = 4kwr^2 - kw^3$ (5)

$\frac{dS}{dw} = 4kr^2 - 3kw^2$ (5)

$= k(4r^2 - 3w^2)$



$(2r)^2 = w^2 + d^2$

$4r^2 = w^2 + d^2$

$d^2 = 4r^2 - w^2$

For maximum or minimum strength ; $\frac{dS}{dw} = 0$

$k(4r^2 - 3w^2) = 0$

$3w^2 = 4r^2$

$w = \frac{2r}{\sqrt{3}}$ (10)

	$0 < w < \frac{2r}{\sqrt{3}}$	$\frac{2r}{\sqrt{3}} < w$
Sign of ds/dw	+	−

(5)

When $w = \frac{2r}{\sqrt{3}}$; strength is maximum. (5)

then $d = \sqrt{4r^2 - \frac{4r^2}{3}}$

$d = \sqrt{\frac{8r^2}{3}} = \frac{2\sqrt{2}r}{\sqrt{3}}$ (5)

35

(15) $\dfrac{1}{x(x^2+1)} = \dfrac{A}{x} + \dfrac{Bx+C}{x^2+1}$

$1 \equiv A(x^2+1) + x(Bx+C)$

$x^2 \rightsquigarrow A+B=0$

$x \rightsquigarrow \underline{\underline{C=0}}$ (5)

constant $\rightsquigarrow \underline{\underline{A=1}}$ (5)

$\underline{\underline{B=-1}}$ (5)

15

$\dfrac{1}{x(x^2+1)} = \dfrac{1}{x} - \dfrac{x}{x^2+1}$

$\displaystyle\int \frac{1}{x(x^2+1)}\,dx = \int \frac{1}{x}\,dx - \int \frac{x}{x^2+1}\,dx$

$\displaystyle = \int \frac{1}{x}\,dx - \frac{1}{2}\int \frac{2x}{x^2+1}\,dx$ (5) (5)

$= \underline{\underline{\ln|x| - \frac{1}{2}\ln|x^2+1| + C}}$ (5)

15

$x = \cos\theta$

$\dfrac{dx}{d\theta} = -\sin\theta$ (5)

$\displaystyle\int \frac{-\sin\theta}{\cos\theta(1+\cos^2\theta)}\,d\theta = \ln|\cos\theta| - \frac{1}{2}\ln|1+\cos^2\theta|$

$\displaystyle\int \frac{2\sin\theta}{\cos\theta(1+\cos^2\theta)}\,d\theta = \ln|1+\cos^2\theta| - 2\ln|\cos\theta| + C$ (5)

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{2\sin\theta}{\cos\theta + \cos^3\theta}\, d\theta = \Big\{ \ln|1+\cos^2\theta| \Big\}_{\pi/4}^{\pi/3} - \Big\{ 2\ln|\cos\theta| \Big\}_{\pi/4}^{\pi/3}$$

$$= \Big\{ \ln|1+\cos^2 \tfrac{\pi}{3}| - \ln|1+\cos^2 \tfrac{\pi}{4}| \Big\} - 2\Big\{ \ln|\cos \tfrac{\pi}{3}| - \ln|\cos \tfrac{\pi}{4}| \Big\} \quad (5)$$

$$= \ln|1+\tfrac{1}{4}| - \ln|1+\tfrac{1}{2}| - 2\Big\{ \ln|\tfrac{1}{2}| - \ln|\tfrac{1}{\sqrt{2}}| \Big\} \quad (5)\ (5)$$

$$= \ln\left|\tfrac{5}{4}\right| - \ln\left|\tfrac{3}{2}\right| - 2\Big\{ \ln\left|\tfrac{1}{2} \times \sqrt{2}\right|$$

$$= \ln\left| \tfrac{5}{4} \times \tfrac{2}{3} \right| - 2\ln\left|\tfrac{1}{\sqrt{2}}\right|$$

$$= \ln\left|\tfrac{5}{6}\right| - \ln\left|\tfrac{1}{2}\right|$$

$$= \ln\left(\tfrac{5}{6} \times 2 \right)$$

$$= \ln\left(\tfrac{5}{3} \right) \quad (5)$$

[30]

b) $$\int e^{-2x} \sin \pi x \, dx = \int \sin \pi x \frac{d}{dx}\left(\frac{e^{-2x}}{-2} \right) dx \quad (5)$$

$$= -\frac{1}{2} e^{-2x} \sin \pi x - \int \frac{e^{-2x}}{2} \cos \pi x \cdot \pi \, dx \quad (5)$$

$$= -\frac{1}{2} e^{-2x} \sin \pi x - \frac{\pi}{2} \int \cos \pi x \frac{d}{dx}\left(\frac{e^{-2x}}{-2} \right) dx \quad (5)$$

$$= -\frac{1}{2} e^{-2x} \sin \pi x - \frac{\pi}{2} \Big\{ -\frac{1}{2} e^{-2x} \cos \pi x + \int \frac{e^{-2x}}{-2} \sin \pi x \cdot \pi \, dx \quad (5)$$

$$= -\frac{1}{2} e^{-2x} \sin \pi x + \frac{\pi}{4} e^{-2x} \cos \pi x - \frac{\pi^2}{4} \int e^{-2x} \sin \pi x \, dx$$

$$\left(1 + \frac{\pi^2}{4}\right) \int e^{-2x} \sin \pi x \, dx = \frac{1}{4} e^{-2x} \left(\pi \cos \pi x - 2 \sin \pi x \right) \quad (5)$$

[35]

$$\int e^{-2x} \sin \pi x \, dx = \frac{e^{-2x}}{\pi^2 + 4} \Big\{ \pi \cos \pi x - 2 \sin \pi x \Big\} \quad (5)$$

c) Theorem

$$\int_0^{\pi} f(x)\,dx = \int_0^{\pi} f(\pi - x)\,dx \quad \boxed{10}$$

$$I = \int_0^{\pi} x \sin^3 x\,dx$$

$$= \int_0^{\pi} (\pi - x) \sin^3(\pi - x)\,dx \quad (5)$$

$$= \int_0^{\pi} (\pi - x) \sin^3 x\,dx$$

$$= \int_0^{\pi} \pi \sin^3 x\,dx - \int_0^{\pi} x \sin^3 x\,dx \quad (5)$$

$$= \pi \int_0^{\pi} \sin^3 x\,dx - I$$

$$2I = \pi \int_0^{\pi} \sin^3 x\,dx \quad (5)$$

$$I = \frac{\pi}{2} \int_0^{\pi} \sin^3 x\,dx$$

$$\therefore I = \frac{\pi}{2} \int_0^{\pi} \sin^3 x\,dx \quad (5)$$

$$I = \frac{\pi}{2} \int_0^{\pi} \sin x\,(1 - \cos^2 x)\,dx \quad (5)$$

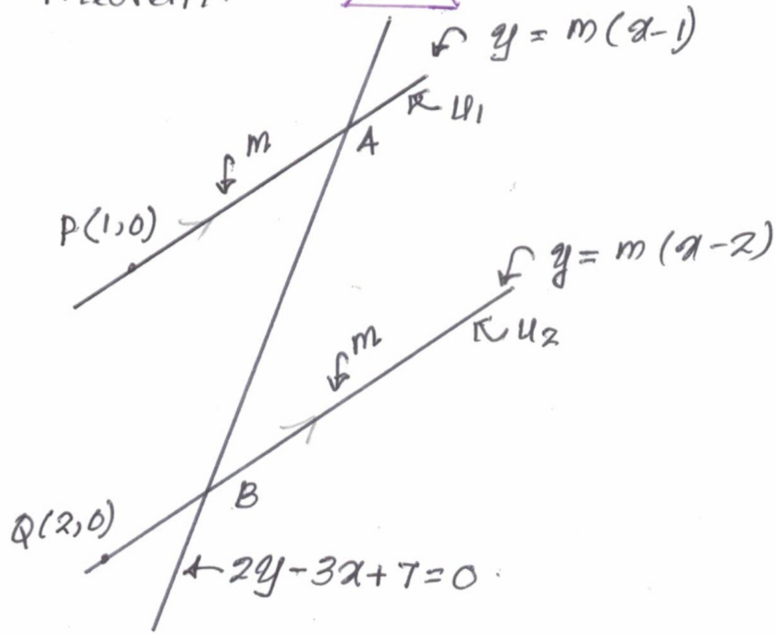
$$= \frac{\pi}{2} \left\{ \int_0^{\pi} \sin x\,dx + \int_0^{\pi} \cos^2 x\,(-\sin x)\,dx \right\} \quad (5)\ (5)$$

$$= \frac{\pi}{2} \left\{ \left(-\cos x\right)_0^{\pi} + \left(\frac{\cos^3 x}{3}\right)_0^{\pi} \right\} \quad (5)\ (5)$$

$$= \frac{\pi}{2} \left\{ \left[-1 - (1)\right] + \frac{1}{3}\left[(-1)^3 - 1\right] \right\}$$

$$= \frac{\pi}{2} \left\{ +2 - \frac{2}{3} \right\} = \frac{\pi}{2} \times \frac{4}{3} = \frac{2\pi}{3} \quad (5) \quad \boxed{50}$$

(16) Theorem. (25)



A

$y = m(x-1)$ (5)

$2y - 3x + 7 = 0$

$2(x-1)m - 3x + 7 = 0$

$(2m-3)x - 2m + 7 = 0$

$x = \dfrac{2m-7}{2m-3}$ (5)

$y = m\left\{\dfrac{2m-7}{2m-3} - 1\right\}$

$y = m\left\{\dfrac{2m-7-2m+3}{2m-3}\right\}$

$y = \dfrac{-4m}{2m-3}$ (5)

$A \equiv \left(\dfrac{2m-7}{2m-3}, \dfrac{-4m}{2m-3}\right)$

B

$y = m(x-2)$ (5)

$2y - 3x + 7 = 0$

$2m(x-2) - 3x + 7 = 0$

$(2m-3)x = 4m - 7$

$x = \dfrac{4m-7}{2m-3}$ (5)

$y = m\left\{\dfrac{4m-7}{2m-3} - 2\right\}$

$y = m\left\{\dfrac{4m-7-4m+6}{2m-3}\right\}$

$y = \dfrac{-m}{2m-3}$ (5)

$B \equiv \left(\dfrac{4m-7}{2m-3}, \dfrac{-m}{2m-3}\right)$

$$(AB)^2 = \left(\frac{2m-7}{2m-3} - \frac{4m-7}{2m-3}\right)^2 + \left(\frac{-4m}{2m-3} + \frac{m}{2m-3}\right)^2 \quad (5)$$

$$(\sqrt{13})^2 = \left(\frac{-2m}{2m-3}\right)^2 + \left(\frac{-3m}{2m-3}\right)^2 \quad (5)$$

$$13(2m-3)^2 = 4m^2 + 9m^2$$

$$13(2m-3)^2 - 13m^2 = 0$$

$$(2m-3)^2 - m^2 = 0$$

$$(2m-3-m)(2m-3+m) = 0$$

$$\underline{\underline{m = 3 \quad \text{or} \quad m = 1}} \quad (5)$$

When $m=1$

$A \equiv \left(\frac{-5}{-1}, \frac{-4}{-1}\right)$ $\qquad$ $B \equiv \left(\frac{-3}{-1}, \frac{-1}{-1}\right)$

$\underline{\underline{A \equiv (5, 4)}}$ (5) $\qquad$ $\underline{\underline{B \equiv (3, 1)}}$ (5)

When $m=3$

$A \equiv \left(\frac{-1}{3}, \frac{-12}{3}\right)$ $\qquad$ $B \equiv \left(\frac{-1}{3}, \frac{-3}{3}\right)$

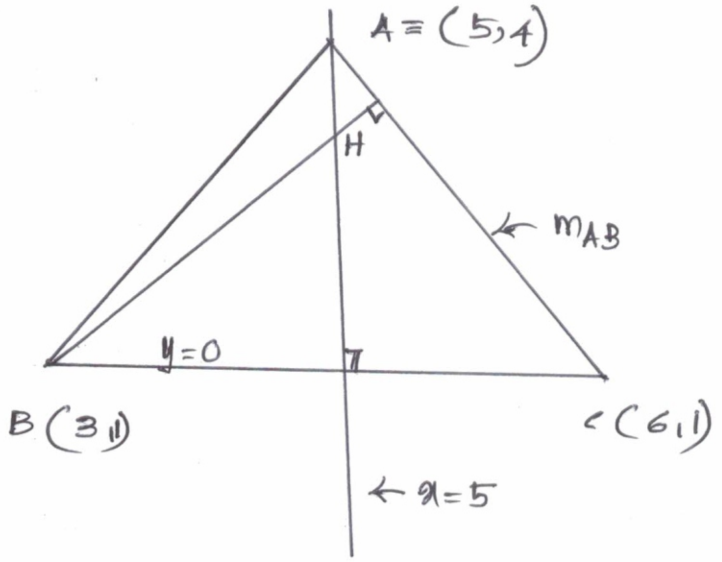
$\underline{\underline{A \equiv \left(-\frac{1}{3}, -4\right)}}$ $\qquad$ $\underline{\underline{B \equiv \left(-\frac{1}{3}, -1\right)}}$

$\qquad$

$a, b \in \mathbb{Z}$ $\qquad$ $a, b \in \mathbb{Z}$ (5)

$\therefore$ $\underline{\underline{A(5, 4) \text{ and } B(3, 1)}}$

60



$$m_{AB} = \frac{4-1}{5-6}$$

$$= -3 \quad (5)$$

$$m_{AB} \cdot m_{BH} = -1$$

$$(-3)\, m_{BH} = -1$$

$$m_{BH} = \frac{1}{3} \quad (5)$$

Equation of BH

$$\frac{y-1}{x-3} = \frac{1}{3} \Rightarrow 3y - 3 = x - 3$$

$$3y = x \quad (5)$$

When $x = 5$ $\quad y = \frac{5}{3}$ (5)

$$\therefore H \equiv \left(5, \frac{5}{3}\right) \quad (5)$$

Finding the equation of the circle whose diameter is AH

$A(5, 4)$ and $H\left(5, \frac{5}{3}\right)$

$$\left(\frac{y-4}{x-5}\right)\left(\frac{y-\frac{5}{3}}{x-5}\right) = -1 \quad (5)$$

$$y^2 - \frac{5}{3}y - 4y + \frac{20}{3} = -x^2 - 25 + 10x$$

$$x^2 + y^2 - 10x - \frac{17}{3}y + \frac{95}{3} = 0 \quad (5)$$

Finding the equation of the circle whose diameter is BC.

B (3,1) and C (6,1)

$$\left(\frac{y-1}{x-3}\right)\left(\frac{y-1}{x-6}\right) = -1$$

$$y^2 - 2y + 1 = -x^2 + 9x - 18$$

$$x^2 + y^2 - 9x - 2y + 19 = 0 \quad (5)$$

$$x^2 + y^2 - 10x - \frac{17}{3}y + \frac{95}{3} = 0 \qquad x^2 + y^2 - 9x - 2y + 19 = 0$$

$$g = -5 \quad f = -\frac{17}{6} \quad c = \frac{95}{3} \quad (5) \qquad g' = -\frac{9}{2} \quad f' = -1 \quad c' = 19 \quad (5)$$

$$2gg' + 2ff' = c + c' \quad (5)$$

$$2(-5)\left(\frac{-9}{2}\right) + 2\left(\frac{-17}{6}\right)(-1) = \frac{95}{3} + 19$$

$$45 + \frac{17}{3} = \frac{95 + 57}{3}$$

$$\frac{152}{3} = \frac{153}{3} \quad (5)$$

(5)

∴ Two circles intersect each other orthogonally

65

(17) a. L.H.S $= \sin^2 5° + \sin^2 10° + \sin^2 15° + \cdots + \sin^2 90$

$= (\sin^2 5° + \sin^2 85°) + (\sin^2 10° + \sin^2 80°) + \cdots$
$+ (\sin^2 40 + \sin^2 50) + \sin^2 45° + \sin^2 90$ (5)

$= (\sin^2 5° + \cos^2 5°) + (\sin^2 10 + \cos^2 10) + \cdots$
$+ (\sin^2 40 + \cos^2 40°) + \frac{1}{2} + 1$ (5)

(5)
$= (1 + 1 + 1 + \cdots 8 \text{ times}) + \frac{3}{2}$

$= 8 + \frac{3}{2} = 9\frac{1}{2}$ (5)

20

b) Theorem – 20

Let $\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13} = k.$

$b + c = 11k$

$c + a = 12k$

$a + b = 13k$

$b = 6k \quad c = 5k \quad a = 7k$

$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ (5)

$= \frac{36k^2 + 25k^2 - 49k^2}{2(6k)(5k)}$

$= \frac{12}{60}$

$= \frac{1}{5}$ (5)

$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$ (5)

$= \frac{49k^2 + 25k^2 - 36k^2}{2(7k)(5k)}$

$= \frac{38}{70}$

$= \frac{19}{35}$ (5)

$\cos C = \frac{a^2 + b^2 - c^2}{2ac}$ (5)

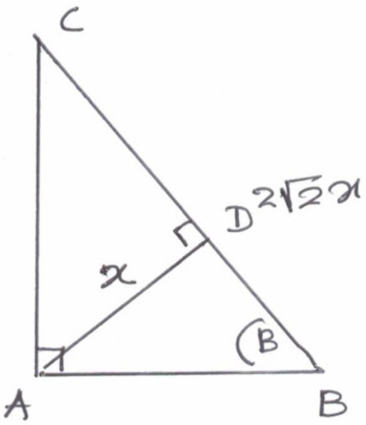
$= \frac{49k^2 + 36k^2 - 25k^2}{2(7k)(6k)}$

$= \frac{60}{84} = \frac{5}{7}$ (5)

$\cos A : \cos B : \cos C =$
$\frac{1}{5} : \frac{19}{35} : \frac{5}{7}$
$= 7 : 19 : 25$ (5)

30

c)



In Δ ABD

$$\cot B = \frac{BD}{x}$$

$$BD = x\cot B \quad (5) \quad \text{—— (1)}$$

In Δ ACD

$$\cot C = \frac{CD}{x}$$

$$CD = x\cot C \quad (5) \quad \text{—— (1)}$$

(1) + (2) $\quad BD + CD = x\cot B + x\cot C$

$$2\sqrt{2}x = x\cot B + x\cot C$$

$$\cot B + \cot C = 2\sqrt{2} \quad (5)$$

$$\frac{\cos B}{\sin B} + \frac{\cos C}{\sin C} = 2\sqrt{2}$$

$$\frac{\sin C\cos B + \cos C\sin B}{\sin B\sin C} = 2\sqrt{2}$$

$$\frac{\sin(B+C) \ (5)}{\sin B\sin C} = 2\sqrt{2}$$

$A + B + C = \pi$

$\frac{\pi}{2} + B + C = \pi$

$B + C = \frac{\pi}{2}$

$$\frac{\sin\left(\frac{\pi}{2}\right) \ (5)}{\sin B\sin C} = 2\sqrt{2}$$

$$\frac{1}{\sin B\sin C} = 2\sqrt{2}$$

$$\sin B\sin C = \frac{1}{2\sqrt{2}}$$

$$\sin B \sin\left(\frac{\pi}{2} - B\right) = \frac{1}{2\sqrt{2}}$$

$$2 \sin B \cos B = \frac{1}{\sqrt{2}} \quad (5)$$

$$\sin 2B = \frac{1}{\sqrt{2}} = \sin\frac{\pi}{4}$$

$$2B = \frac{\pi}{4} \quad ; 0 < B < \frac{\pi}{2}$$

$$B = \frac{\pi}{8} \quad (5)$$

$$\therefore \quad C = \frac{\pi}{2} - \frac{\pi}{8} = \frac{3\pi}{8} \quad (5)$$

40

d)

$$81^{\sin^2 x} + 81^{\cos^2 x} = 30$$

$$81^{\sin^2 x} + 81^{1-\sin^2 x} = 30 \quad (5)$$

$$81^{\sin^2 x} + \frac{81}{81^{\sin^2 x}} = 30$$

let $y = 81^{\sin^2 x}$

$$y + \frac{81}{y} = 30$$

$$y^2 - 30y + 81 = 0 \quad (5)$$

$$(y-3)(y-27) = 0$$

$$y = 3 \quad \text{or} \quad y = 27 \quad (5)$$

$$81^{\sin^2 x} = 3$$

$$(3^4)^{\sin^2 x} = 3$$

$$3^{4\sin^2 x} = 3^1$$

$$81^{\sin^2 x} = 27$$

$$81^{\sin^2 x} = 3^3$$

$$(3^4)^{\sin^2 x} = 3^3$$

$$3^{4\sin^2 x} = 3^3$$

$4\sin^2 x = 1$ (5)

$\sin x = \pm\frac{1}{2}$

$\sin x = \sin\left(\pm\frac{\pi}{6}\right)$

$x = n\pi + (-1)^n\left(\pm\frac{\pi}{6}\right);$

$x = n\pi \pm (-1)^n\frac{\pi}{6};\ n \in \mathbb{Z}$ (5)

$4\sin^2 x = 3$ (5)

$\sin x = \pm\frac{\sqrt{3}}{2}$

$\sin x = \sin\left(\pm\frac{\pi}{3}\right)$

$x = n\pi \pm (-1)^n\frac{\pi}{3};\ n \in \mathbb{Z}$ (5)

When $n=0$

$x = \pi/6$

When $n=1$

$x = \pi - \pi/6$

$= 5\pi/6$

When $n=0$

$x = \pi/3$

When $n=1$

$x = \pi - \pi/3$

$= 2\pi/3$

$x = \left\{\pi/6, \pi/3, 2\pi/3, 5\pi/6\right\}$ (5)

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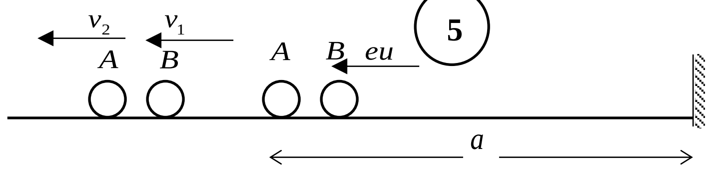
Provincial Department of Education – NWP

Third Term Test Grade 13 – 2023 Combined Mathematics II

Marking Scheme

Part A

1.



(5)

$I = \Delta(mV)$

$mv_1 + mv_2 - m(eu) = 0$ (5)

$v_1 + v_2 = (eu)$ —— (1)

By Newton's Experimental Law,

$v_2 - v_1 = e^2u$ —— (2) (5)

(1) + (2) $\Longrightarrow$ $v_2 = \frac{e(1+e)u}{2}$ (5)

The required time $= \frac{a}{2u} + \frac{a}{eu}$

$\frac{a}{u}\left(\frac{1}{2} + \frac{1}{e}\right)$ (5)

25

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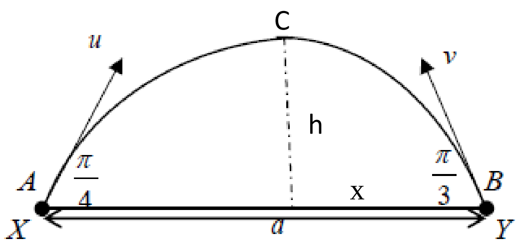
2. From A to C $S = ut + 1/2at^2$

$\uparrow$ $h = \frac{u}{\sqrt{2}}T - \frac{1}{2}gT^2$ (5)

From B to C $\uparrow$ $h = \frac{v\sqrt{3}}{2}T - \frac{1}{2}gT^2$ (5)

$\frac{u}{\sqrt{2}}T = \frac{v\sqrt{3}}{2}T$

$v = \sqrt{\frac{2}{3}}\,u$ (5)



From A to C and From B to C $S = ut$

$\rightarrow$ $a - x = \frac{u}{\sqrt{2}}T$, $\leftarrow$ $x = \frac{v}{2}T$ (5)

$a = \frac{v}{2}T + \frac{u}{\sqrt{2}}T$

$a = \frac{1}{2}\sqrt{\frac{2}{3}}uT + \frac{u}{\sqrt{2}}T$

$a = \frac{(1+\sqrt{3})uT}{\sqrt{6}}$

$T = \left(\frac{\sqrt{6}}{\sqrt{3}+1}\right)\frac{a}{u}$ (5)

25

3. Applying F=ma:

P, ↑ $T - 2mg = 2mf$ (5)

Q, ↓ $3mg - T = 3mf$ (5)

$f = g/5$

$a = \frac{1}{2}\frac{g}{5}t^2$

$t = \sqrt{\frac{10a}{g}}$

$v = \frac{g}{5} \times \sqrt{\frac{10a}{g}}$

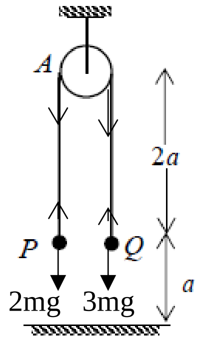
$v = \frac{\sqrt{10ag}}{5}$ (5)

$v^2=u^2+2as$ ↑ $0 = \frac{10ag}{25} - 2gh$ (5)

$h = \frac{a}{5}$

$= \frac{a}{5} + a$

$= \frac{11a}{5}$ (5)



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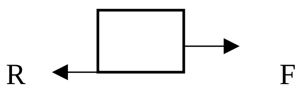
4.



F=ma: $F - R = 1200a$ (5)

$F=1200\times3+400$

$F=4000$ N (5)

P=FV : $P=4000\times15$

$P=60000w=60kw$ (5)

$60000=F^1 \times 12$

$F^1 = 5000N$ (5)

$F^1-1200\times10\times1/2-400=1200a$ (5)

25

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5. Applying F=ma

↑ $(T_1 - T_2)\cos\alpha = mg$ (5)

$T_1 - T_2 = \frac{mg}{\cos\alpha}$

← $(T_1+T_2)\sin\alpha = m\,2a\sin\alpha\,\omega^2$ (5)

$T_1 + T_2 = 2ma\omega^2$

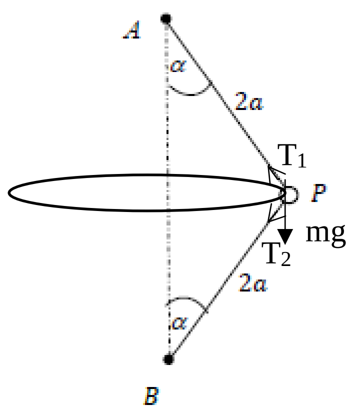
$2T_2 = m\left(2a\omega^2 - \frac{g}{\cos\alpha}\right)$

$T_2 = \frac{m}{2}\left(2a\omega^2 - \frac{g}{\cos\alpha}\right)$ (5)

$T_2 > 0$ (5)

$2a\omega^2 - \frac{g}{\cos\alpha} > 0$

$\omega > \sqrt{\frac{g}{2a\cos\alpha}}$ (5)



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6.

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$ (5)

$= -(2\underline{a} + 3\underline{b}) + \frac{1}{3}\underline{a} + \frac{3}{4}\underline{b}$

$= -\frac{5}{3}\underline{a} - \frac{9}{4}\underline{b}$ (5)

$\overrightarrow{BC} = \overrightarrow{BO} + \overrightarrow{OC}$

$= -(\frac{1}{3}\underline{a} + \frac{3}{4}\underline{b}) + k\underline{a} + 2\underline{b}$

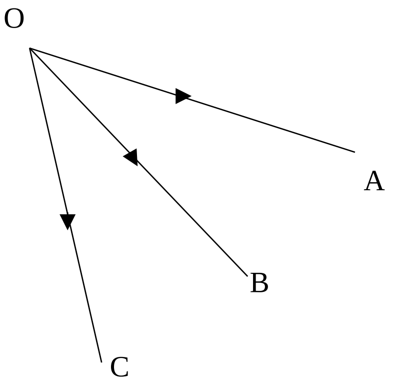
$= (k - \frac{1}{3})\underline{a} + \frac{5}{4}\underline{b}$ (5)

$\frac{k - 1/3}{-5/3} = \frac{5/4}{-9/4}$ (5)

$9(k - \frac{1}{3}) = 5 \times \frac{5}{3}$

$9k - 3 = \frac{25}{3}$

$k = \frac{34}{27}$ (5)



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7. ↺A

$4W \times 2a\cos\alpha = W \times a\sin\alpha$ (5)

$\tan\alpha = 8$

$\alpha = \tan^{-1}(8)$ (5)

Using Lami Theorum;

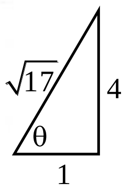
$\frac{R}{\sin 90^\circ} = \frac{W}{\sin(90^\circ+\theta)} = \frac{4W}{\sin(180^\circ-\theta)}$ (5)

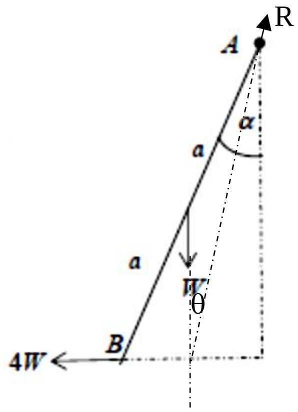
$\frac{R}{1} = \frac{W}{\cos\theta} = \frac{4W}{\sin\theta}$

$\tan\theta = 4$ (5)

$R = \frac{W}{\cos\theta}$

$R = \sqrt{17}W$ (5)





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8.

→

$F = S\sin\theta$ (5)

$F = S \times \frac{3}{5}$

$F = \frac{3S}{5}$

$F = \mu R$

$\frac{3S}{5} = \frac{1}{2}R$

$R = \frac{6S}{5}$ (5)

↑ $R + S\cos\theta = W$ (5)

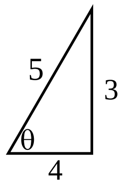
$\frac{6S}{5} + S \times \frac{4}{5} = W$

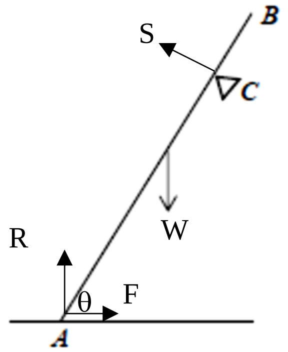
$10S = 5W$

$S = \frac{W}{2}$ (5)

↺A $S \times AC = W \times 2a \times \frac{4}{5}$ (5)

$AC = \frac{16a}{5}$





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9. $p(A' \cup B') = p(A \cap B)' = 1 - p(A \cap B)$ (5)

$\frac{9}{10} = 1 - p(A \cap B)$

$p(A \cap B) = \frac{1}{10}$ (5)

$p(A \cap B) = p(A) \cdot p(B)$

$\frac{1}{10} = p(A)\ \frac{1}{4}$ (5)

$p(A) = \frac{2}{5}$

$p(A \cup B) = p(A) + p(B) - p(A \cap B)$

$= \frac{2}{5} + \frac{1}{4} - \frac{1}{10}$

$p(A \cup B) = \frac{11}{20}$ (5)

$p(B \mid A \cup B) = \frac{p[B \cap (A \cup B)]}{p(A \cup B)} = \frac{p(B)}{p(A \cup B)} = \frac{1/4}{11/20} = \frac{5}{11}$ (5)

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10. $Z\text{-score} = \frac{x_i - \bar{x}}{\sigma}$

$1.6 = \frac{70 - \bar{x}}{10}$ (5)

$\bar{x} = 54$ (5)

$\bar{x} = \frac{\sum x_i}{n}$ (5)

$\sum x_i = 54 \times 100 - 70 + 60 = 5390$ (5)

$\bar{x}_{True} = \frac{5390}{100} = 53.9$ (5)

25

Part B

11. (a) $\tan\alpha = \frac{u}{t_1} = 3f$

$t_1 = \frac{u}{3f}$ (5)

$\frac{1}{2} \times t_1 \times u = 2a$ (5)

$u = \frac{4a}{t_1}$

$t_1 = 2\sqrt{\frac{a}{3f}}$ (5)

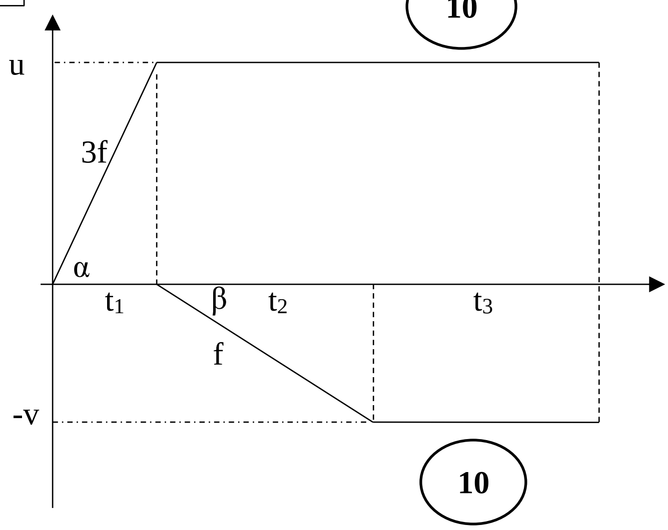
$\tan\beta = \frac{v}{t_2} = f$

$t_2 = \frac{v}{f}$ (5)

$\frac{1}{2} \times t_2 \times v = 3a$ (5)

$\frac{1}{2} \times t_2 \times f t_2 = 3a$

$t_2 = \sqrt{\frac{6a}{f}}$ (5)



(10)

(10)

50

$\frac{1}{2} \times \frac{u}{3f} \times u = 2a$

$u = \sqrt{12fa}$ (5)

$\frac{1}{2} \times \frac{v}{f} \times v = 3a$

$v = \sqrt{6fa}$ (5)

$2a + (t_2 + t_3)u + 3a + t_3 v = 16a$ (10)

$(u + v)t_3 = 11a - ut_2$

$(u+v)t_3 = 11a - \sqrt{\frac{6a}{f}} \times \sqrt{12fa}$

$t_3 = \frac{11a - \sqrt{72}\,a}{\sqrt{12fa} + \sqrt{6af}}$

$t_3 = \left(\frac{11-6\sqrt{2}}{2\sqrt{3}+\sqrt{6}}\right)\sqrt{\frac{a}{f}}$ s (5)

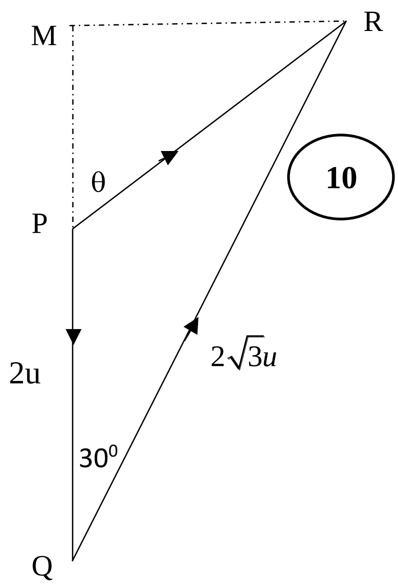
25

(b) $V_{A,E} = \downarrow 2u$ (5)

$V_{B,A} =$ [arrow at 30° to vertical, $2\sqrt{3}u$]

$V_{B,E} = V_{B,A} + V_{A,E}$ (5)

$\vec{PR} =$ [Q to R at 30°, $2\sqrt{3}u$] $+$ [P to Q, $\downarrow 2u$] (5)



$PR^2 = PQ^2 + QR^2 - 2\,PQ\,QR \cdot \cos 30^\circ$ (5)

$= 4u^2 + 12u^2 - 2 \times 2u \times 2\sqrt{3}u \times \frac{\sqrt{3}}{2}$ (5)

$= 4u^2$

$PR = 2u$

$\therefore V_{B,E} = 2u$ (5)

$PQ = QR$

$\therefore \theta = 60^\circ$, $V_{B,E} =$ [arrow at 60° to vertical, $2u$] (5)

45

$AM = a\sin 60^\circ = \frac{\sqrt{3}a}{2}$ (5)

$XM^2 = AX^2 - AM^2$

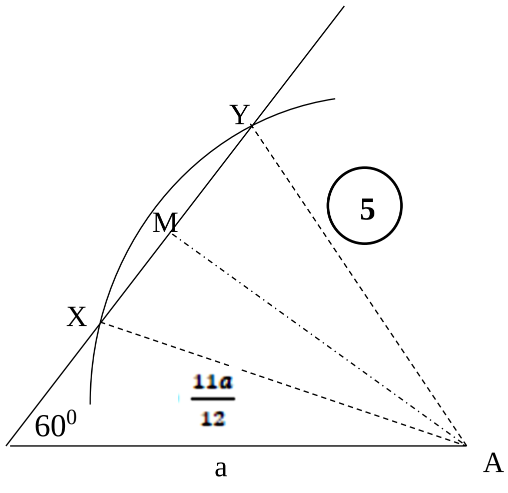
$= \left(\frac{11a}{12}\right)^2 - \left(\frac{\sqrt{3}a}{2}\right)^2$

$= \frac{121a^2 - 108a^2}{144}$

$XM = \frac{\sqrt{13}}{12}a$ (5)

$\therefore XY = 2 \times \frac{\sqrt{13}}{12}a = \frac{\sqrt{13}a}{6}$ (5)

$= \frac{XY}{2\sqrt{3}u} = \frac{\sqrt{13}a/6}{2\sqrt{3}u}$ (5)



(5)

Risky time of ship B = $\frac{\sqrt{39}a}{36u}$ h (5)

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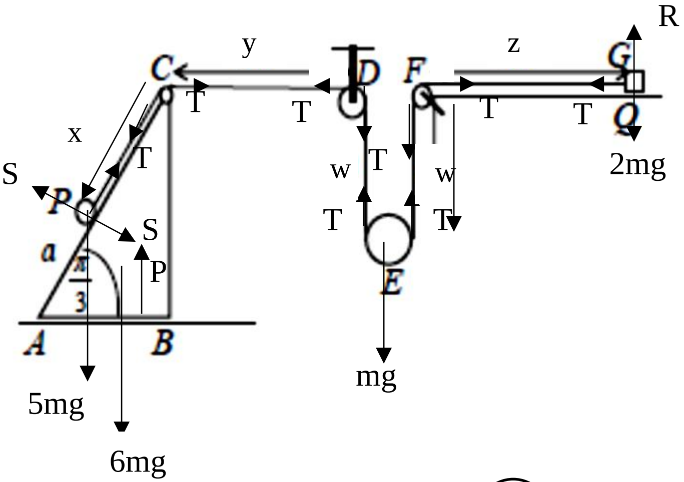
12. (a)



(10)

x+y+z+2w = constant. (5)

$\ddot{x} + \ddot{y} + \ddot{z} + 2\ddot{w} = 0$ (5)

Applying F=ma

(E) $\downarrow$ $mg - 2T = m\ddot{w}$ (10)

(Q) $\rightarrow$ $-T = 2m\ddot{z}$ (10)

(P) $\swarrow$ $5mg\sin\pi/3 - T = 5m(\ddot{x} + \ddot{y}\cos\pi/3)$ (15)

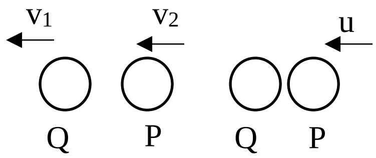
(P,6m) $\leftarrow$ $-T = 6m\ddot{y} + 5m(\ddot{y} + \ddot{x}\cos\frac{\pi}{3})$ (15)

(P) $\swarrow$ $S = ut + \frac{1}{2}at^2$

$a = \frac{1}{2}\ddot{x}t^2$ (5)

75

(b)



I=Δ(mV) $mv_1 + mv_2 = mu$ (5)

$v_1 + v_2 = u$

By Newton's Experimental Law $v_1 - v_2 = \frac{u}{2}$ (5)

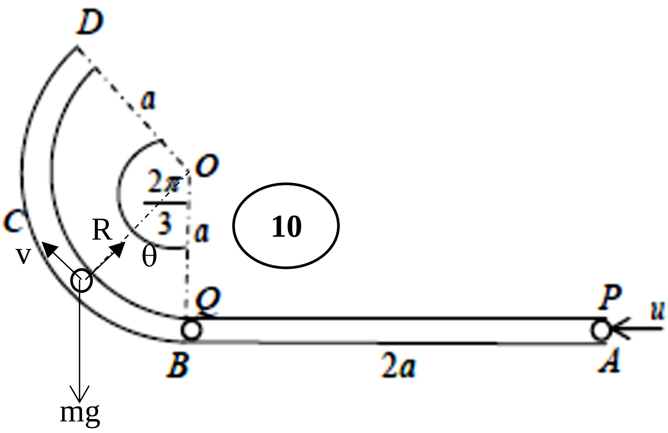
$v_1 = \frac{3u}{4}$ (5)

By the Conservation of energy

$\frac{1}{2}mv^2 - mga\cos\theta = \frac{1}{2}m\left(\frac{3u}{4}\right)^2 - mga$ (20)

$\frac{1}{2}v^2 = \frac{9u^2}{32} - ga(1-\cos\theta)$

$v^2 = \frac{9u^2}{16} - 2ga(1-\cos\theta)$



(10)

$\nearrow F = ma$

$R - mg\cos\theta = \frac{mv^2}{a}$ (10)

$R = \frac{m}{a}\left\{\frac{9u^2}{16} - 2ga(1-\cos\theta)\right\} + mg\cos\theta$

$R = \frac{m}{a}\left\{\frac{9u^2}{16} - ga(2 - 3\cos\theta)\right\}$ (5)

If Q leaves at D, v>0 (5)

$\frac{9u^2}{16} - 2ga\left(1-\cos\frac{2\pi}{3}\right) > 0$

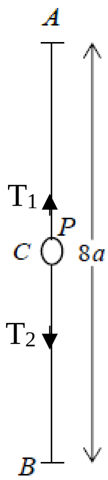
$\frac{9u^2}{16} > 2ga\left(1+\frac{1}{2}\right)$ (5)

$u^2 > \frac{16ga}{3}$

$u > 4\sqrt{\frac{ga}{3}}$ (5)

75

13.



$T_1 = \frac{mg(AC-a)}{a}$ (5)

$T_2 = \frac{mg(8a-AC-3a)}{3a} = \frac{mg(5a-AC)}{3a}$ (5)

For the equilibrium of P

$\uparrow T_1 = T_2 + mg$ (10)

$\frac{mg(AC-a)}{a} = \frac{mg(5a-AC)}{3a} + mg$ (5)

$3(AC-a) = 5a - AC + 3a$

$AC = \frac{11a}{4}$ (5)

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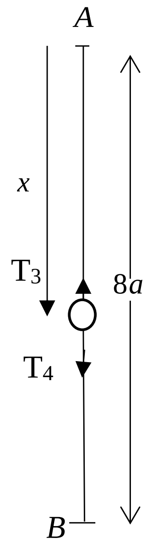


$$T_3 = \frac{mg(x-a)}{a}$$ (5)

$$T_4 = \frac{mg(5a-x)}{3a}$$ (5)

Applying F=ma

$$\uparrow \quad T_3 - T_4 - mg = -m\ddot{x}$$ (10)

$$\frac{mg(x-a)}{a} - \frac{mg(5a-x)}{3a} - mg = -m\ddot{x}$$ (5)

$$\frac{g}{3a}[3x - 3a - 5a + x - 3a] = -\ddot{x}$$

$$\ddot{x} + \frac{4g}{3a}\left(x - \frac{11a}{4}\right) = 0$$ (5)

$$y = x - \frac{11a}{4}$$

$$\dot{y} = \dot{x}$$ (5)

$$\ddot{y} = \ddot{x}$$

$$\ddot{y} + \frac{4g}{3a}y = 0 \longrightarrow \text{(a)}$$ (5)

40

$$y = A\cos\omega t + B\sin\omega t$$

$$\dot{y} = -A\omega\sin\omega t + B\omega\cos\omega t$$ (5)

$$\ddot{y} = -A\omega^2\cos\omega t - B\omega^2\sin\omega t$$ (5)

$$\ddot{y} = -\omega^2(A\cos\omega t + B\sin\omega t)$$

$$\ddot{y} + \omega^2 y = 0 \longrightarrow \text{(b)}$$ (5)

Comparing (a) and (b)

$$\omega = \sqrt{\frac{4g}{3a}}$$ (5)

When $x = 5a, \quad t = 0$ (5)

$$y = 5a - \frac{11a}{4}$$

$$y = \frac{9a}{4}$$ (5)

$$\frac{9a}{4} = A\cos 0 + B\sin 0$$

$$A = \frac{9a}{4}$$ (5)

When $t = 0 \quad \dot{x} = 0$ (5)

$$\dot{y} = 0$$

$$0 = -A\omega\sin 0 + B\omega\cos 0$$

$$B = 0$$ (5)

45

When $x = a$

$y = a - \frac{11a}{4} = -\frac{7a}{4}$ (5)

$-\frac{7a}{4} = \frac{9a}{4}\cos\omega t_1$

$\cos\omega t_1 = -\frac{7}{9}$ (5)

$\cos\omega t_1 = \cos(\pi - \theta)$

$\omega t_1 = \pi - \theta$ (5)

$\cos\theta = \frac{7}{9}$

$\sin\theta = \frac{4\sqrt{2}}{9}$ (5)

$\dot{y} = -\frac{9a}{4}\sqrt{\frac{4g}{3a}}\sin(\pi - \theta)$

$= -\frac{9a}{4}\sqrt{\frac{4g}{3a}} \times \frac{4\sqrt{2}}{9}$

$= -\sqrt{\frac{8ga}{3}}$ (5)

Velocity of P is $\sqrt{\frac{8ga}{3}}$ (5)

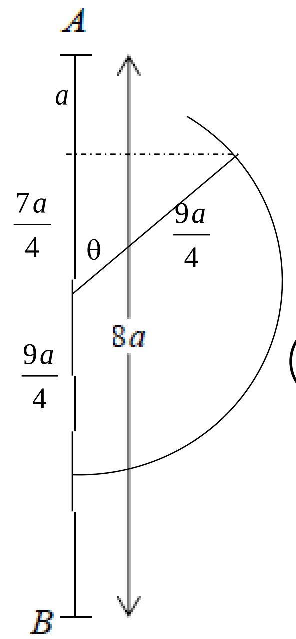
$\omega t_1 = \pi - \theta$

$t_1 = \frac{\pi - \theta}{\omega}$

The required time $= \sqrt{\frac{3a}{4g}}\left[\pi - \cos^{-1}\left(\frac{7}{9}\right)\right]$ (5)

35

Aliter



(5)

The centre of this S.H.M. is $x = \frac{11a}{4}$ (5)

Amplitude is $= 5a - \frac{11a}{4} = \frac{9a}{4}$ (5)

$\cos\theta = \frac{7a/4}{9a/4}$

$\theta = \cos^{-1}\left(\frac{7}{9}\right)$ (5)

$v^2 = \omega^2(a^2 - x^2)$

$= \frac{4g}{3a}\left[\left(\frac{9a}{4}\right)^2 - \left(\frac{7a}{4}\right)^2\right]$ (5)

$= \frac{4g}{3a} \times \frac{32a^2}{16}$

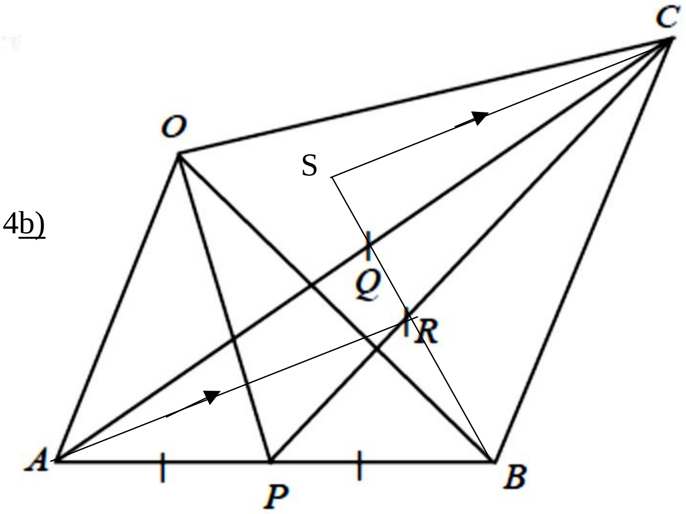
Velocity of P is $\sqrt{\frac{8ga}{3}}$ (5)

Time $= \frac{\pi - \theta}{\omega}$

The required time $= \sqrt{\frac{3a}{4g}}\left[\pi - \cos^{-1}\left(\frac{7}{9}\right)\right]$ (5)

35

14. (a) (i)



$\overrightarrow{OQ} = \overrightarrow{OA} + \overrightarrow{AQ}$ (5)

$= 2\underline{a} - \underline{b} + \frac{1}{2}\overrightarrow{AC}$

$= 2\underline{a} - \underline{b} + \frac{1}{2}(\overrightarrow{AO} + \overrightarrow{OC})$

$= 2\underline{a} - \underline{b} + \frac{1}{2}(-2\underline{a} + \underline{b} + -\underline{a} + 4\underline{b})$

$= 2\underline{a} - \underline{b} + \frac{1}{2}(-3\underline{a} + 5\underline{b})$

$= \frac{\underline{a} + 3\underline{b}}{2}$ (5)

$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$

$= 2\underline{a} - \underline{b} + \frac{1}{2}\overrightarrow{AB}$

$= 2\underline{a} - \underline{b} + \frac{1}{2}(\overrightarrow{AO} + \overrightarrow{OB})$ (5)

$= 2\underline{a} - \underline{b} + \frac{1}{2}(-2\underline{a} + \underline{b} + 4\underline{a} + 5\underline{b})$

$= 2\underline{a} - \underline{b} + \frac{1}{2}(2\underline{a} + 6\underline{b})$

$= 3\underline{a} + 2\underline{b}$ (5)

$\overrightarrow{OR} = \overrightarrow{OP} + \overrightarrow{PR}$

$= \overrightarrow{OP} + \frac{1}{3}\overrightarrow{PC}$

$= 3\underline{a} + 2\underline{b} + \frac{1}{3}(\overrightarrow{PO} + \overrightarrow{OC})$ (5)

$= 3\underline{a} + 2\underline{b} + \frac{1}{3}(-3\underline{a} - 2\underline{b} - \underline{a} + 4\underline{b})$

$= 3\underline{a} + 2\underline{b} + \frac{1}{3}(-4\underline{a} + 2\underline{b})$

$= \frac{5}{3}\underline{a} + \frac{8}{3}\underline{b}$ (5)

[30]

(ii) $\overrightarrow{BR} = \overrightarrow{BP} + \overrightarrow{PR}$

$= -\frac{1}{2}(2\underline{a} + 6\underline{b}) + \frac{1}{3}(-4\underline{a} + 2\underline{b})$

$\overrightarrow{BR} = -\frac{7}{3}(\underline{a} + \underline{b})$ (5)

$\overrightarrow{PQ} = \overrightarrow{PA} + \overrightarrow{AQ}$

$= -\frac{1}{2}(2\underline{a} + 6\underline{b}) + \frac{1}{2}(-3\underline{a} + 5\underline{b})$

$= -\frac{1}{2}(5\underline{a} + \underline{b})$ (5)

$\overrightarrow{RQ} = \overrightarrow{RP} + \overrightarrow{PQ}$

$= -\frac{1}{3}(-4\underline{a} + 2\underline{b}) + -\frac{1}{2}(5\underline{a} + \underline{b})$

$= \frac{1}{6}(8\underline{a} - 4\underline{b} - 15\underline{a} - 3\underline{b})$

$= \frac{1}{6}(-7\underline{a} - 7\underline{b})$

$= -\frac{7}{6}(\underline{a} + \underline{b})$ (5)

$\overrightarrow{RQ} = \frac{1}{2} \times -\frac{7}{3}(\underline{a}+\underline{b})$

$\overrightarrow{RQ} = \frac{1}{2} \times \overrightarrow{BR}$

$\overrightarrow{BR} = 2\overrightarrow{RQ}$

$BR : RQ = 2 : 1$ (5)

20

(iii)

$\overrightarrow{BQ} = \overrightarrow{BR} + \overrightarrow{RQ}$

$= 3\overrightarrow{RQ}$

$= 3 \times -\frac{7}{6}(\underline{a}+\underline{b})$

$= -\frac{7}{2}(\underline{a}+\underline{b})$ (5)

$\overrightarrow{BS} = k\overrightarrow{BQ}$

$\overrightarrow{BS} = k \times -\frac{7}{2}(\underline{a}+\underline{b})$

$\overrightarrow{SC} = \mu\overrightarrow{AR}$

$= \mu(\overrightarrow{AB} + \overrightarrow{BR})$

$= \mu\left[2\underline{a} + 6\underline{b} - \frac{7}{3}(\underline{a}+\underline{b})\right]$

$\overrightarrow{SC} = \mu\left(\frac{-\underline{a} + 11\underline{b}}{3}\right)$ (5)

$\overrightarrow{AS} = \overrightarrow{AC} + \overrightarrow{CS}$

$= \overrightarrow{AO} + \overrightarrow{OC} + \overrightarrow{CS}$

$= -2\underline{a} + \underline{b} - \underline{a} + 4\underline{b} - \frac{\mu}{3}(-\underline{a} + 11\underline{b})$

$= -3\underline{a} + 5\underline{b} + \frac{\mu}{3}(\underline{a} - 11\underline{b})$

$= \frac{(-9+\mu)\underline{a} + (15-11\mu)\underline{b}}{3}$ (5)

$\overrightarrow{BS} = \overrightarrow{BA} + \overrightarrow{AS}$

$= -2\underline{a} - 6\underline{b} + \overrightarrow{AS}$

$= -2\underline{a} - 6\underline{b} + \frac{(-9+\mu)\underline{a} + (15-11\mu)\underline{b}}{3}$

$= \frac{-6\underline{a} - 18\underline{b} + (-9+\mu)\underline{a} + (15-11\mu)\underline{b}}{3}$

$= \frac{(\mu-15)\underline{a} + (-3-11\mu)\underline{b}}{3}$ (5)

$-\frac{7k}{2}(\underline{a}+\underline{b}) = \frac{1}{3}\left[(\mu-15)\underline{a} + (-3-11\mu)\underline{b}\right]$

$(2\mu - 30 + 21k)\underline{a} + (-6 - 22\mu + 21k)\underline{b} = \underline{0}$ (5)

$2\mu - 30 + 21k = 0$

$-6 - 22\mu + 21k = 0$

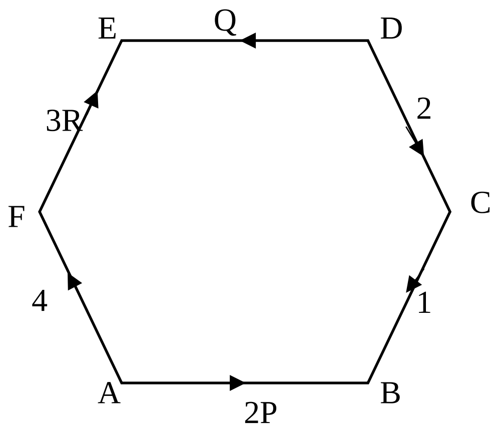
$24\mu - 24 = 0$

$\mu = 1$

$k = \frac{4}{3}$ (5)

30

(b)



(i)

→ $2P - Q + (3R + 2 - 1 - 4)\cos\frac{\pi}{3} = 0$ (5)

$2P - Q + \frac{3R}{2} - \frac{3}{2} = 0$

$4P - 2Q + 3R = 3$

↑ $(3R + 4 - 1 - 2)\sin\frac{\pi}{3} = 0$ (5)

$R = -\frac{1}{3}$ (5)

E ↺ $2P \times 4a\cos 30^\circ - 1 \times 4a\cos 30^\circ - 2 \times 2a\cos 30^\circ - 4 \times 2a\cos 30^\circ = 0$ (5)

$8P - 4 - 4 - 8 = 0$

$8P = 16$

$P = 2$ (5)

$4 \times 2 - 2Q + 3 \times -\frac{1}{3} = 3$

$-2Q = -4$ (5)

$Q = 2$

30

(ii)

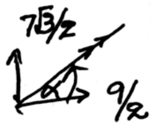
→ $X = 2 \times 2 - 1 + 3 \times 2 \times \frac{1}{2} - \frac{3}{2}$

$= 6 - \frac{3}{2}$

$X = \frac{9}{2}$ (5)

↑ $Y = (3 \times 2 + 4 - 1 - 2)\frac{\sqrt{3}}{2}$

$Y = \frac{7\sqrt{3}}{2}$ (5)



$R = \sqrt{\left(\frac{9}{2}\right)^2 + \left(\frac{7\sqrt{3}}{2}\right)^2}$

$= \sqrt{\frac{81}{4} + \frac{147}{4}}$

$= \sqrt{\frac{228}{4}}$

$= \sqrt{57}$ (5)

$\tan\alpha = \frac{7\sqrt{3}/2}{9/2}$

$\alpha = \tan^{-1}\left(\frac{7\sqrt{3}}{9}\right)$ (5)

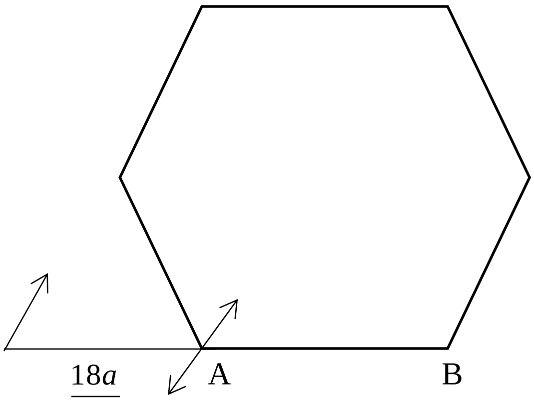
A ↺ $\frac{7\sqrt{3}}{2} d = -1 \times 2a\sin\frac{\pi}{3} - 2 \times 4a\sin\frac{\pi}{3} + 1 \times 4a\sin\frac{\pi}{3} - 6 \times 2a\sin\frac{\pi}{3}$ (5)

$\frac{7\sqrt{3}d}{2} = (-2a - 8a + 4a - 12a)\frac{\sqrt{3}}{2}$

$7d = -18a$

$d = \frac{-18a}{7}$ (5)

30



$\frac{18a}{7}$

Magnitude of the resultant $\sqrt{57}$ (5) both

Direction $\alpha = \tan^{-1}\left(\frac{7\sqrt{3}}{9}\right)$

Coulple of Magnitude

$H = \frac{7\sqrt{3}}{2} \times \frac{18a}{7}$

$= 9\sqrt{3}a$ (5) both

Contraclockwise sense

10

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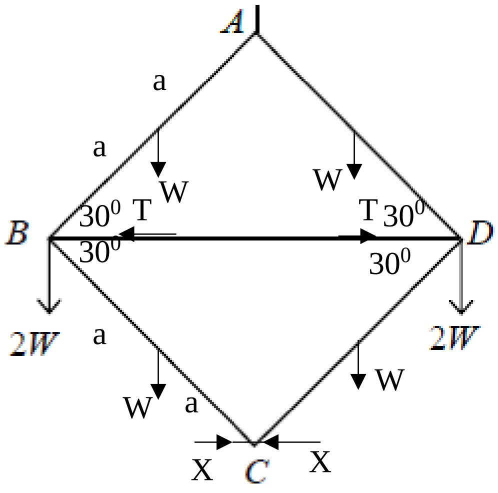
15. (a)



(10)

By symmetry Y=0 (5)

For BC

B ↻ $X \times 2a\sin 30^\circ = W \times a\cos 30^\circ$ (10)

$X = \frac{\sqrt{3}W}{2}$ (5)

For ABC

A ↻ $T \times 2a\sin 30^\circ = 2W \times a\cos 30^\circ + 2W \times 2a\cos 30^\circ + X \times 4a\sin 30^\circ$ (15)

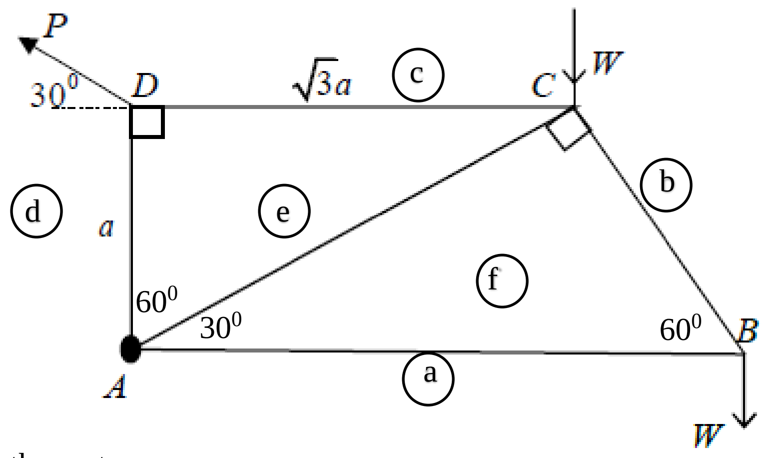
$T \times 2a \times \frac{1}{2} = 2W \times a \times \frac{\sqrt{3}}{2} + 2W \times 2a \times \frac{\sqrt{3}}{2} + \frac{\sqrt{3}W}{2} \times 4a \times \frac{1}{2}$ (5)

$T = \sqrt{3}W + 2\sqrt{3}W + \sqrt{3}W$

$T = 4\sqrt{3}W$ (5)

55

(b)



For the system

$A \circlearrowright \quad P\cos 30^\circ \times a = W \times \sqrt{3}a + W \times \frac{4a}{\sqrt{3}}$ (10)

$P\frac{\sqrt{3}}{2} = \frac{7W}{\sqrt{3}}$

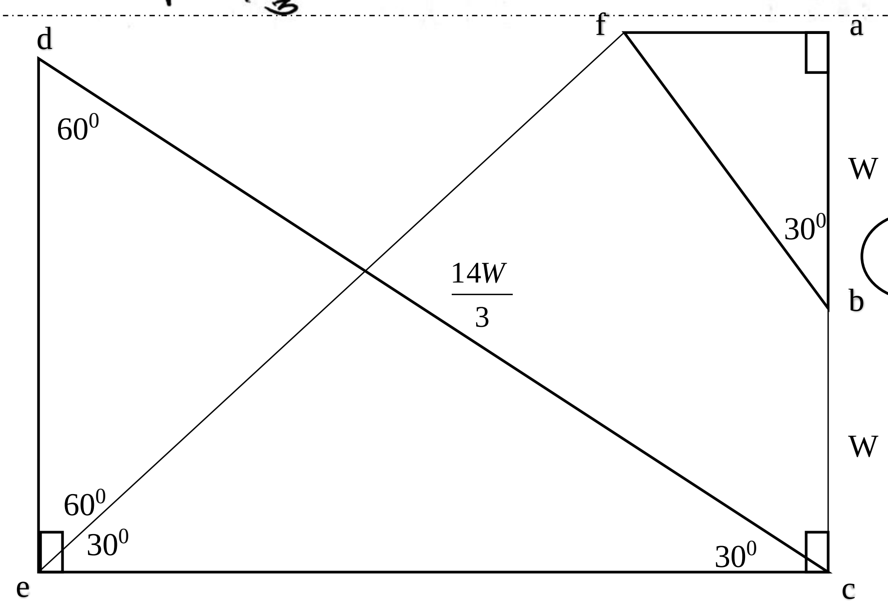
$P = \frac{14W}{3}$ (5)

$\cos 30^\circ = \frac{AC}{AB}$

$\frac{\sqrt{3}}{2} = \frac{2a}{AB}$

$AB = \frac{4a}{\sqrt{3}}$

15



(10) + (10) + (10)

(Each joint 10)

30

Rod	Tension	Thrust
AB		$\frac{W}{\sqrt{3}}$
BC	$\frac{2W}{\sqrt{3}}$	
AC		$4W$
CD	$\frac{7W}{\sqrt{3}}$	
AD	$\frac{7W}{3}$	

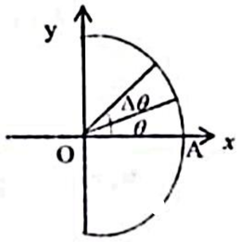
(10) (10) (10) (10) (10)

(Each tension or thrust (5) , magnitude (5))

50

16.

By symmetry, centre of mass lies on the x- axis (5)



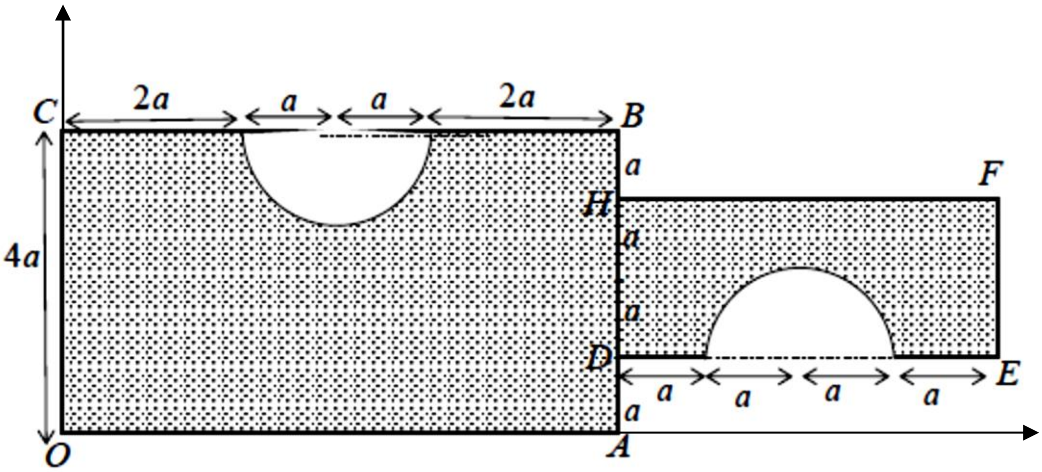
$$\Delta m = \frac{1}{2}a^2 \Delta\theta \times \sigma$$

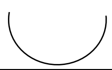
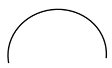
$$\bar{x} = \frac{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{2}a^2\sigma \cdot \frac{2}{3}a\cos\theta\, d\theta}{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{2}a^2\sigma \cdot d\theta}$$ (5) (5)

$$= \frac{\frac{2}{3}a^3 \sin\theta \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}}{\frac{1}{2}a^2\theta \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}}$$ (5) (5)

$$= \frac{4a}{3\pi}$$ (5)

30



Object	Mass	Distance from OC	Distance from OA
OABC	$24a^2\sigma$ (5)	$3a$ (5)	$2a$ (5)
DEFH	$8a^2\sigma$ (5)	$8a$ (5)	$2a$ (5)
	$\frac{\pi a^2\sigma}{2}$ (5)	$3a$ (5)	$4a - \frac{4a}{3\pi}$ (5)
	$\frac{\pi a^2\sigma}{2}$ (5)	$8a$ (5)	$a + \frac{4a}{3\pi}$ (5)
Composite body	$(32-\pi)a^2\sigma$ (5)	$\bar{x}$	$\bar{y}$

65

$$(32-\pi)a^2\sigma\bar{x} = 24a^2\sigma\times 3a + 8a^2\sigma\times 8a - \frac{\pi a^2\sigma}{2}\times 3a - \frac{\pi a^2\sigma}{2}\times 8a$$ (15)

$$(32-\pi)\bar{x} = \left(136 - \frac{11\pi}{2}\right)a$$

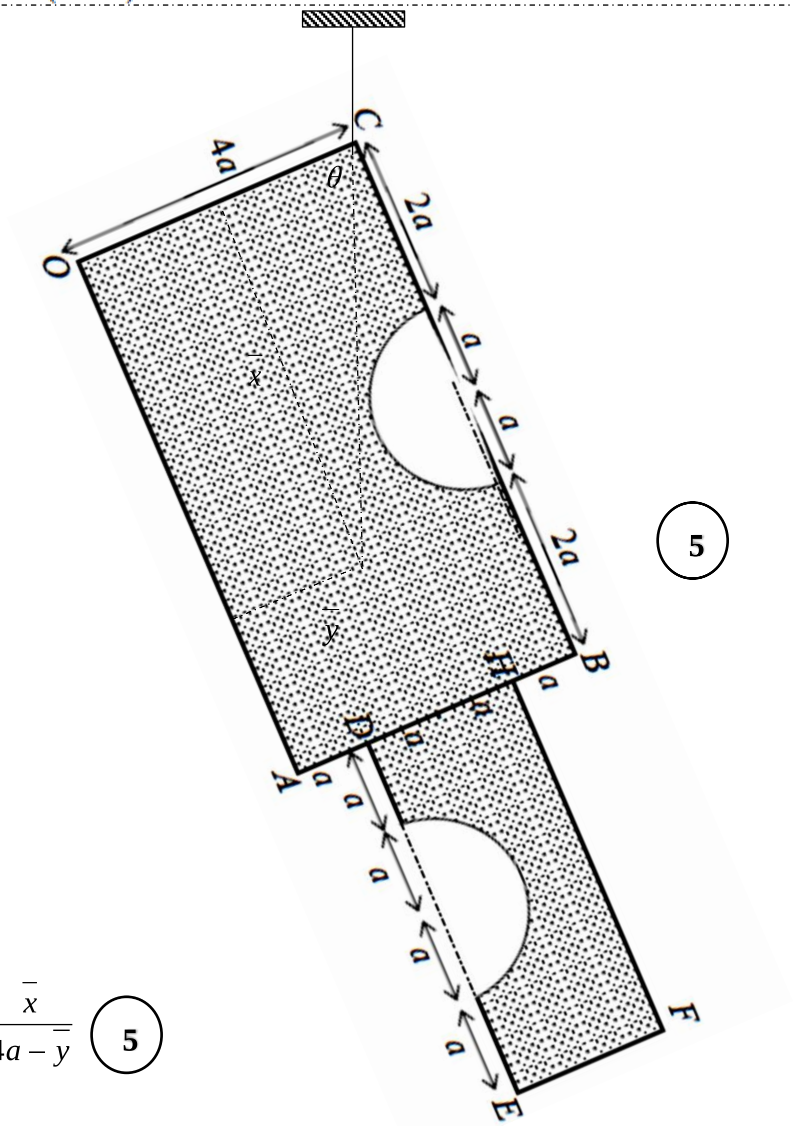
$$\therefore\ \bar{x} = \frac{(272-11\pi)a}{2(32-\pi)}$$ (5)

$$(32-\pi)a^2\sigma\bar{y} = 24a^2\sigma\times 2a + 8a^2\sigma\times 2a - \frac{\pi a^2\sigma}{2}\times\left(4a - \frac{4a}{3\pi}\right) - \frac{\pi a^2\sigma}{2}\times\left(a + \frac{4a}{3\pi}\right)$$ (15)

$$(32-\pi)\bar{x} = \left(64 - \frac{5\pi}{2}\right)a$$

$$\bar{y} = \frac{(128-5\pi)a}{2(32-\pi)}$$ (5)

40



(5)

$$\tan\theta = \frac{\bar{x}}{4a-\bar{y}}$$ (5)

$$\theta = \tan^{-1}\left(\frac{272-11\pi}{128-3\pi}\right)$$ (5)

15

---

17. (a) Let X be the event two blue balls are drawn.

$P(X) = P(X \mid A)P(A) + P(X \mid B)P(B) + P(X \mid C)P(C)$ (10)

$P(A) = P(B) = P(C) = \frac{1}{3}$ (5)

$P(X) = \frac{1}{3}\times\frac{7}{10}\times\frac{6}{9} + \frac{1}{3}\times\frac{6}{10}\times\frac{5}{9} + \frac{1}{3}\times\frac{2}{10}\times\frac{1}{9}$ (10) + (10) + (10)

$= \frac{42}{270} + \frac{30}{270} + \frac{2}{270}$

$= \frac{37}{135}$ (10)

By Bayes' Theorem

$P(B \mid X) = \frac{P(X \mid B)P(B)}{P(X)}$ (5)

$P(B \mid X) = \frac{\frac{1}{3}\times\frac{6}{10}\times\frac{5}{9}}{\frac{37}{135}} = \frac{15}{37}$ (10)

70

---

(5) (5) (5)

Class Interval	Mid point x	f	fx	fx^2
1 - 5	3	5	15	45
5 - 9	7	7	49	343
9 - 13	11	12	132	1452
13 - 17	15	10	150	2250
17 - 21	19	6	114	2166
		$\sum f = 40$	$\sum fx = 460$	$\sum fx^2 = 6256$

(5) (5)

Mean, $\bar{x} = \frac{\sum fx}{\sum f} = \frac{460}{40} = 11.5$ (5)

(5) (5)

Mode, $M_0 = L_1 + \frac{\Delta_1 C}{\Delta_1 + \Delta_2} = 9 + \frac{(12-7)\times 4}{(12-7)+(12-10)} = 9 + \frac{20}{7} \simeq 11.86$ (5)

(5) (5) (5)

Standard deviation, $\sigma = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2} = \sqrt{\frac{6256}{40} - 11.5^2} = \sqrt{156.4 - 132.25} = \sqrt{24.15} \simeq 4.91$ (5)

The coefficient of skewness $= \frac{\bar{x} - M_0}{\sigma} = \frac{11.5 - 11.86}{4.91} = -0.073$ (5)

(5) (5)

80

Aliter

(5) (5) (5)

Class Interval	Mid point x	f	u	fu	fu^2
1 - 5	3	5	-2	-10	20
5 - 9	7	7	-1	-7	7
9 - 13	(11)	12	0	0	0
13 - 17	15	10	1	10	10
17 - 21	19	6	2	12	24
		$\sum f = 40$		$\sum fu = 5$	$\sum fu^2 = 61$

(5) (5)

$\bar{x} = A + c\bar{u}$ (5)

$= 11 + 4 \times \frac{5}{40}$

$= 11.5$ (5)

$$\sigma = c\sqrt{\frac{\sum fu^2}{\sum f} - \bar{u}^2} = 4\sqrt{\frac{61}{40} - \left(\frac{5}{40}\right)^2} = 4\sqrt{1.525 - 0.125^2} = 4\sqrt{1.525 - 0.0156} \simeq 4\sqrt{1.5094} = 4 \times 1.228 \simeq 4.91$$

(5) (5) (5)

* * *

---Ajith Premalal

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පෙර පාසලේ සිට උසස් පෙළ දක්වා සියළුම ප්‍රශ්න පත්‍ර,
කෙටි සටහන්, වැඩ පොත්, අතිරේක කියවීම් පොත්, සඟරා
සිංහල සහ ඉංග්‍රීසි මාධ්‍යයෙන් ගෙදරටම ගෙන්වා ගැනීමට
www.LOL.lk වෙබ් අඩවිය වෙත යන්න