Confidential

NEW/OLD

Department of Examinations - Sri Lanka

G.C.E. (A/L) Examination - 2020

07 - Mathematics II

NEW/OLD Syllabus

Marking Scheme

This document has been prepared for the use of Marking Examiners. Some changes would be made according to the views presented at the Chief Examiners' meeting.

Amendments to be included

G. C. E (Advanced Level) Examination - 2020

07 - Mathematics II (New/Old Syllabus)

Distribution of Marks

Paper II

Part A	$= 10 \times 25$	$=$	250
Part B	$= 05 \times 150$	$=$	750

Total $= \dfrac{1000}{10}$

Final marks $= 100$

Common Techniques of Marking Answer Scripts.

It is compulsory to adhere to the following standard method in marking answer scripts and entering marks into the mark sheets.

1. Use a red color ball point pen for marking. (Only Chief/Additional Chief Examiner may use a mauve color pen.)
2. Note down Examiner's Code Number and initials on the front page of each answer script.
3. Write off any numerals written wrong with a clear single line and authenticate the alterations with Examiner's initials.
4. Write down marks of each subsection in a $\triangle$ and write the final marks of each question as a rational number in a $\square$ with the question number. Use the column assigned for Examiners to write down marks.

Example: **Question No. 03**

(i) ....
.... $\checkmark$ $\frac{4}{5}$
....

(ii) ....
.... $\checkmark$ $\frac{3}{5}$
....

(iii) ....
.... $\checkmark$ $\frac{3}{5}$
....

03 (i) $\frac{4}{5}$ + (ii) $\frac{3}{5}$ + (iii) $\frac{3}{5}$ = $\boxed{\frac{10}{15}}$

MCQ answer scripts: (Template)

1. Marking templets for G.C.E.(A/L) and GIT examination will be provided by the Department of Examinations itself. Marking examiners bear the responsibility of using correctly prepared and certified templates.
2. Then, check the answer scripts carefully. If there are more than one or no answers Marked to a certain question write off the options with a line. Sometimes candidates may have erased an option marked previously and selected another option. In such occasions, if the erasure is not clear write off those options too.
3. Place the template on the answer script correctly. Mark the right answers with a 'V' and the wrong answers with a 'X' against the options column. Write down the number of correct answers inside the

cage given under each column. Then, add those numbers and write the number of correct answers in the relevant cage.

Structured essay type and assay type answer scripts:

1. Cross off any pages left blank by candidates. Underline wrong or unsuitable answers. Show areas where marks can be offered with check marks.
2. Use the right margin of the overland paper to write down the marks.
3. Write down the marks given for each question against the question number in the relevant cage on the front page in two digits. Selection of questions should be in accordance with the instructions given in the question paper. Mark all answers and transfer the marks to the front page, and write off answers with lower marks if extra questions have been answered against instructions.
4. Add the total carefully and write in the relevant cage on the front page. Turn pages of answer script and add all the marks given for all answers again. Check whether that total tallies with the total marks written on the front page.

Preparation of Mark Sheets.

Except for the subjects with a single question paper, final marks of two papers will not be calculated within the evaluation board this time. Therefore, add separate mark sheets for each of the question paper. Write paper 01 marks in the paper 01 column of the mark sheet and write them in words too. Write paper II Marks in the paper II Column and wright the relevant details. For the subject 51 Art, marks for Papers 01, 02 and 03 should be entered numerically in the mark sheets.

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G.C.E. (A/L) Examination - 2020

07 - Mathematics II (New Syllabus)

1. Let $a, b, c \in \mathbb{R}$.

Show that $\begin{vmatrix} a & a & 2a+b+c \\ b & a+2b+c & b \\ a+b+2c & c & c \end{vmatrix} = -2(a+b+c)^3$.

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$$\begin{vmatrix} a & a & 2a+b+c \\ b & a+2b+c & b \\ a+b+2c & c & c \end{vmatrix} \xrightarrow{r_1 \to r_1+r_2+r_3} \begin{vmatrix} 2(a+b+c) & 2(a+b+c) & 2(a+b+c) \\ b & a+2b+c & b \\ a+b+2c & c & c \end{vmatrix}$$

$$\downarrow$$

$$2(a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 0 & a+b+c & 0 \\ a+b+2c & c & c \end{vmatrix} \xleftarrow{r_2 \to r_2 - br_1} 2(a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ b & a+2b+c & b \\ a+b+2c & c & c \end{vmatrix}$$

5 5

$$\downarrow$$

$$2(a+b+c)^2\begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ a+b+2c & c & c \end{vmatrix} \longrightarrow 2(a+b+c)^2\begin{vmatrix} 1 & 1 \\ a+b+2c & c \end{vmatrix} = 2(a+b+c)^3.$$

5 5

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2. Let $\mathbf{A} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 3 & 2 \\ -1 & 2 \\ 1 & 0 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} -1 & 0 \\ 1 & 3 \end{pmatrix}$. Find $\mathbf{AB}$ and $\mathbf{BC}$.

Verify that $\mathbf{A(BC)} = \mathbf{(AB)C}$.

$$AB = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 3 & 2 \\ -1 & 2 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 3-1+0 & 2+2+0 \\ 0-1+1 & 0+2+0 \end{pmatrix}$$ 5

$$= \begin{pmatrix} 2 & 4 \\ 0 & 2 \end{pmatrix}$$ 5

$$BC = \begin{pmatrix} 3 & 2 \\ -1 & 2 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} -3+2 & 0+6 \\ 1+2 & 0+6 \\ -1+0 & 0+0 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 6 \\ 3 & 6 \\ -1 & 0 \end{pmatrix}. \quad 5$$

$$\text{Now, } A(BC) = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} -1 & 6 \\ 3 & 6 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 12 \\ 2 & 6 \end{pmatrix} \text{ ---------- } (1)$$

5

$$(AB)C = \begin{pmatrix} 2 & 4 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 2 & 12 \\ 2 & 6 \end{pmatrix} \text{ ---------- } (2)$$

By (1) and (2),

$A\,(BC) = (AB)C.$ 5

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3. The mean and the standard deviation of a set of 10 observations are 5 and 10, respectively. Find the sum and the sum of squares of these observations.
If another observation of value 5 is added to this set, find the new values of the mean and the standard deviation.

$n = 10, \qquad \mu = 5, \quad \sigma = 10.$

$$\mu = \frac{\sum_{i=1}^{10} x_i}{10} \Rightarrow \sum_{i=1}^{10} x_i = 5 \times 10 = 50 \quad 5$$

$$\sigma^2 = \frac{\sum_{i=1}^{10} x_i^2}{n} - \mu^2$$

$$100 = \frac{\sum_{i=1}^{10} x_i^2}{10} - 25.$$

$$\sum_{i=1}^{10} x_i^2 = 100 \times 10 + 10 \times 25 = 1250. \quad 5$$

new values of $\sum_{i=1}^{10} x_i$ and $\sum_{i=1}^{10} x_i^2$, are (50+5) and $(1250+5^2)$ respectively. 5

New mean $\frac{55}{11} = 5.$ 5

New standard deviation $= \sqrt{\frac{1275-11\times 25}{11}} = \sqrt{\frac{1000}{11}} = \sqrt{90.9}.$ 5

25

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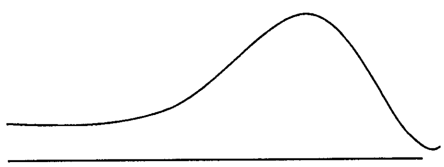
4. The mean, median and the standard deviation of a distribution are 28, 32 and 5, respectively. Calculate Karl Pearson's coefficient of skewness and describe the shape of the distribution.
Is the mean a fair measurement of the central tendency for this distribution? Give reasons for your answer.

Karl Pearson's coefficient of skewness

$= \frac{3(\text{mean} - \text{median})}{\text{standard deviation}}$ 5

$= \frac{3\,(28-32)}{5} = -2.4.$ 5

This distribution is negatively skewed.



More data of this distribution lie closer to the right tail of the distribution. 5

Mean of this distribution is not a fair measure of central tendency. 5

The reason is that the given distribution is not a symmetric distribution. 5

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5. The speed of vehicles travelling on a certain section of a highway, is normally distributed with mean 90 km h^{-1} and standard deviation 10 km h^{-1}. Find the probability that the speed of a randomly selected vehicle is between 85 km h^{-1} and 100 km h^{-1}.

Let X be the random variable that represents the speed of a randomly selected vehicle.

$X \sim N(90, 10^2)$

$P(85 < X < 100)$ 5

$= P\left(\frac{85-90}{10} < Z < \frac{100-90}{10}\right)$ 5

$= P(-0.5 < Z < 1)$

$= P(Z < 1) - P(Z < -0.5)$ 5

$= 0.8413 - 0.3085$ 5

$= 0.5328$ 5

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6. It is found from previous records that 10% of the bolts produced by a machine are defective. If 5 bolts produced by this machine are chosen at random, find the probability that
   (i) exactly 3 bolts are defective,
   (ii) more than 2 bolts are **non-defective**.

(i) P (getting a defective bolt) = 0.1.

Let X be the number of defective bolts out of 5.

$X \sim B(5,\ 0.1)$

$P(X = 3) = {}^5C_3(0.1)^3(0.9)^2$ 5

$= \frac{5!}{3!2!} \times 0.001 \times 0.81$

$= 10 \times 0.001 \times 0.81 = 0.0081$ 5

10

(ii) P (getting a non-defective bolt) $= 0.9$

More than 2 non-defective is the same as "at most 2 defective"

$$P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$$ 5

$$= {}^5C_0(0.1)^0(0.9)^5 + {}^5C_1(0.1)(0.9)^4 + {}^5C_2(0.1)^2(0.9)^3$$ 5

$$= 0.59049 + 5\times(0.1)\times 0.6561 + 10\times(0.01)\times 0.729$$

$$= 0.99144$$ 5

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7. In a group consisting of 30 cricketers, 20 have played for club A and 15 have played for club B. Every cricketer has played for at least one of these clubs. Find the probability that a cricketer selected at random has played for club B, given that he has played for club A.

Let X be the event that a cricketer plays for club A and B be the event that a cricketer plays for club Y.

$$P(X) = \frac{20}{30} \qquad P(Y) = \frac{15}{30}$$ 5

$$P(X \cup Y) = \frac{30}{30}$$ 5

$$P(X \cap Y) = P(X) + P(Y) - P(X \cup Y)$$ 5

$$= \frac{20}{30} + \frac{15}{30} - \frac{30}{30}$$

$$= \frac{1}{6} \quad 5$$

$$P(Y \mid X) = \frac{P(X \cap Y)}{P(X)} = \frac{1/6}{2/3} = \frac{1}{4} \quad 5$$

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8. Let A and B be two events of a sample space S such that $P(A) = \frac{3}{8}$, $P(A \cap B) = \frac{1}{8}$ and $P(A \cup B) = \frac{3}{4}$.
Find
(i) $P(B)$, (ii) $P(A' \cap B)$ and (iii) $P(A' | B)$.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \quad 5$$

$$\left.\begin{aligned} P(B) &= P(A \cup B) - P(A) + P(A \cap B) \\ &= \frac{3}{4} - \frac{3}{8} + \frac{1}{8} = \frac{6-3+1}{8} = \frac{1}{2} \end{aligned}\right\} \quad 5$$

$$P(A' \cap B) = P(B) - P(A \cap B) \quad 5$$

$$= \frac{1}{2} - \frac{1}{8} = \frac{4-1}{8} = \frac{3}{8} \quad 5$$

$$P(A' \mid B) = \frac{P(A' \cap B)}{P(B)} = \frac{3/8}{1/2} = \frac{3}{4} \quad 5$$

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9. The probability mass function of a discrete random variable X is given below:

x	1	2	3	4	5
$P(X=x)$	p	$2p$	p	$2p$	p

Find the value of the constant p and show that $E(X) = 3$.
Let Y be the random variable given by $3X-4$. Find $P(Y>X)$.

$\sum P(X=x)=1$ 5

$\therefore p+2p+p+2p+p=1$

$\therefore p=\frac{1}{7}.$ 5

$E(X)=\sum xP(X=x)$ 5

$=1\times p+2\times 2p+3\times p+4\times 2p+5\times p$

$=21\times\frac{1}{7}=3.$ 5

Since $Y=3X-4,\ P(Y>X)=P(X>2)$

$=1-P(X=1)-P(X=2)$

$1-\frac{1}{7}-\frac{2}{7}=\frac{4}{7}$ 5

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10. A continuous random variable X has probability density function $f(x)$ given by

$$f(x)=\begin{cases} kx-x^2 & , \text{ if } 0\le x\le 1, \\ 0 & , \text{ otherwise}, \end{cases}$$

where k is a constant.

Show that $k=\frac{8}{3}$ and find $E(X)$.

$$\int_{-\infty}^{\infty} f(x)dx=1 \quad 5$$

$$\therefore \int_{-\infty}^{0} 0dx+\int_{0}^{1}(kx-x^2)dx+\int_{1}^{\infty}0dx=1 \quad 5$$

$$\therefore \left[\frac{kx^2}{2}-\frac{x^3}{3}\right]_0^1=1$$

Hence, $\frac{k}{2}-\frac{1}{3}-0=1$ and so

$$k=\frac{8}{3}.$$

$$E(X)=\int_0^1 xf(x)dx \quad 5$$

$$=\int_0^1 (kx^2-x^3)dx$$

$$=\left[\frac{kx^3}{3}-\frac{x^4}{4}\right]_0^1 \quad 5$$

$$=\frac{8}{9}-\frac{1}{4}=\frac{23}{36}. \quad 5$$

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Part B

11. A factory manufactures tables and chairs. The production of each item requires three operations: cutting, assembling and finishing.

For cutting, assembling and finishing, the maximum number of hours that can be used are 600, 160 and 280, respectively. The following table gives the number of hours required for each operation in producing each item and the profit per item sold.

	Number of hours for cutting	Number of hours for assembling	Number of hours for finishing	Profit (in thousands of rupees)
Table	5	1	1	12
Chair	6	2	4	15

The factory wishes to maximize the profit.

(i) Formulate this as a linear programming problem.

(ii) Sketch the feasible region.

(iii) Using the graphical method, find the solution of the problem formulated in part (i) above.

(iv) Due to shortage of storage space, the factory has to limit the total number of tables and chairs produced to at most 108. Find the decrease in the profit due to above limitation, if the factory still wishes to maximize the profit.

(i) Let x be the number of tables to be manufactured and y be the number of chairs to be manufactured.

The linear programming problem:

Need to maximize $z = 12x + 15y$ (10)

subject to the following conditions;

$5x + 6y \leq 600,$ (10)

$x + 2y \leq 160,$ (10)

$x + 4y \leq 280,$ (10)

$x \geq 0\,,\ y \geq 0.$ (10)

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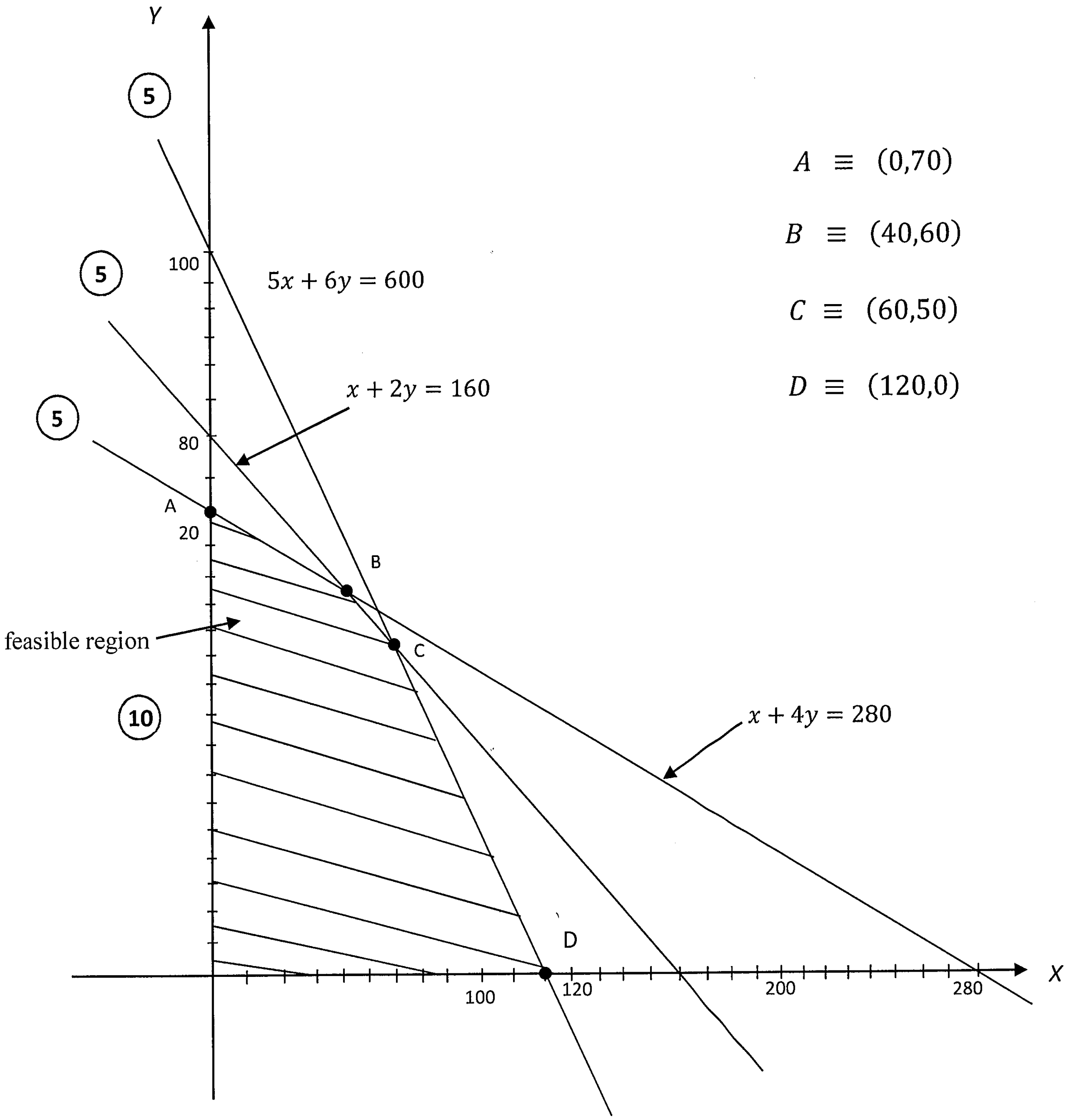
(ii)



$A \equiv (0,70)$

$B \equiv (40,60)$

$C \equiv (60,50)$

$D \equiv (120,0)$

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(iii)

Point	The value of $z = 12x + 15y$
$A \equiv (0, 70)$	$12 \times 0 + 15 \times 70 = 1050$
$B \equiv (40, 60)$	$12 \times 40 + 15 \times 60 = 1380$
$C \equiv (60, 50)$	$12 \times 60 + 15 \times 50 = 1470$
$D \equiv (120, 0)$	$12 \times 120 + 15 \times 0 = 1440$

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maximum $z = 1470$ and it occurs at the point $C \equiv (60, 50)$. 5

To maximize the profit, the factory should manufacture 60 tables and 50 chairs. 10

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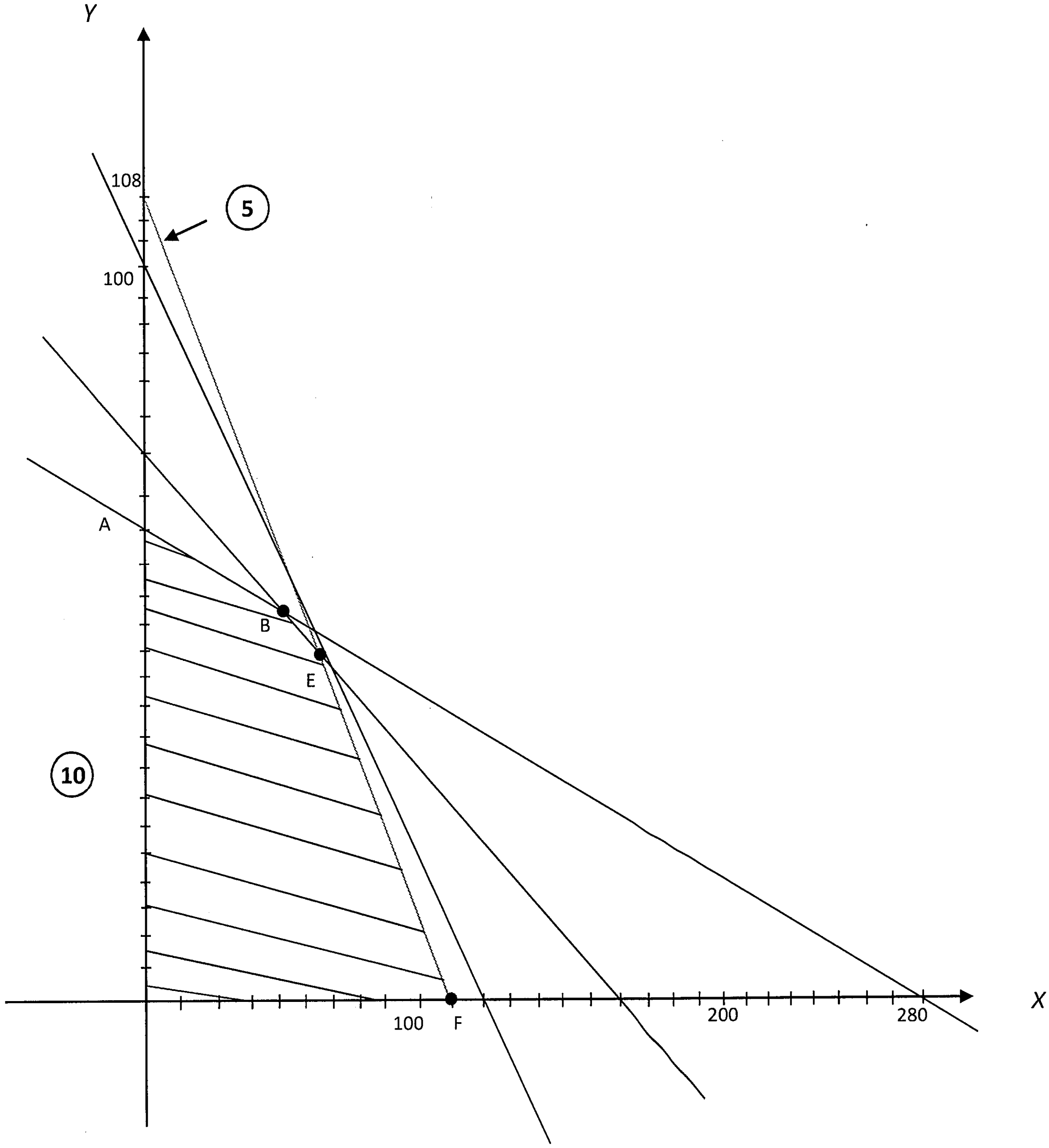
(iv) With the new constraint $x + y \leq 108$, the feasible region is given below:



Point	The value of z
$A \equiv (0, 70)$	1050
$B \equiv (40, 60)$	1380
$E \equiv (56, 52)$	1452
$F \equiv (108, 0)$	1296

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maximum $z = 1452$ and it occurs at the point $E \equiv (56, 52)$ 5

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$\therefore$ The decrease in the profit = 1470 - 1452 = 18 Thousand rupees 5

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12. (*a*) Let $\mathbf{A} = \begin{pmatrix} 4 & 7 \\ -1 & -2 \end{pmatrix}$. Write down $\mathbf{A}^{-1}$.

Let $\mathbf{B} = \begin{pmatrix} -1 & 3 \\ 0 & 1 \end{pmatrix}$.

Find the matrix $\mathbf{C}$ such that $\mathbf{AC} = \mathbf{B}$ and show that

$$\mathbf{AC} - \mathbf{CA} = \begin{pmatrix} 20 & 43 \\ -11 & -20 \end{pmatrix}.$$

Find the matrix $\mathbf{D}$ such that $\mathbf{AC} - \mathbf{DA} = \mathbf{O}$, where $\mathbf{O}$ is the zero matrix of order 2.

(*b*) Let $a \in \mathbb{R}$. Write the pair of **simultaneous** equations

$$(a-5)x + 3y = a$$
$$-4x + (a+2)y = 1$$

in the form $\mathbf{PX} = \mathbf{Q}$, where $\mathbf{X} = \begin{pmatrix} x \\ y \end{pmatrix}$, and $\mathbf{P}$ and $\mathbf{Q}$ are matrices to be determined.

Express $\Delta = \begin{vmatrix} (a-5) & 3 \\ -4 & (a+2) \end{vmatrix}$ as a quadratic function of a.

Show that the roots of the equation $\Delta = 0$ are $a = 1$ and $a = 2$.

Show that the above pair of equations has

(i) infinitely many solutions when $a = 1$,

(ii) no solution when $a = 2$,

(iii) a unique solution when $a = 3$.

(a) Since $A = \begin{pmatrix} 4 & 7 \\ -1 & -2 \end{pmatrix}$, $A^{-1} = \frac{1}{(-8+7)}\begin{pmatrix} -2 & -7 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} 2 & 7 \\ -1 & -4 \end{pmatrix}$. 10

Now, $B = \begin{pmatrix} -1 & 3 \\ 0 & 1 \end{pmatrix}$.

Note that the matrix C such that $AC = B$ is given by $C = A^{-1}B$. 10

$\therefore\ C = \begin{pmatrix} 2 & 7 \\ -1 & -4 \end{pmatrix}\begin{pmatrix} -1 & 3 \\ 0 & 1 \end{pmatrix}$ 5

$= \begin{pmatrix} -2 & 13 \\ 1 & -7 \end{pmatrix}$. 10

Next, $AC - CA = \begin{pmatrix} 4 & 7 \\ -1 & -2 \end{pmatrix}\begin{pmatrix} -2 & 13 \\ 1 & -7 \end{pmatrix} - \begin{pmatrix} -2 & 13 \\ 1 & -7 \end{pmatrix}\begin{pmatrix} 4 & 7 \\ -1 & -2 \end{pmatrix}$

5 5

$= \begin{pmatrix} -1 & 3 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} -21 & -40 \\ 11 & 21 \end{pmatrix}$

$= \begin{pmatrix} 20 & 43 \\ -11 & -20 \end{pmatrix}$. 10

The matrix D such that $AC - DA = 0$ is given by $B - DA = 0$. 5

$\therefore\ DA = B$ and therefore, $D = BA^{-1}$.

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$\therefore\ D = \begin{pmatrix} -1 & 3 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 2 & 7 \\ -1 & -4 \end{pmatrix}$ 5

$= \begin{pmatrix} -5 & -19 \\ -1 & -4 \end{pmatrix}$. 10

85

(b) The pair of simultaneous equations

$(a - 5)\, x + 3y = a$

$-4x + (a + 2)y = 1$

is equivalent to

$$\begin{pmatrix} a-5 & 3 \\ -4 & a+2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a \\ 1 \end{pmatrix}. \quad 10$$

This is of the form **PX** = **Q**, where **P** $= \begin{pmatrix} a-5 & 3 \\ -4 & a+2 \end{pmatrix}$ and Q $= \begin{pmatrix} a \\ 1 \end{pmatrix}$. 5

5

Now, $\Delta = \begin{vmatrix} (a-5) & 3 \\ -4 & (a+2) \end{vmatrix} = (a-5)(a+2) + 12$ 5

$= a^2 - 3a + 2.$ 5

$\Delta = 0 \Leftrightarrow a^2 - 3a + 2 = 0$ 5

$\Leftrightarrow (a-2)(a-1) = 0$

$\Leftrightarrow a = 2$ or $a = 1.$

Therefore, the roots of $\Delta = 0$ are $a = 2$ and $a = 1$ 5

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(i) $a = 1$:

In this case, the pair of equations becomes

$-4x + 3y = 1.$ 5

The solutions are given by $x = t,\ y = \frac{1}{3}(1 + 4t)$, where $t \in \mathbb{R}$. 5

$\therefore$ The pair of equations has infinitely many solutions.

(ii) $a = 2$:

In this case, the pair of equations become

$$\left.\begin{aligned} -3x+3y=2 \\ -4x+4y=1 \end{aligned}\right\}$$

Since, these give us

$$\left.\begin{aligned} -x+y=\frac{2}{3} \\ -x+y=\frac{1}{4} \end{aligned}\right\}$$, the pair of equations has no solution. 5

(iii) $a = 3$:

In this case, the pair of equations become

$$\left.\begin{aligned} -2x+3y=3 \\ -4x+5y=1 \end{aligned}\right\}$$ 5

and hence, $x = 6$ and $y = 5$. 5

The pair of equations has a unique solution.

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13. (*a*) An unbiased cubic die with faces marked 1, 2, 2, 3, 3, 4 is tossed twice. Let A be the event that the sum of the numbers obtained is 4 and B be the event that the sum of the numbers obtained is even.

Find $P(A)$, $P(B)$ and $P(A\,|\,B)$.

(*b*) Four digits from the set of digits $\{1, 2, 3, 4, 5, 6\}$ are chosen without replacement and a 4-digit number is made.

(i) How many different 4-digit numbers can be made?

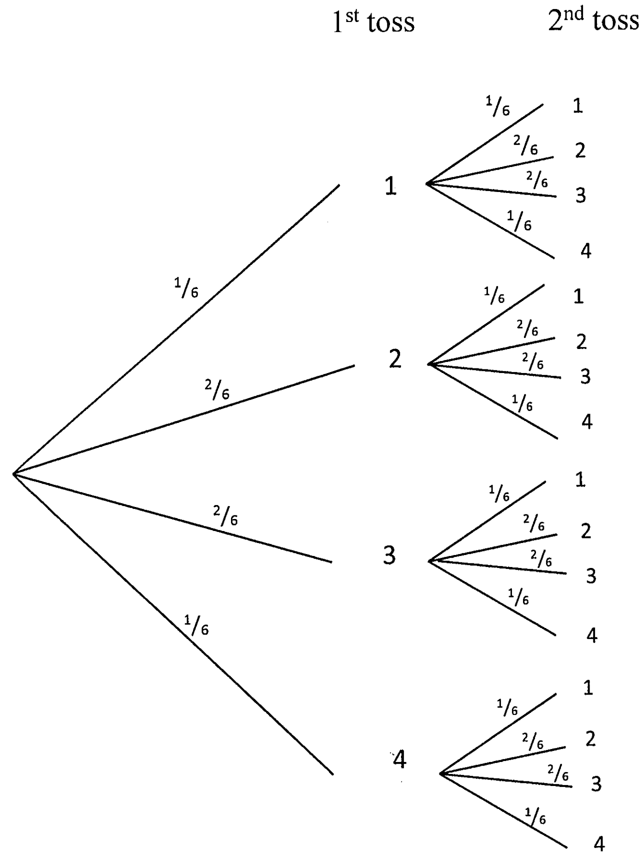
(ii) How many of these 4-digit numbers start with 3 or 5?

(*c*) A team of four people must be selected from a group of four males and two females.

(i) How many different teams of four people can be selected?

(ii) Find the probability that both females are selected to these teams.

Let A be the event that the sum of the numbers obtained is equal to 4 and B be the event that the sum of the numbers obtained is an even number.



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$$P(A)=\left(\frac{1}{6}\times\frac{2}{6}\right)+\left(\frac{2}{6}\times\frac{2}{6}\right)+\left(\frac{2}{6}\times\frac{1}{6}\right)=\frac{10}{36}=\frac{5}{18}$$ 5

$$P(B)=+\left(\frac{1}{6}\times\frac{1}{6}\right)+\left(\frac{1}{6}\times\frac{2}{6}\right)+\left(\frac{2}{6}\times\frac{2}{6}\right)+\left(\frac{2}{6}\times\frac{1}{6}\right)+\left(\frac{2}{6}\times\frac{1}{6}\right)+\left(\frac{2}{6}\times\frac{2}{6}\right)+\left(\frac{1}{6}\times\frac{2}{6}\right)+\left(\frac{1}{6}\times\frac{1}{6}\right)$$ 20

$$=\frac{1}{36}+\frac{2}{36}+\frac{4}{36}+\frac{2}{36}+\frac{2}{36}+\frac{4}{36}+\frac{2}{36}+\frac{1}{36}$$

$$=\frac{18}{36}=\frac{1}{2}.$$ 5

$$P(A|B)=\underbrace{\frac{P(A\cap B)}{P(B)}}_{5}=\underbrace{\frac{P(A)}{P(B)}}_{10}=\frac{5/18}{1/2}=\frac{5}{9}$$ 5

75

(b) (i) The required answer $= {}^6P_4 = \dfrac{6!}{2!} = 6\times5\times4\times3 = 360.$ 5

10

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(ii) The number of ways in which the last 3 digits can be made $= {}^5P_3 = \dfrac{5!}{2!} = 5\times4\times3 = 60$ 10

$\therefore$ The number of 4–digit numbers starting with 3 or 5 $= 2\times60 = 120.$ 10

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(c) (i) The number of different 4-member teams that can be selected out of 6 people

$$= {}^6C_4 = \frac{6!}{4!2!} = \frac{6\times5}{2\times1} = 15$$ 5

10

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(ii) The number of ways in which both females can be included $= 1.$

The number of of ways in which 2 out of 4 males can be selected

$$= {}^4C_2 = \frac{4!}{2!2!} = 6.$$ 10

$\therefore$ The number of 4–member teams with both females $= 6.$ 5

$\therefore$ The probability that the both females are selected $= \dfrac{6}{15} = \dfrac{2}{5}.$ 10

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14. A box X contains 4 red cards and 6 blue cards. A box Y contains 3 red cards and 2 blue cards. A biased coin with $\frac{2}{3}$ as the probability of getting a head is tossed. If the outcome is a head, 2 cards are drawn from the box X, at random without replacement, and if it is a tail, 2 cards are drawn from the box Y, at random without replacement. Find the probability that

(i) both cards drawn are red,

(ii) at least one of the cards drawn is red,

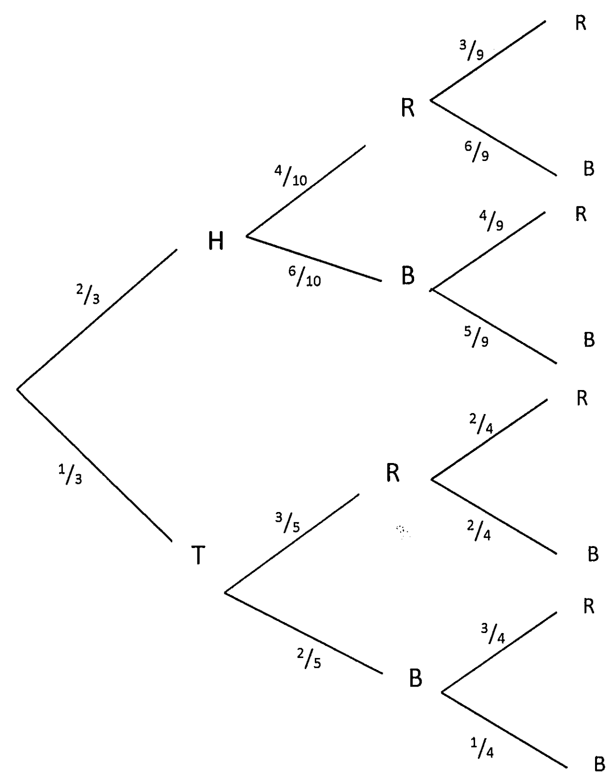
(iii) the two cards drawn are of different colours,

(iv) the two cards drawn are of different colours, given that at least one of the cards drawn is red.

(i) Let H be the event of getting a head and T be the event of getting a Tail.

Also, let R be the event of drawing a red card and B be the event of drawing a blue card.

Since $P(H) = \frac{2}{3}$, we have $P(T) = \frac{1}{3}$. 10



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$$P \text{ (drawing 2 red cards)} = \left(\frac{\cancel{2}}{\cancel{3}} \times \frac{4}{\cancel{10}_5} \times \frac{\cancel{3}}{9}\right) + \left(\frac{1}{\cancel{3}} \times \frac{\cancel{3}}{5} \times \frac{2}{4}\right) \quad 15$$

$$= \frac{4}{45} + \frac{2}{20}$$

$$= \frac{17}{90}. \quad 10$$

$\boxed{55}$

(ii) P (drawing at least 1 red card)

$$= \left(\frac{\cancel{2}}{\cancel{3}}\right)\left(\frac{4}{\cancel{10}_5}\right)\left(\frac{\cancel{3}}{9}\right) + \left(\frac{\cancel{2}}{\cancel{3}}\right)\left(\frac{4}{\cancel{10}_5}\right)\left(\frac{\cancel{6}^2}{9}\right) + \left(\frac{\cancel{2}}{\cancel{3}}\right)\left(\frac{\cancel{6}^2}{\cancel{10}_5}\right)\left(\frac{4}{9}\right) + \left(\frac{1}{\cancel{3}}\right)\left(\frac{\cancel{3}}{5}\right)\left(\frac{\cancel{2}}{\cancel{4}_2}\right) + \left(\frac{1}{\cancel{3}}\right)\left(\frac{\cancel{3}}{5}\right)\left(\frac{\cancel{2}}{\cancel{4}_2}\right) + \left(\frac{1}{\cancel{3}}\right)\left(\frac{\cancel{2}}{5}\right)\left(\frac{\cancel{3}}{\cancel{4}_2}\right) \quad 5$$

30

$$= \frac{4}{45} + \frac{8}{45} + \frac{8}{45} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} \quad 5$$

$$= \frac{20}{45} + \frac{3}{10} = \frac{\cancel{335}^{67}}{\cancel{450}_{90}} = \frac{67}{90}. \quad 5$$

$\boxed{45}$

(iii) P (drawing two cards of different colours)

$$= \left(\frac{\cancel{2}}{3}\right)\left(\frac{4}{\cancel{10}_5}\right)\left(\frac{\cancel{6}^2}{9}\right) + \left(\frac{\cancel{2}}{\cancel{3}}\right)\left(\frac{\cancel{6}^2}{\cancel{10}_5}\right)\left(\frac{4}{9}\right) + \left(\frac{1}{\cancel{3}}\right)\left(\frac{\cancel{3}}{5}\right)\left(\frac{\cancel{2}}{\cancel{4}_2}\right) + \left(\frac{1}{\cancel{3}}\right)\left(\frac{\cancel{2}}{5}\right)\left(\frac{\cancel{3}}{\cancel{4}_2}\right) \quad 20$$

$$= \frac{8}{45} + \frac{8}{45} + \frac{1}{10} + \frac{1}{10} \quad = \frac{25}{45}. \quad 10$$

$\boxed{30}$

(iv) A - getting at least one red card.

C- getting two cards of different colours.

$$A \cap C = C \quad 5$$

$$P(C|A) = \frac{P(A\cap C)}{P(A)} = \frac{P(C)}{P(A)} = \frac{25/45}{67/90} = \frac{50}{67}. \quad 5$$

5 5

20

Alternative Method

(i) Let H be the event of getting a head while T be the event of getting a Tail

$$P(H) = \frac{2}{3} \; and \; P(T) = \frac{1}{2} \quad 10$$

P (drawing two red cards)

$= P$ (drawing two red cards from box X)

$+ P$ (drawing two red cards from box Y) 10

10 10

$$= \frac{2}{3} \times \frac{{}^4C_2}{{}^{10}C_2} + \frac{1}{3} \times \frac{{}^3C_2}{{}^5C_2}$$

$$= \frac{2}{\cancel{3}} \times \frac{\cancel{6}^2}{45} + \frac{1}{\cancel{3}} \times \frac{\cancel{3}}{10} = \frac{17}{90}. \quad 5$$

5 5

55

(ii) P (drawing at least 1 red card)

$= P$ (drawing two red cards from box X)

$+ P$ (drawing 1 red and 1 blue card from box X) 10

$+ P$ (drawing 2 red cards from box Y)

$+ P$ (drawing 1 red and 1 blue card from box Y)

5 5 5 5

$$=\frac{2}{3}\times\frac{{}^4C_2}{{}^{10}C_2}+\frac{2}{3}\times\frac{{}^4C_1\times{}^6C_1}{{}^{10}C_2}+\frac{1}{3}\times\frac{{}^3C_2}{{}^5C_2}+\frac{1}{3}\times\frac{{}^3C_1\times{}^2C_1}{{}^5C_2}$$

$$=\frac{2}{\cancel{3}}\times\frac{\cancel{6}^{2}}{45}+\frac{2}{\cancel{3}}\times\frac{4\times\cancel{6}^{2}}{45}+\frac{1}{\cancel{3}}\times\frac{\cancel{3}}{10}+\frac{1}{\cancel{3}}\times\frac{\cancel{3}\times 2}{10}$$ 10

$$\frac{4}{45}+\frac{16}{45}+\frac{1}{10}+\frac{2}{10}=\frac{67}{90}.$$ 5

45

(iii) P (drawing two cards of different colours)

$= P$ (drawing 1 red and 1 blue card from box X) 10

$+ P$ (drawing 1 red and 1 blue card from box Y)

5 5

$$\frac{2}{3}\times\frac{{}^4C_1\times{}^6C_1}{{}^{10}C_2}+\frac{1}{3}\frac{{}^3C_1\times{}^2C_1}{{}^5C_2}$$

$$=\frac{2}{3}\times\frac{4\times 6}{45}+\frac{1}{3}\times\frac{3\times 2}{10} \qquad =\frac{16}{45}+\frac{2}{20}=\frac{25}{45}$$ 10

30

(iv). Same as previous answer

20

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15. (a) The time X, measured in minutes, between consecutive arrivals of buses to a certain bus stop is exponentially distributed with probability density function

$$f(x)=\begin{cases}\lambda e^{-\lambda x} & , \quad x>0,\\ 0 & , \quad \text{otherwise},\end{cases}$$

where $\lambda\,(>0)$ is a parameter.

If the mean number of buses that arrive at the bus stop in an hour is 12, find the value of λ.

(i) After a bus arrives at the bus stop, find the probability that the time taken for the next bus to arrive at the bus stop is

(α) between one minute to three minutes,

(β) less than five minutes.

(ii) If it is given that five minutes has already passed from the arrival of a bus to the bus stop, find the probability that it takes at least an additional two minutes for the next bus to arrive.

12 busses for 60 minutes.

Average time between busses $=\dfrac{60}{12}=5.$ 5

$\therefore \lambda=\dfrac{1}{5}=0.2$ 5

10

(i) ($\propto$) Let X be the time between busses.

$P(1<X<3) = \int_1^3 \lambda e^{-\lambda x}dx$ 5

5

$= \left.\dfrac{\lambda e^{-\lambda x}}{-\lambda}\right|_1^3$ 5

$= \left.-e^{-\lambda x}\right|_1^3$

$= -e^{-3\lambda}+e^{-\lambda} = -e^{-0.2}-e^{-0.6}$ 10

25

(β) $P(X<5)=\int_{-\infty}^{5}\lambda e^{-\lambda x}dx$ 5

5

$$=\int_{-\infty}^{5}\lambda e^{-\lambda x}dx=-e^{-\lambda x}\Big|_{0}^{5}$$ 5

$$=1-\frac{1}{e}$$ 5

20

---

5 5

(ii) $P(X>5+2|X>5)$ $\frac{20}{d}=\frac{20}{20-d}\Rightarrow 40-20d=20d$

$$=\frac{P(X>7)}{P(X>5)}$$ 5

$$P(X>7)=1-P(X\le 7)=1-\int_{o}^{7}\lambda e^{-\lambda x}dx$$

5 5

5 $$=1-\left[-e^{-\lambda x}\right]_0^7=1+\left[e^{-\lambda x}\right]_0^7$$

$$=1+\left[-e^{-\lambda x}-1\right]=e^{-1.4}$$ 10

$$\therefore P(X>5+2|X>5)=\frac{e^{-1.4}}{1-(1-\frac{1}{e})}=\frac{e}{e^{1.4}}$$

$$=\frac{1}{e^{0.4}}$$ 10

50

(b) A continuous random variable X is uniformly distributed over the interval $[a, b]$. Find the values of a and b such that $P(X < 16) = 0.4$ and $P(X > 21) = 0.2$.

$X \sim$ uniform on $[a, b]$

$f(x) = \frac{1}{b-a}$ for $a \leq x \leq b$. 5

$P(X < 16) = \int_a^{16} f(x) = \int_a^{16} \frac{1}{b-a} dx = \frac{1}{b-a} [x]_a^{16}$ 5

5

$= \frac{16 - a}{6 - a}$ 5

It is given that $P(X < 16) = 0.4$.

$\therefore \frac{16-a}{b-a} = 0.4 \Rightarrow 16 - a = 0.4b - 0.4a.$

$\therefore 0.6a + 0.4b = 16$ ———— (1)

5

$P(X > 21) = \int_{21}^{b} \frac{1}{b-a} dx = \frac{1}{b-a} (b - 21)$ 5

It is given that $P(X > 21) = 0.2$.

$\therefore \frac{b-21}{b-a} = 0.2 \Rightarrow b - 21 = 0.2b - 0.2a.$

5

$\therefore 0.2a + 0.8b = 21$ ———— (2)

Form (1) and (2), $a = 11$ and $b = 23.5$.

5 5

45

16. Hundred students faced an entrance test. The frequency distribution of the marks they obtained is given in the following table:

Marks	frequency
0 – 20	15
20 – 40	20
40 – 60	40
60 – 80	15
80 – 100	10

(i) Estimate each of the following:

(a) the mean,

(b) the standard deviation,

(c) the median,

(d) the inter quartile range and

(e) the mode

of the marks.

(ii) After rescrutiny, it was discovered that the marks of two answer scripts should be changed as follows:

Marks before rescrutiny	Marks after rescrutiny
50	62
70	75

Find the mean of the new distribution of marks.

(a)

Marks	Mid-Point x_i	Frequency f_i	$f_i x_i$	$f_i x_i^2$
0 - 20	10	15	150	1500
20 - 40	30	20	600	18000
40 - 60	50	40	2000	100000
60 - 80	70	15	1050	73500
80 - 100	90	10	900	81000
Total		100	4700	274,000

5 5 10

Mean $= \dfrac{\Sigma f_i x_i}{\Sigma f_i} = \dfrac{4700}{100} = 47$ 5 5 5

35

(b) $\sigma^2 = \dfrac{1}{\Sigma f_i}\left(\Sigma f_i x^2 - \Sigma f_i \left(\dfrac{\Sigma f_i x_i}{\Sigma f_i}\right)^2\right)$ 5

$= \dfrac{1}{100}(274,000 - 100 \times 47^2)$ 5

$= \dfrac{53,100}{100} = 531.$ 5

$\sigma = \sqrt{531}.$ 5

20

(c)

$\dfrac{m-40}{60-40} = \dfrac{50-35}{75-35}$, where m is the median. 10

10

$m = \left(\dfrac{15}{\cancel{40}_2}\right) \times \cancel{20} + 40 = 7.5 + 40 = 47.5$

20

(d) Let ℓ_1 be the lower quartile and ℓ_2 be the upper quartile

10

$\dfrac{\ell_1 - 20}{40 - 20} = \dfrac{25 - 15}{35 - 15}$

$\ell_1 = \dfrac{10}{20} \times 20 + 20 = 30$ 5

ℓ_2 is the 75th observation

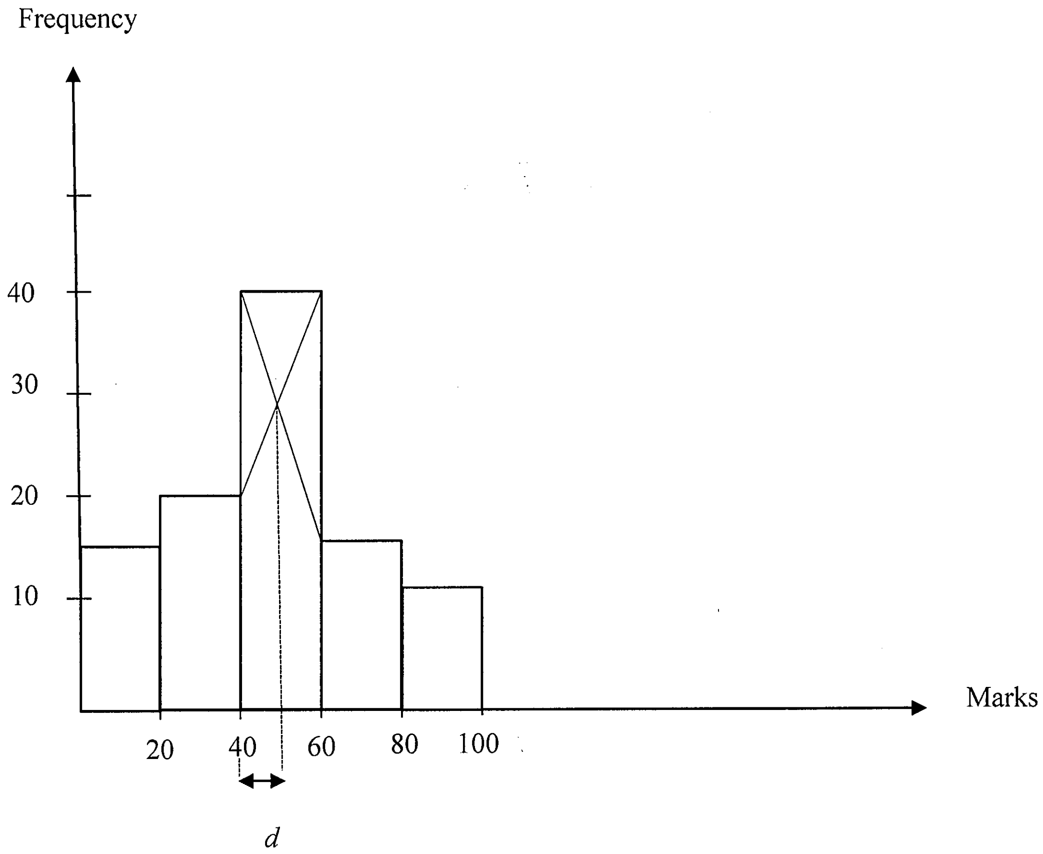
$\therefore \ell_2 = 60$ 5

Inter quartile range = $\ell_1 - \ell_2 = 60 - 30 = 30$ 5 5

30

(e)

The modal class is 40 - 60 5



$$\frac{20}{d} = \frac{20}{20-d} \Rightarrow 40 - 20d = 20d$$ 10

$\therefore \ d = 1$ 5

$\therefore$ The mode = $40 + d = 41$ 5

25

5 5

Marks	Mid-Point x_i	Frequency f_i	$f_i x_i$
0 - 20	10	15	150
20 - 40	30	20	600
40 - 60	50	39	1950
60 - 80	70	16	1120
80 - 100	90	10	900
Total		100	4720

New Mean $= \frac{4720}{100} = 47.2$ 10

20

---

17. The duration of activities of a project and the flow of activities are given in the following table.

Activity	**Preceding Activity (Activities)**	**Duration (in Weeks)**
A	–	03
B	A	08
C	A	05
D	A	03
E	B	06
F	C	03
G	E, F	04
H	D, F	06
I	G, H	03

(i) Construct the project network.

(ii) Prepare an activity schedule that includes earliest start time, earliest finish time, latest start time, latest finish time and float for each activity.

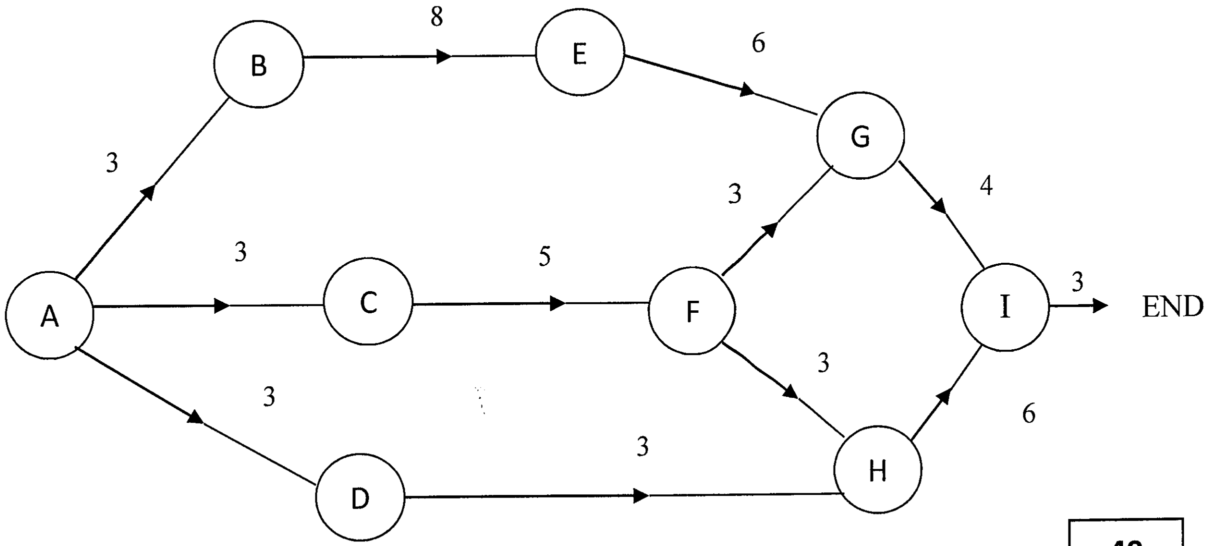
(iii) Find the total duration of the project.

(iv) What are the activities that can be delayed without extending the total duration of the project?

(v) Write down the critical path of this project.

(vi) Suppose that the duration of the activity D has to be extended by two weeks due to an unexpected matter. Determine whether the project could still be completed within the total duration calculated in part (iii) above.

(i)



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(ii)

Assuming that project starts on week 1

Activity	ES	EF	LS	LF	Float
(A)	1	1 + 3 - 1 = 3	3 - 3 + 1 = 1	4 - 1 = 3	0
(B)	3 + 1 = 4	4 + 8 - 1 = 11	11 - 8 + 1 = 4	12 - 1 = 11	0
C	3 + 1 = 4	4 + 5 - 1 = 8	12 - 5 + 1 = 8	13 - 1 = 12	4
D	3 + 1 = 4	4 + 3 - 1 = 6	15 - 3 + 1 = 13	16 - 1 = 15	9
(E)	11 + 1 = 12	12 + 6 - 1 = 17	17 - 6 + 1 = 12	18 - 1 = 17	0
F	8 + 1 = 9	9 + 3 - 1 = 11	15 - 3 + 1 = 13	16 - 1 = 15	4
(G)	17 + 1 = 18	18 + 4 - 1 = 21	21 - 4 + 1 = 18	22 - 1 = 21	0
H	11 + 1 = 12	12 + 6 - 1 = 17	21 - 6 + 1 = 16	22 - 1 = 21	4
(I)	21 + 1 = 22	22 + 3 - 1 = 24	24 - 3 + 1 = 22	24	0

10 for each column

50

(iii)

Total duration of the project (from the table) = 24

$\boxed{10}$

(iv)

C, D, F and H

$\boxed{15}$

(v)

A B E G I

$\boxed{15}$

(vi)

Activity D has a floating time of 9 weeks. 10

Therefore, the project can still be completed even if the activity D is delayed by 2 weeks. 10

$\boxed{20}$

G.C.E. (A/L) Examination - 2020

07 - Mathematics II (Old Syllabus)

1. Find all real values of x satisfying the inequality $x \geq \dfrac{6}{x+1}$.

$$x \geq \frac{6}{x+1}$$

$$\Leftrightarrow x - \frac{6}{x+1} \geq 0$$

$$\Leftrightarrow \frac{x(x+1)-6}{x+1} \geq 0$$

$$\Leftrightarrow \frac{x^2+x-6}{(x+1)} \geq 0$$

$$\Leftrightarrow \frac{(x+3)(x-2)}{(x+1)} \geq 0.$$ 5

	$-\infty < x < -3$	$-3 < x < -1$	$-1 < x < 2$	$2 < x < \infty$
Sign of $\dfrac{(x+2)(x-2)}{(x+1)}$	$\dfrac{(-)(-)}{(-)} = (-)$	$\dfrac{(+)(-)}{(-)} = (+)$	$\dfrac{(+)(-)}{(+)} = (-)$	$\dfrac{(+)(+)}{(+)} = (+)$

15

=0 undefined =0

The solutions are given by $-3 \leq x < 1$ or $2 \leq x < \infty$.

The solution set $= \{x \in \mathbb{R} : -3 \leq x < -1\} \cup \{x \in \mathbb{R} : 2 \leq x < \infty\}$. 5

$= [-3, -1) \cup [2, \infty)$.

25

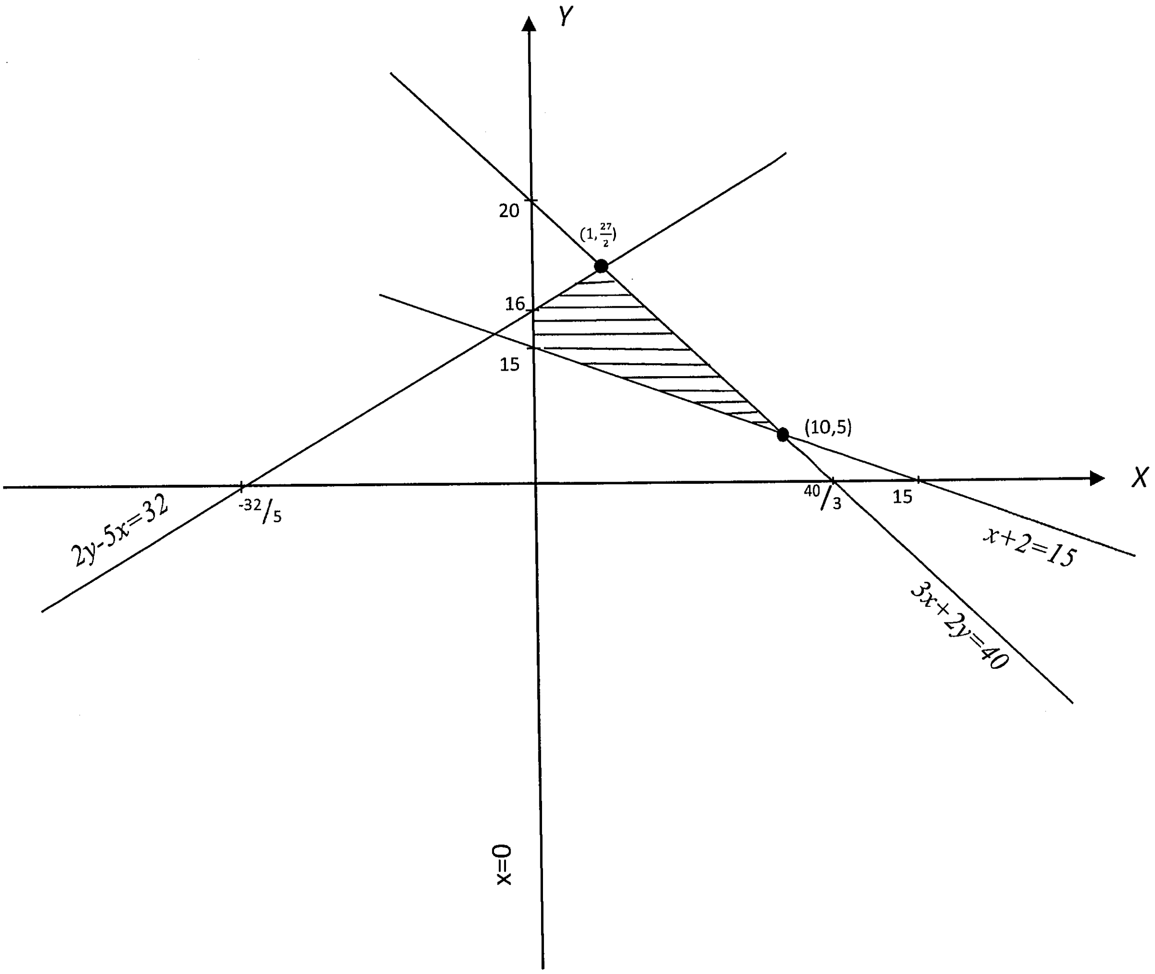
2. Sketch the region in the xy-plane satisfying the inequalities $3x + 2y \leq 40$, $2y - 5x \leq 32$, $x + y \geq 15$ and $x \geq 0$ indicating the coordinates of the vertices.

$3x+2y \leq 40$

$2y-5x \leq 32$

$x+y \geq 15$

$x \geq 0$



Region (10)

Vertices (15)

25

3. Express $\sqrt{3}\sin x - \cos x$ in the form $R\sin(x-\alpha)$, where $R(>0)$ and $\alpha\left(0<\alpha<\frac{\pi}{2}\right)$ are real constants to be determined.
Hence, solve the equation $\sqrt{3}\sin x - \cos x = \sqrt{3}$ for $0<x<2\pi$.

$$\sqrt{3}\sin x - \cos x$$

$$=2\left[\frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x\right] \quad 5$$

$$=2\left[\sin x.\cos\frac{\pi}{6} - \cos x.\sin\frac{\pi}{6}\right]$$

5

$$=2\sin\left(x-\frac{\pi}{6}\right) \text{ with } R=2 \text{ and } \alpha=\frac{\pi}{6}.$$

Now, $\sqrt{3}\sin x - \cos x = \sqrt{3}$

$$\Leftrightarrow 2\sin\left(x-\frac{\pi}{6}\right)=\sqrt{3}$$

$$\Leftrightarrow \sin\left(x-\frac{\pi}{6}\right)=\sin\frac{\pi}{3} \quad 5$$

$$\Leftrightarrow x-\frac{\pi}{6}=n\pi+(-1)^n\frac{\pi}{3} \text{ for } n\in\mathbb{Z}.$$

Hence, the solutions in $0<x<2\pi$ are $x=\frac{\pi}{2}$ or $\frac{5\pi}{6}$. 5

5

25

4. Find the values of the real constants A and B such that $\dfrac{x+2}{(x-1)(x-2)} = \dfrac{A}{x-1} + \dfrac{B}{x-2}$ for $x \neq 1,\ 2.$

Hence, find $\displaystyle\int_3^4 \frac{x+2}{(x-1)(x-2)}\,\mathrm{d}x$.

$$\frac{x+2}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2} = \frac{A(x-2)+B(x-1)}{(x-1)(x-2)}$$

$$= \frac{(A+B)x-(2A+B)}{(x-1)(x-2)}$$

By comparing coefficients:

$$\left.\begin{array}{l} A+B=1 \\ -(2A+B)=2 \end{array}\right\} \Leftrightarrow \begin{array}{l} A=-3 \quad 5 \\ B=4 \quad 5 \end{array}$$

$$\frac{x+2}{(x-1)(x-2)} = \frac{-3}{x-1} + \frac{4}{x-2}$$

$$\int_3^4 \frac{x+2}{(x-1)(x-2)}dx = \int_3^4 \frac{-3}{(x-1)}dx + \int_3^4 \frac{4}{(x-2)}dx$$

$$= -3\ln|x-1|\Big|_3^4 + 4\ln|x-2|\Big|_3^4 \qquad 10$$

$$= -3[\ln 3 - \ln 2] + 4[\ln 2 - \ln 1] \qquad 5$$

$$= -3\ln 3 - 3\ln 2 + 4\ln 3$$

$$= -3\ln 3 + \ln 2$$

$$= \ln\left(\frac{2}{27}\right).$$

25

5. Using the method of integration by parts, find $\int (3x+5)\cos(2x)\,\mathrm{d}x$.

$$\int (3x+5)\cos 2x \; dx.$$

Let $u = 3x+5$ and $dv = \cos 2x \; dx.$

Then, $du = 3 \; dx$ and $v = \dfrac{\sin 2x}{2}.$ 5

$$\int (3x+5)\cos 2x \; dx = (3x+5)\frac{\sin 2x}{2} - \int \frac{\sin 2x}{2} \times 3dx$$ 10

$$= (3x+5)\frac{\sin 2x}{2} - \frac{3}{2}\frac{(-\cos 2x)}{2} + C$$ 10

$$= \frac{1}{2}(3x+5)\sin 2x + \frac{3}{4}\cos 2x + C,$$ where C in an arbitrary constant.

25

12. (i) Solve $(\sin\theta - \cos\theta)(2\sin\theta - 1) = 0$.

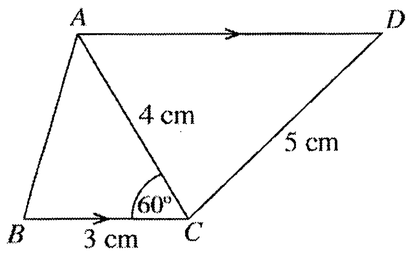
(ii) Starting with $\cos 3\theta = \cos(\theta + 2\theta)$ and stating the trigonometric identifies that you use, show that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$.

Hence, solve the equation $8\cos^3\theta - 6\cos\theta - 1 = 0$ for $0 < \theta < \pi$.

(iii) In the above figure $AC = 4$ cm, $BC = 3$ cm, $CD = 5$ cm, $A\hat{C}B = 60^\circ$ and AD is parallel to BC.

Using the **Cosine Rule** for the triangle ABC, find the length of AB.

Using the **Sine Rule** for the triangle ACD, find the angle ADC.



(i) $(\sin\theta - \cos\theta)(2\sin\theta - 1) = 0$

$\Leftrightarrow \sin\theta - \cos\theta = 0$ or $2\sin\theta - 1 = 0$. 5

$\sin\theta = \cos\theta$ or $\sin\theta = \dfrac{1}{2}$.

$\therefore \tan\theta = 1$ ($\because \cos\theta \neq 0$) or $\sin\theta = \dfrac{1}{2}$. 5

$\therefore \tan\theta = \tan\dfrac{\pi}{4}$ or $\sin\theta = \sin\dfrac{\pi}{6}$. 10

$\therefore \theta = n\pi + \dfrac{\pi}{4}$ or $\theta = n\pi + (-1)^n\dfrac{\pi}{6}$, where $n \in \mathbb{Z}$. 15

$\boxed{35}$

(ii)

$\cos(3\theta) = \cos(\theta + 2\theta)$

$= \cos\theta.\cos 2\theta - \sin\theta.\sin 2\theta$ 5 $\quad$ ($\because \cos(A+B) = \cos A\cos B - \sin A\sin B$) 5

$= \cos\theta(2\cos^2\theta - 1) - \sin\theta - 2\sin\theta\cos\theta$ 5 5 $\quad$ ($\because \cos 2\theta = 2\cos^2\theta - 1$ and 5 $\sin 2\theta = 2\sin\theta\cos\theta$) 5

$= 2\cos^3\theta - \cos\theta - 2\sin^2\theta.\cos\theta$ 5

$= 2\cos^3\theta - \cos\theta - 2\cos\theta(1 - \cos^2\theta)$ 5 $\quad$ ($\because \sin^2\theta + \cos^2\theta = 1$) 5

$= 2\cos^3\theta - \cos\theta - 2\cos\theta + 2\cos^3\theta$

$= 4\cos^3\theta - 3\cos\theta$. 5

$\boxed{50}$

$$8\cos^3\theta - 6\cos\theta - 1 = 0$$

$$\Leftrightarrow 2(4\cos^3\theta - 3\cos\theta) - 1 = 0 \quad 5$$

$$\Leftrightarrow 2\cos 3\theta - 1 = 0 \quad 5$$

$$\Leftrightarrow \cos 3\theta = \frac{1}{2}$$

$$\Leftrightarrow \cos 3\theta = \cos\frac{\pi}{3} \quad 5$$

$$\Leftrightarrow 3\theta = 2n\pi \pm \frac{\pi}{3}, \text{ where } n \in \mathbb{Z}. \quad 10$$

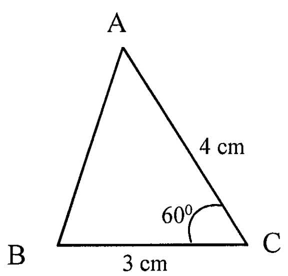
But $0 < \theta < \pi \Rightarrow$

$$3\theta = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}$$

$$\theta = \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}. \quad 10$$

35

(iii)



Applying the Cosine Rule for the triangle ABC:

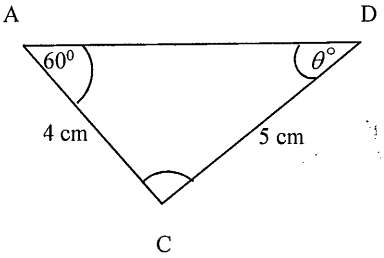
$$AB^2 = 4^2 + 3^2 - 2\times 4\times 3\times \cos 60^\circ \quad 10$$

$$AB^2 = 16 + 9 - 24\times\frac{1}{2} = 25 - 12$$

$$AB = \sqrt{13} \text{ cm.} \quad 5$$

15

Applying the Sine Rule for the triangle ACD:



$$\frac{\sin\theta}{4} = \frac{\sin 60^\circ}{5} \quad 10$$

$$\sin\theta = \frac{4}{5}\times\frac{\sqrt{3}}{2}$$

$$\theta = \sin^{-1}\left(\frac{2\sqrt{3}}{5}\right). \quad 5$$

15

---

13. (*a*) Shade the region enclosed by the curves, $x^2 + y^2 = 2$ and $y = x^2$.
Find the area of the shaded region.

(*b*) The following table gives the values of the function, $f(x) = \sqrt{1+x^4}$ correct to four decimal places, for values of x between 0 to 1.5 at intervals of length 0.25.

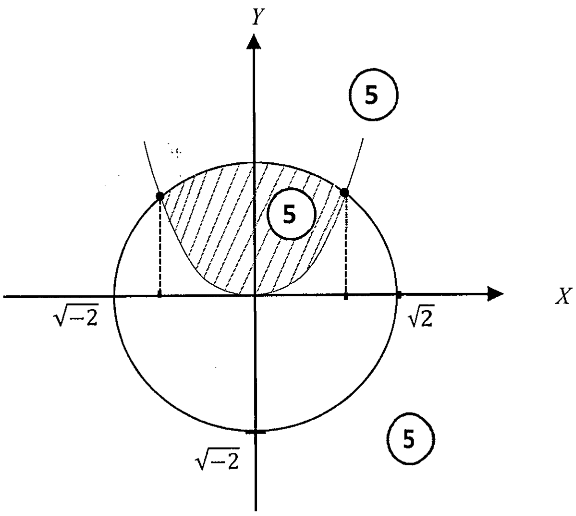
x	0	0.25	0.5	0.75	1.0	1.25	1.5
$f(x)$	1	1.0020	1.0308	1.1473	1.4142	1.8551	2.4622

Using **Simpson's Rule**, find approximate value for $\int_0^{1.5} \sqrt{1+x^4}\, dx$, correct to three decimal places.

Hence, find an approximate value for, $\int_0^{1.5} \left(1+\sqrt{1+x^4}\right)^2 dx$.

(a) $x^2 + y^2 = 2$

$y = x^2$



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The area of the shaded region $= 2\int_0^1 \left[\sqrt{2-x^2} - x^2\right] dx$ 20

Let $x = \sqrt{2}\sin\theta$. 5

Then, $\frac{dx}{d\theta} = \sqrt{2}\cos\theta$ and

$dx = \sqrt{2}\cos\theta \, d\theta$. 5

$x = 0 \Rightarrow \theta = 0$

$x = 1 \Rightarrow \theta = \frac{\pi}{4}$ 5

$$= 4\int_0^{\frac{\pi}{4}} \cos^2\theta \, d\theta - 2\int_0^1 x^2 dx$$ 10

$$= \frac{4}{2}\int (1+\cos 2\theta)\, d\theta - 2\left.\frac{x^3}{3}\right|_0^1$$ 10

$$= 2\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{4}} - \frac{2}{3}$$ 10

$$= 2\left[\frac{\pi}{4} + \frac{1}{2}\right] - \frac{2}{3}$$ 5

$$= \frac{\pi}{2} + \frac{1}{3}.$$

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(b) $f(x) = \sqrt{1+x^4}$, $h = 0.25$ 5

x	0	0.25	0.50	0.75	1.0	1.25	1.5
$f(x)$	1	1.0020	1.0308	1.1473	1.4142	1.8551	2.4622
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

$$\therefore \int_0^{1.5} \sqrt{1+x^4}\,dx \approx \frac{h}{3}\left[y_0 + 4(y_1+y_3+y_5) + 2(y_2+y_4) + y_6\right] \quad (10)$$

$$= \frac{0.25}{3}\left[1 + 4(1.0020 + 1.1473 + 1.8551) + 2(1.0308 + 1.4142) + 2.4622\right] \quad (10)$$

$$= \frac{0.25}{3}(1 + 4(4.0044) + 2(2.445) + 2.4622)$$

$$= 2.031 \quad (10)$$

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Now,

$$\int_0^{1.5}\left(1+\sqrt{1+x^4}\right)^2 dx = \int_0^{1.5}(1 + 2\sqrt{1+x^4} + 1 + x^4)\,dx \quad (10)$$

$$= \int_0^{1.5}\left(2 + 2\sqrt{1+x^4} + x^4\right)dx$$

$$= 2\int_0^{1.5} dx + \int_0^{1.5} x^4\,dx + 2\int_0^{1.5}\sqrt{1+x^4}\,dx \quad (5)$$

$$\approx 2\left[x\right]_0^{1.5} + \left[\frac{x^5}{5}\right]_0^{1.5} + 2\times 2.031 \quad (5)$$

$$= 2\times 1.5 + \frac{1.5^5}{5} + 4.062 \quad (5)$$

$$= 14.656 \quad (5)$$

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16. The travel times of the manager and the assistant manager of a company from their homes to the office are normally distributed with means and standard deviations as given in the following table:

	Mean (minutes)	Standard deviation (minutes)
Manager	45	5
Assistant Manager	55	6

The office starts at 8.30 am and the travel times of the manager and the assistant manager are independent. Find the probability of

(i) the manager is late, if he leaves home at 7.45 am,

(ii) the manager reaches office between 8.20 am and 8.30 am, if he leaves home at 7.30 am,

(iii) the assistant manager reaches the office on time or earlier, if he leaves home at 7.29 am,

(iv) the assistant manager is late to the office given that the manger is also late, if the manager and the assistant manager leave their homes at 7.45 am and 7.29 am respectively,

on a randomly selected day.

Let X_1 and X_2 be the travel times of the manager and the assistant manager, respectively.

(i) $P(X_1 > 45) = 1 - P(X_1 \leq 45)$ 20

$= 1 - P\left(Z \leq \frac{45-45}{5}\right)$ 10

$= 1 - P(Z \leq 0)$ 5

$= 1 - 0.5 = 0.5$ 5

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(ii) $P(50 < X_1 < 60) = P\left(\frac{50-45}{5} < Z < \frac{60-45}{5}\right)$ 25

$= P(1 < Z < 3)$ 5

$= P(Z < 3) - P(Z \leq 1)$ 10

$= (0.5+0.4987) - (0.5+0.3413)$

$= 0.1574$ 10

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(iii) $P(X_2 \le 61) = P\left(Z \le \dfrac{61-55}{6}\right)$ 20

$= P(Z \le 1)$ 5

$= 0.5 + 0.3413$

$= 0.8413$ 5

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(iv) Let A and B represent the events that the manager and assistant manager are late to office, respectively.

Now, $P(B \mid A) = \dfrac{P(B \cap A)}{P(A)}$ 5

Since, A and B are independent, 5

$P(B \cap A) = P(B)P(A)$. 5

Hence, $P(B \mid A) = P(B)$. 5

$P(B) = P(X_2 > 61)$

$= 1 - P(X_2 \le 61)$ 5

$= 1 - 0.8413$

$= 0.1587$ 5

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