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ANANDA COLLEGE - COLOMBO 10

10 E

Second Term Test - November 2022
G.C.E. (Adv. Level) Examination - 2023

Combined Mathematics **Grade 12** **03 hours**

Name : .....

For candidate's use only:

- Answer all questions in **Part A** and only 5 questions in **Part B.**
- Answer all questions in **Part A** within the space provided.

For marking examiner's use only.

Part A

Question Numbers	Marks
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Total	

Part B.

Question Numbers	Marks
11	
12	
13	
14	
15	
16	
17	
Total	

	Marks
Part A	
Part B	
Total	

22 A/L අපි [papers group]

PART - A

* **Answer all questions.**

01. Let $y = \dfrac{1-kx}{(x-1)(x-2)}$; $x \neq 1, 2$.

Show that $\dfrac{1}{2} < k < 1$, such that y can get any real value for two real distinct values of x.

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22 A/L අපි [papers group]

02. Find the set of real values of x satisfying the inequality $\dfrac{3x^2+4x+3}{2x^2+x-1} < 1$.

.....

03. Find the domain of the function $f(x) = -\sqrt{\frac{2}{x+1} - 1}$.

Sketch the graph of $f(x)$ without using the differentiation.

Hence, sketch the graph of $f^{-1}(x)$ in the same diagram and deduce the domain of $f^{-1}(x)$.

22 A/L අපි [papers group]

04. Evaluate. $\lim_{x \to \frac{\pi}{4}} \frac{\tan^3 x - \tan x}{x^2 \cos\left(x + \frac{\pi}{4}\right)}$

05. Find the co - ordinate of C, which divides the line segment AB, such that the points A $\equiv (a, b)$, B $\equiv (2a, 3b)$ in the ratio $-\frac{2}{3}$. Here a, b are real constants.

Find the locus of the point which moves a distance 'a' from the point C. Hence show that the point $(-2a, -3b)$ lies on the locus.

22 A/L අපි [papers group]

06. Show that $4x^2 - 4xy\cos\alpha + y^2 = 4\sin^2\alpha$, if $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$; $o < \alpha < \pi$

07. Vehicle A which is at rest, moves with an accelaration 2f from the point P, which is 'd' metres apart from point Q. P and Q are in the same horizontal road. Vehicle B, moves from point Q with the velocity u and decelaration 'f' towards the vehicle A.

Sketch the velocity time graphs for the motions of the vehicles A and B in the same diagram. Hence, find the time taken for the collision if the vehicles A and B collide while they are moving.

22 A/L අපි [papers group]

08. A ship sails towards south with the velocity u at the point $(-d, \sqrt{3}\,d)$ at time $t = 0$ in the OXY cartiesian plane. At $t = 0$ a boat which is at O, sails with a constant velocity and meets the ship at the time $\frac{2d}{\sqrt{3}\,u}$. Find the velocity of the boat relative to earth and the direction of it.

09. Let three position vectors be $\underline{a} = 7\underline{i} + \underline{j}$, $\underline{b} = \underline{i} + \underline{j}$ and $\underline{c} = \underline{i} + \lambda\underline{j}$

(i) Find λ, if $\underline{a}$ and $\underline{c}$ are perpendicular to each other.

(ii) If the angle between $\underline{a}$ and $\underline{b}$ be θ and the angle between $\underline{b}$ and $\underline{c}$ be 2θ, show that $\lambda = -k \pm \sqrt{k^2 - 1}$. Here $k = \frac{625}{527}$.

22 A/L අපි [papers group]

10. A uniform rod AB of length 2ℓ, weight w is in equillibrium such that A leans against a smooth vertical wall while C is on a rough peg which lies below A away from the wall. Rod is perpendicular to the plane of the wall and it makes an angle θ $(0 < \theta < \pi/2)$, to the down ward vertical . $CB = \ell/2$. Find the magnitude of the reaction of the rod with the wall in terms of w and θ. Find the angle which makes the reaction on the rod at C with the vertical.

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Combined Mathematics **Grade 12**

Answer five questions selecting at least two questions out of 15, 16 and 17.

PART - B

11. (a) Let $a \neq 0$ and $a, b, c \in \mathbb{R}$. Let α and β be the roots of the quadratic equation $ax^2 + bx + c = 0$.

Show that $\alpha + \beta = \frac{-b}{a}$ and $\alpha\beta = \frac{c}{a}$.

Show that the roots of the quadratic equation, $n(n+1)x^2 + (2n^2 + 2n - 1)x + n^2 + n - 2 = 0$, are real when $n \in Z - \{0, -1\}$.

Write down the quadratic equation with the coefficients in terms of m, by taking $m = n(n+1)$.

If roots of this quadratic equation are α and β,

(i) Show that the quadratic equation, $m^3x^2 - (1-2m)(m+1)^2x + m^2(m-2) = 0$. Whose roots are $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$.

(ii) What can you say about the sign of roots α and β when $3 < m$.

(b) Let $f(x) = ax^4 + bx^3 + x^2 + cx + 10$. The remainders when f(x) is divided by $(1 + x)$ and $(3 - x)$ are 2 and 10 respectively. $f(2) = 26$.

Find the constants a, b, c and find the quotion and the remainder when f(x) is divided by $(1 + x)(3 - x)$.

Furthermore, deduce that the quetient and the remainder when $f(x + 2)$ is divided by $x^2 + 4x + 6$.

12. (a) Show that the range of the function $f(x) = \frac{x^2 + 2x + 5}{x^2 + 2x + 3}$; $x \in \mathbb{R}$ is $[1, 2]$.

(b) Resolve into partial fractions $\frac{x^3 + 1}{x^3 - 1}$.

22 A/L අපි [papers group]

22 A/L අපි [papers group]

(c) Sketch the graphs of $y = x^2 - 2|x| + 1$ and $y = 3\,\big|1 - |x|\big|$ in the same diagram.

Find all real values of x satisfying the inequality,

$$x^2 - 2|x| + 1 < 3\,\big|1 - |x|\big|.$$

Hence find all real values of a satisfying the inequality,

$$4x^2 - 4|x| + 1 < 3\,\big|1 - 2|x|\big|.$$

13. (a) Let $f(x) = \dfrac{ax + b}{(x+2)^2}$; $x \neq -2$.

There is a turning point at the point $(0, \frac{1}{4})$ for the function $y = f(x)$.

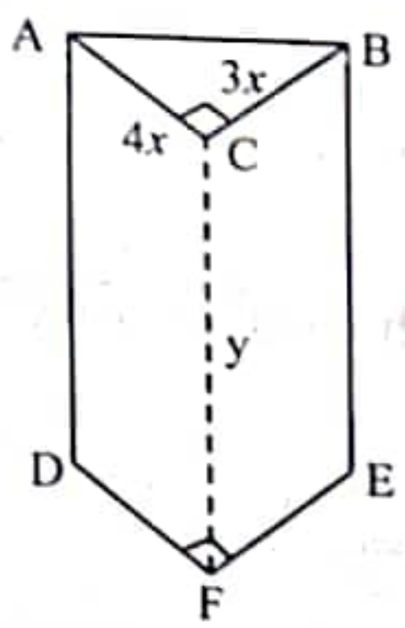
Find the constants 'a' and b'.

Show that $f''(x) = \dfrac{2(x-1)}{(x+2)^4}$; $x \neq -2$.

Sketch the graph of $y = f(x)$, indicating the asymptotes, turning points and the infletion points.

Here $f''(x)$ is the second derivative of $f(x)$.

(b)



Triangular part of ABC of the uniform cross section of the solid prizm is perpendicular to the edges AD, CF and BE as shown in the figure. Length of parallel edges are y cm and the faces ABC and DEF are right angle triangles. Here AC = $4x$ cm, BC = $3x$ cm and the volume of solid prizm is 1500 cm^3. Find an expression for the entire area (S) of the prizm in terms of x. Hence find y such that S is minimum.

Furthermore, find the rate of increase of S when the rate of increase of x is 0.3 cm s^{-1} when $x = 6$ cm.

14. (a) Write down cos (A + B) and cos (A - B) in terms of cos A, cos B, sin A, sin B.

Hence, deduce that, $\cos C + \cos D \equiv 2\cos\left(\dfrac{C+D}{2}\right)\cos\left(\dfrac{C-D}{2}\right)$.

If $A + B + C = \pi$ and $\cos B + \cos C = 4\sin^2 A/2$, show that the value of $\tan\dfrac{B}{2}\tan\dfrac{C}{2}$ is $\dfrac{1}{3}$.

(b) If $\sin^4 x + \cos^4 x + \sin 2x + k = 0$, show that $\dfrac{-3}{2} \leq k \leq \dfrac{1}{2}$ for x to be real.

Show that $\sin 2x = 1 - \sqrt{3}$ for $k = 0$ and hence write down the general solution.

(c) (i) Express $f(x) = \sin^2 x - 2\sqrt{3}\sin x \cos x - \cos^2 x$ in the form $f(x) = R\cos(2x - \alpha)$. R and α are the constants to be determined.

(ii) Find maximum and minimum values of $f(x)$ for $o < \alpha < \pi/2$.

(iii) Sketch the graph of $f(x)$ in the range $-\pi/2 \le x \le \pi/2$.

(d) In the equation, $\tan^{-1}(x+a) - \tan^{-1}(x+b) = \frac{\pi}{4}$, if there are real solutions for x, prove that $(a - b + 2)^2 \ge 8$.

22 A/L අපි [papers group

15. (a) Two trains A and B move along parallel lines to the same direction. Train A has uniform acceleration f_1 and the train B has uniform deceleration f_2. At a certain moment when the two trains at their front faces together the velocities of A and B are v_1 and v_2 respectively, ($v_1 < v_2$). After the movement of distance d, the trains again locate their front faces together with the velocities v_3 and v_4 respectively. Draw velocity time graphs for the trains in the same diagram.

Find an expression for f_1 in terms v_1, v_3 and d also find an expression for f_2 in terms of v_2, v_4 and d. At the moment when their velocities become equal which is V_0, the train A travels a distance d_1 and the remaining distance say d_2, then show that $2d_1(v_0 + v_3) = (v_1 + v_0)[t(v_0 + v_3) - 2d_2]$. At this moment when their velocities equal, the train B is distance d_3 ahead of A. Show that $d_3 : d_1 = (v_2 - v_1) : (v_1 + v_0)$.

(b) A, B and C are three airports. B is due east of A at a distance a km. C is due north of B at a distance a km. An aeroplane starts the journey from A and moves along AB, BC to reach C. Assume that the time elapse for the aeroplane to take the turn at B is negligible. A wind blows to north - west with uniform velocity $u\ km\,h^{-1}$. The aeroplane moves with a velocity $v\ km\,h^{-1}$ relative to the wind. Sketch the velocity triangles for the motion from A to B and for the motion from B to C in the same diagram.

Hence, show that the total time elapse for the aeroplanes to move from A to C is $\frac{a\sqrt{2(2v^2 - u^2)}}{v^2 - u^2}$, where $v > u$.

16. (a) In the triangle ABC the points P, Q, R lie on the lines AB, BC, CA such that AP : PB = 5 : 2, BQ : QC = 3 : 4, AR : RC = 1 : 6 respectively. Position vectors of the points B and C are $\underline{b}$ and $\underline{c}$ respectively relative to the vertex A.

Find the vector $\overrightarrow{AP}$, $\overrightarrow{BQ}$ and $\overrightarrow{CR}$.

Find the vector $\overrightarrow{AP} + \overrightarrow{BQ} + \overrightarrow{CR}$ in terms of $\underline{b}$ and $\underline{c}$.

If the point D is located on the side AB, such that the vector $\overrightarrow{AP} + \overrightarrow{BQ} + \overrightarrow{CR}$ is parallel to $\overrightarrow{CD}$, then find the location of D.

Using the scalar product for the above triangle ABC, show that $AC^2 + AB^2 - BC^2 = 2\,AB.AC\cos A$

(b) In a rhombus ABCD, the forces $2p\underline{i}$, $3p\underline{i}+3\sqrt{3}p\underline{j}$, $-4p\underline{i}$, $-4\sqrt{3}p\underline{j}$ act at the vertices A, B, C, D with the co-ordinates $(0,0)$, $(2,0)$, $(3,\sqrt{3})$, $(1,\sqrt{3})$ respectively. $\underline{i}$ and $\underline{j}$ are unit vectors along the axes OX and OY. Find the magnitude and the direction of the resultant force of the system of forces and the distance from which AB intersects line of action of the resultant from B. When an extra force is added to this system of forces at B along DB the new resultant force act parallel to CB, find this new force in the form of $a\underline{i}+b\underline{j}$. Find the magnitude, direction of the new resultant force and the distance from which AB intersects the line of action of this resultant from B. Find also the equation of line of action of the resultant force. Now a couple of forces G is added to the system in order to act the resultant at B, find the magnitude and direction of G.

22 A/L අපි [papers group]

17. (a) Particle P is projected from O on a horizontal plane with a velocity u at an angle α to the horizontal. Find the equation of the locus of P in terms of x and y.

A particle is to be projected with a velocity of u at an angle α to the horizontal so as just to pass through three equal rings of diameter d, placed in parallel vertical plane at a distance a apart, with their highest points in a horizontal straight line at a height H above the point of projection. Find the greatest height H and also the maximum range R, in terms of u and α. Show that $\tan\alpha = \dfrac{2\sqrt{Hd}}{a}$.

(b) Write down the sufficient and necessary statements for the equillibrium of set of forces act on a rigid body.

An uniform solid cone of weight w, radius a, vertical height 8a and the semi vertical angle θ, placed with its curved surface on a smooth plane inclined at an angle α to the horizontal. It is in equillibrium by the means of an inextensible string of length $\sqrt{65}$ a connected to the heighest point of its circular part and to a point on the plane.

Find the tension in the string and the reaction of the curved surface and the plane. Furthermore show that the distance from the reaction and the connected point on the plane is,

$$\frac{63a\,[78\cos\alpha+6\sin\alpha]}{\sqrt{65}\,[63\cos\alpha+16\sin\alpha]}.$$

Centre of gravity of a solid cone is $\frac{h}{4}$ from the bottom. h is the vertical height of the cone.

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