

Part A

01. Using the principle of mathematical induction, prove that $\sum_{r=1}^n (3r-2) = \frac{n(3n-1)}{2}$ for all $n \in \mathbb{Z}^+$

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02. Sketch the graph of $y = |x - 3|$ and $y = 5 - |x|$ in the same diagram. Hence or otherwise, find all real values of x satisfying the inequality $2|x| + |2x - 3| > 5$.

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07. If $lx + my = 1$ is normal to the parabola $y^2 = 4ax$ then show that $al^3 + 2alm^2 = m^2$.

08. A variable line, drawn through the point of intersection of the straight lines $\frac{x}{a} + \frac{y}{b} = 1$ and

$\frac{x}{b} + \frac{y}{a} = 1$, meets the coordinate axes in A & B . Show that the locus of the mid-point of AB is

the curve $2xy(a+b) = ab(x+y)$

09. The straight line $2y + x - 1 = 0$ passes through the x-axis and y-axis at the points A and B respectively. Find equation of the circle which touches the line AB at A and passes through the origin. Find the equation of the other tangent drawn to the circle from B and show that it is perpendicular to AB.

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10. If $\tan^{-1} \frac{1}{a-1} = \tan^{-1} \frac{1}{x} + \tan^{-1} \frac{1}{a^2 - x + 1}$, find the values of x in terms of a .



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Practice Test - Grade 13 - 2022

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Combined Mathematics - I

Time: 03 Hours

Part B

❖ Answer only five questions.

11. a. The equation $x^2 + mx + 15 = 0$ has roots α and β and the equation $x^2 + hx + k = 0$ has roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$.

- Write down the value of k .
- Find an expression for h in terms of m .
- Find the two possible values of α such that $\beta - 2\alpha = 1$
- Hence, find the two possible values of m and h

b. A curve has equation $y = x^3 + 2x^2 - 11x - m$, where m is a positive integer. The curve crosses the x - axis at the point $(-4, 0)$

- Find the value of m
- For that value of m , factorise $x^3 + 2x^2 - 11x - m$ completely. The curve also crosses the x - axis at two other points.
- Find the coordinates of those points.

c. Let $f(x) = ax^4 + bx^3 + x^2 + cx - 14$, where $a, b, c \in R$. If $(x-1)$ and $(x-2)$ are factors of $f(x)$ and when $f(x)$ is divided by $(x+1)$, the remainder is 6. Find the values of a, b, c . Write $f(x)$ as a product of linear factors and find the remainder when $f(x)$ is divided by $(3x-1)$.

12. a. Find the number of arrangements of the letters of the word "INDEPENDENCE". In how many of these arrangements,

- do the words start with P
- do all the vowels always occur together
- do the vowels never occur together
- do the words begin with I and end in P?

b. If $f(r) = \{r(r+1)\}^2$, prove that $f(r) - f(r-1) = 4r^3$

Hence, deduce that $\sum_{r=1}^n r^3 = \frac{n^2(n+1)^2}{4}$.

Write down the r^{th} term U_r of the series $\frac{3}{1^3} + \frac{5}{1^3+2^3} + \frac{7}{1^3+2^3+3^3} + \dots$

Find $v(r)$ such that $U(r) = v(r) - v(r+1)$.

Show that $\sum_{r=1}^n U_r = 4 - \frac{4}{(n+1)^2}$.

Show that the infinite series $\sum_{r=1}^{\infty} U_r$ is convergent.

13. a. Given $A = \begin{pmatrix} 5 & 2 \\ -7 & -3 \end{pmatrix}$

i. Find A^2 and find B such that $B = 3I + A - A^2$.

ii. Calculate AB .

iii. Deduce A^{-1} .

b. The complex number w is defined by $w = \frac{22+4i}{(2-i)^2}$. Find w in the form $w = x + iy$

It is given that p is a real number such that $\frac{1}{4}\pi \leq \arg(w+p) \leq \frac{3}{4}\pi$. Find the set of possible values of p .

The complex conjugate of w is denoted by $\bar{w}$. The complex numbers w and $\bar{w}$ are represented in an Argand diagram by the points S and T respectively. Find, in the form $|z-a| = k$, the equation of the circle passing through S, T and the origin.

c. State the De Moivre's Theorem for positive integer index

By writing $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$, show that

$$\text{i. } \sin\left(\frac{\pi}{12}\right) = \frac{1}{4}(\sqrt{6} - \sqrt{2}) \quad \text{ii. } \cos\left(\frac{\pi}{12}\right) = \frac{1}{4}(\sqrt{6} + \sqrt{2})$$

Hence find the exact values of z for which

$$z^4 = 4\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) \text{ Give your answers in the form } Z = a + ib \text{ where } a, b \in \mathbb{R}^+$$

14. a. Let $f(x) = \frac{x^2 + 1}{(x-1)^2}$ for $x \neq 1$.

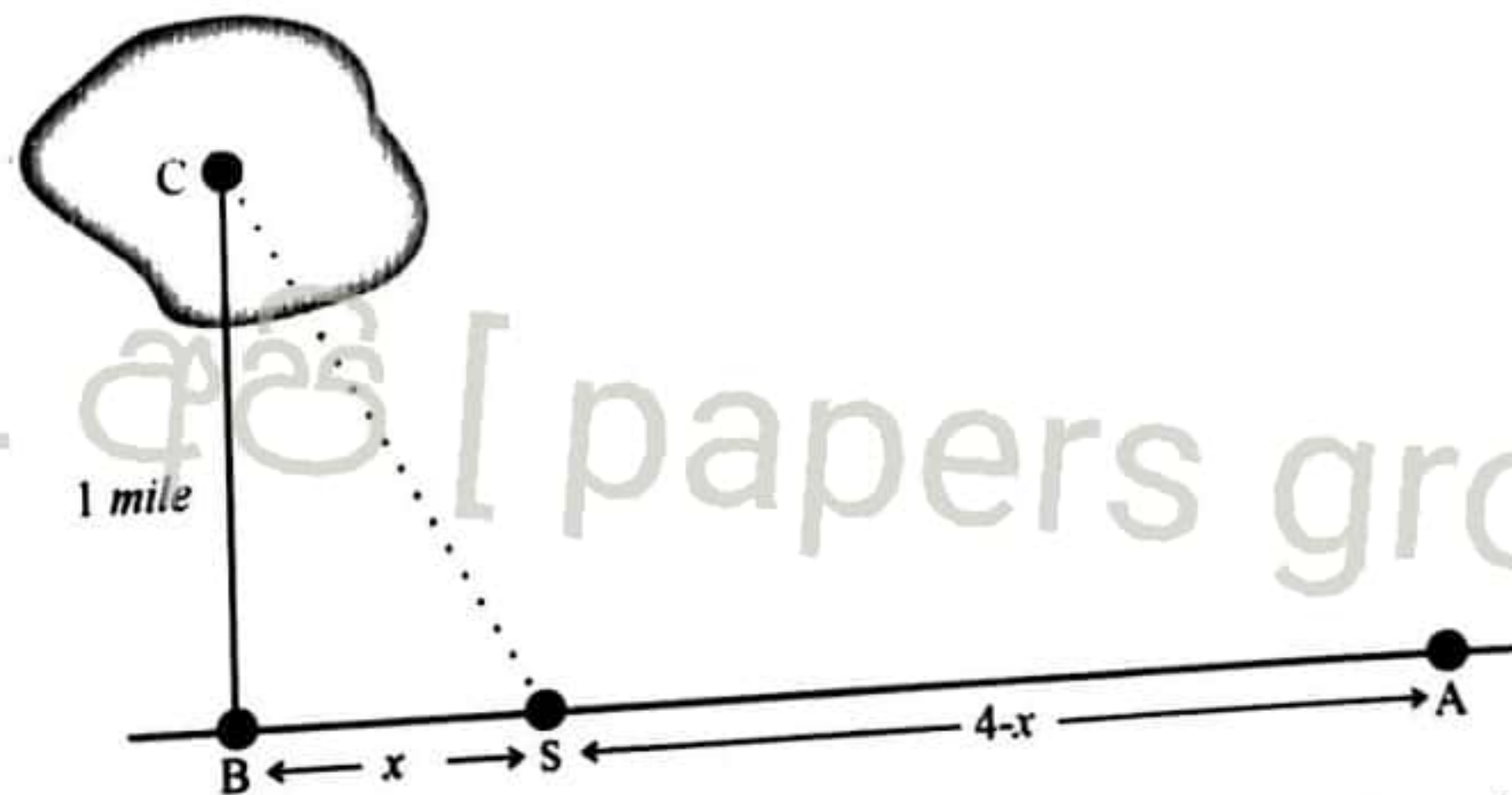
Show that $f'(x)$, the derivative of $f(x)$, is given by $f'(x) = \frac{-2(x+1)}{(x-1)^3}$ for $x \neq 1$.

It is given that $f''(x) = \frac{4(x+2)}{(x-1)^4}$ for $x \neq 1$.

Find the x -coordinates of the points of inflection of the graph of $y = f(x)$.

Sketch the graph of for $y = f(x)$ indicating the asymptotes, y -intercept and the turning points.

b.



A power line is to be constructed from a power station at point A on the shore to an island at point C. B is the nearest point on the shore from C. Also B is 4 miles away from A. It costs \$5000 per mile to lay the power line under water and \$3000 per mile to lay the line underground.

If a point S is located on BA, x miles away from B, find $C(x)$ as the total cost for the power line from A to C across S.

Examine the sign of $\frac{d}{dx} C(x)$ as x increases from 0 to 4 miles.

Find the most suitable position of S on BA in order to minimize cost.

15. a. Find the value of the constants A, B, C and D such that

$$\frac{2x^2 + 3}{(x^2 - 1)(x^2 + 4)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2+4}.$$

Hence integrate $\frac{2x^2 + 3}{(x^2 - 1)(x^2 + 4)}$ with respect to x .

b. Using integration by parts, Find $\int_1^3 \sqrt{x} \tan^{-1} \sqrt{x} dx$

c. Show that $\int_0^{\frac{\pi}{2}} f(x) dx = \int_0^{\frac{\pi}{2}} f\left(\frac{\pi}{2} - x\right) dx$.

Let $I = \int_0^{\frac{\pi}{2}} \frac{\cos x}{1 + \cos x + \sin x} dx$ and $J = \int_0^{\frac{\pi}{2}} \frac{\sin x}{1 + \cos x + \sin x} dx$. Show that $I = J$.

Hence show that $I = \frac{\pi}{4} - \frac{1}{4} \int_0^{\frac{\pi}{2}} \frac{\sec^2 \frac{x}{2}}{1 + \tan \frac{x}{2}} dx$.

Using the substitution $\tan \frac{x}{2} = t$ or otherwise, show that $I = \frac{\pi}{4} - \frac{1}{2} \ln 2$.

d. Using the substitution $x = u^6$, Find $\int \frac{\sqrt{x}}{1 + \sqrt[3]{x}} dx$

16. a. Obtain the equation of the straight line that makes intercepts a and b on the x and y axes respectively.

The straight line $u \equiv 4x + 3y - 12 = 0$ meets the x and y axes at the points L and M . Find the coordinates of L and M .

A straight line perpendicular to $u = 0$ meets x and y axes at the points P and Q respectively. If MP and LQ lines intersect at T , show that the locus of point T is given by, $s \equiv x^2 + y^2 - 3x - 4y = 0$.

Show that LM is a diameter of the circle $s = 0$. O is the origin. If the point P lies on the bisector of $\angle LMO$, find the coordinates of P and Q .

Find also the circle that passes through L, M and Q .

17. a. If $\operatorname{cosec} \theta + \cot \theta = \frac{11}{2}$, then find the value of $\tan \theta$.

- b. State and prove the **sine rule** for any triangle ABC in the usual notation.

Show that

$$(b^2 - c^2) \cot A + (c^2 - a^2) \cot B + (a^2 - b^2) \cot C = 0 \text{ and}$$

If $a^2 + c^2 = 2b^2$, then $\cot A + \cot C = 2 \cot B$.

c. Find the solution of the equation $\sin x + \sin \frac{\pi}{8} \sqrt{(1 - \cos x)^2 + \sin^2 x} = 0$.



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