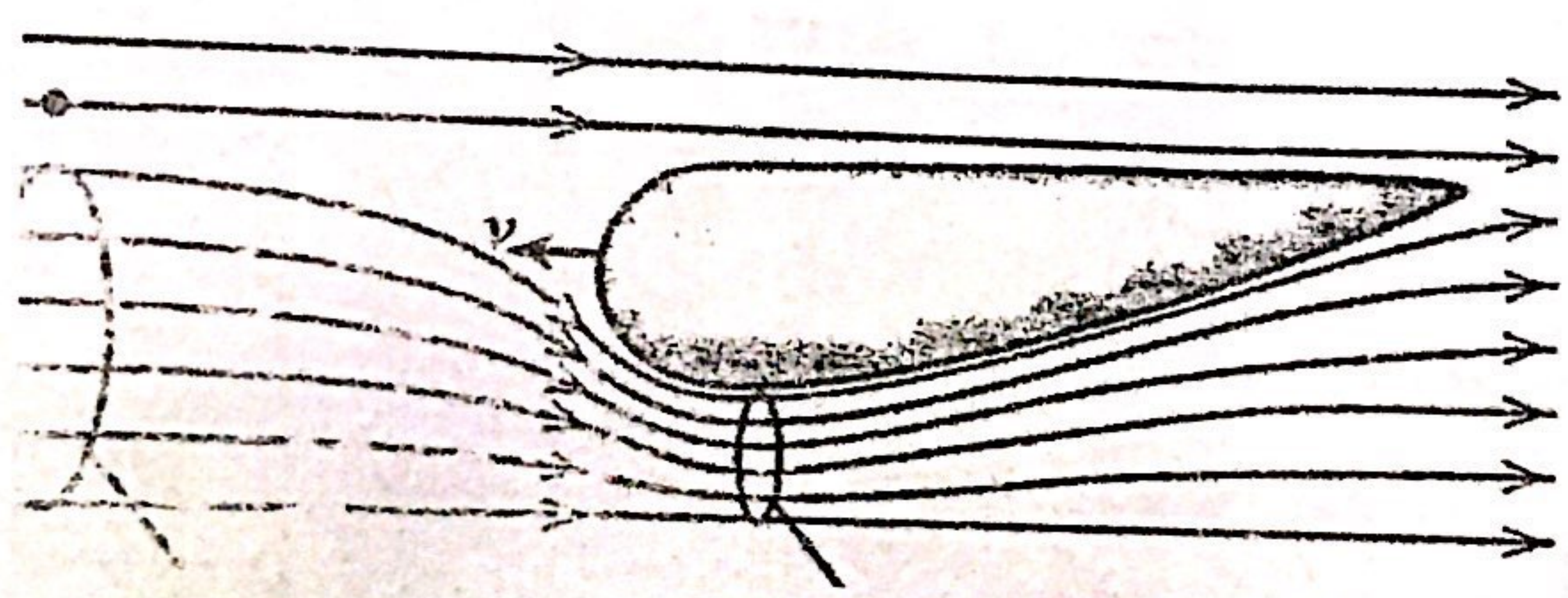
Confidential

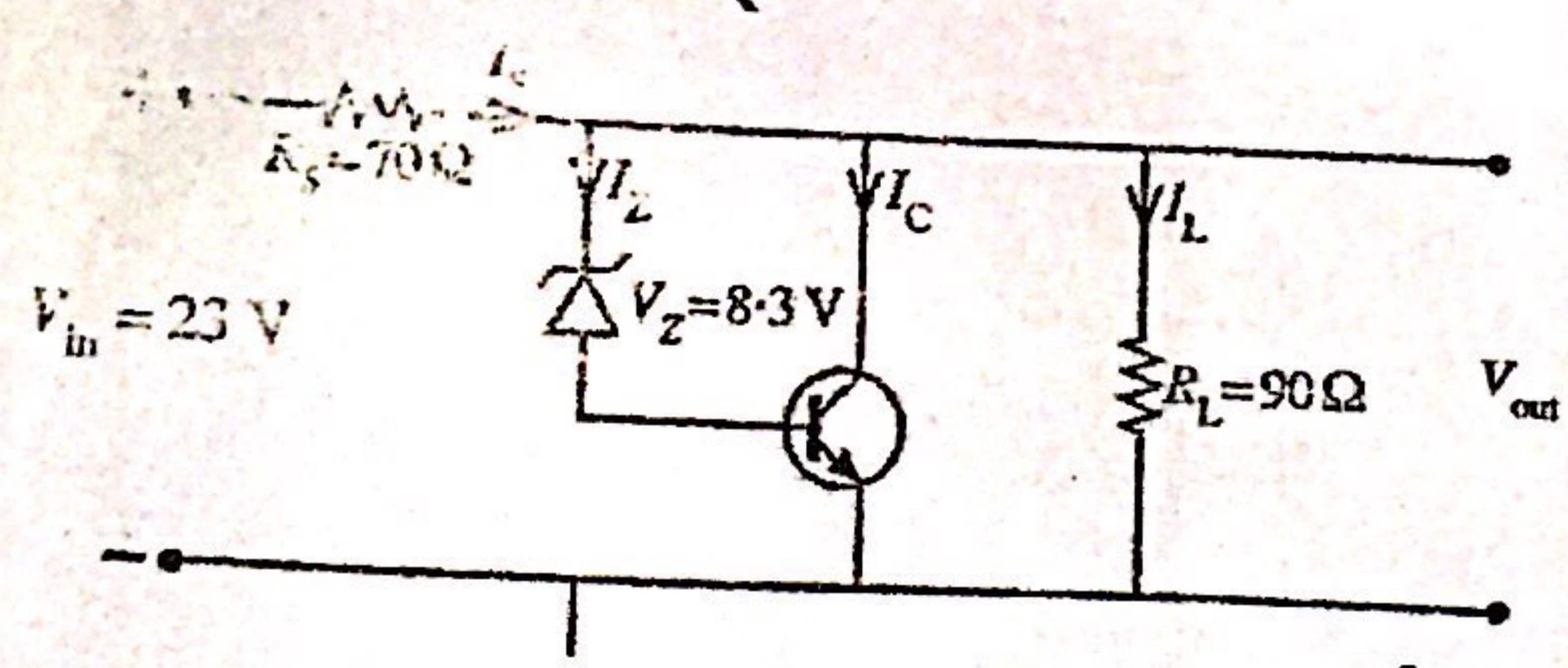
Department of Examinations – Sri Lanka

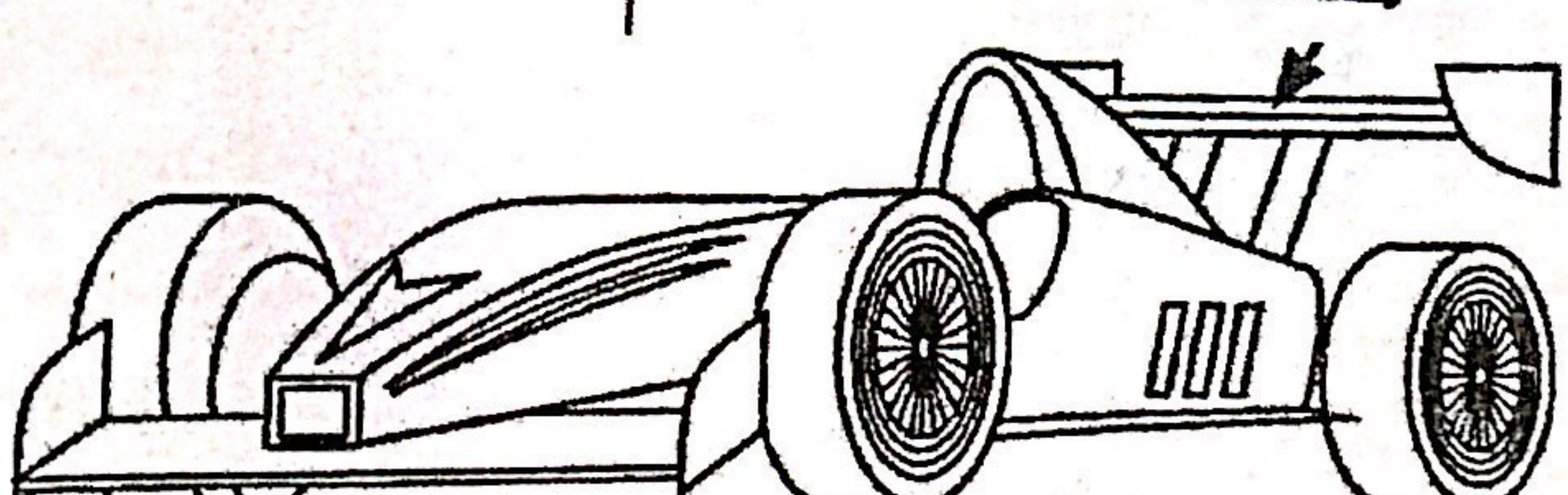
G.C.E. (A/L) Examination – 2022(2023)

01 – Physics

Marking Scheme







This has been prepared for the use of marking examiners

ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව

இலங்கைப் பரீட்சைத் திணைக்களம்

අ.පො.ස. (උ.පෙළ) විභාගය/ க.பொ.த. (உயர் தர)ப் பரீட்சை - 2022(2023)

විෂය අංකය பாட இலக்கம் **01** **විෂයය** பாடம் **Physics**

ලකුණු දීමේ පටිපාටිය / புள்ளி வழங்கும் திட்டம்

I පත්‍රය / பத்திரம் I

ප්‍රශ්න අංකය வினா இல.	පිළිතුරු අංකය விடை இல.	ප්‍රශ්න අංකය வினா இல.	පිළිතුරු අංකය விடை இல.	ප්‍රශ්න අංකය வினா இல.	පිළිතුරු අංකය விடை இல.	ප්‍රශ්න අංකය வினா இல.	පිළිතුරු අංකය விடை இல.	ප්‍රශ්න අංකය வினா இல.	පිළිතුරු අංකය விடை இல.
01.	**04**	11.	**03**	21.	**02**	31.	**05**	41.	**05**
02.	**05**	12.	**03**	22.	**03**	32.	**04**	42.	**01**
03.	**05**	13.	**01**	23.	**04**	33.	**03**	43.	**02**
04.	**01**	14.	**05**	24.	**02**	34.	**04**	44.	**03**
05.	**04**	15.	**02**	25.	**02**	35.	**04**	45.	**01**
06.	**05**	16.	**05**	26.	**04**	36.	**02**	46.	**02**
07.	**04**	17.	**04**	27.	**03**	37.	**04**	47.	**01**
08.	**02**	18.	**02**	28.	**01**	38.	**05**	48.	**04**
09.	**02**	19.	**01**	29.	**05**	39.	**01**	49.	**03**
10.	**03**	20.	**01**	30.	**03**	40.	**02**	50.	**02**

✪ විශේෂ උපදෙස් / விசேட அறிவுறுத்தல் :

එක් පිළිතුරකට / ஒரு சரியான விடைக்கு ලකුණු 01 බැගින් / புள்ளி வீதம்

මුළු ලකුණු / மொத்தப் புள்ளிகள் 1 × 50 = 50

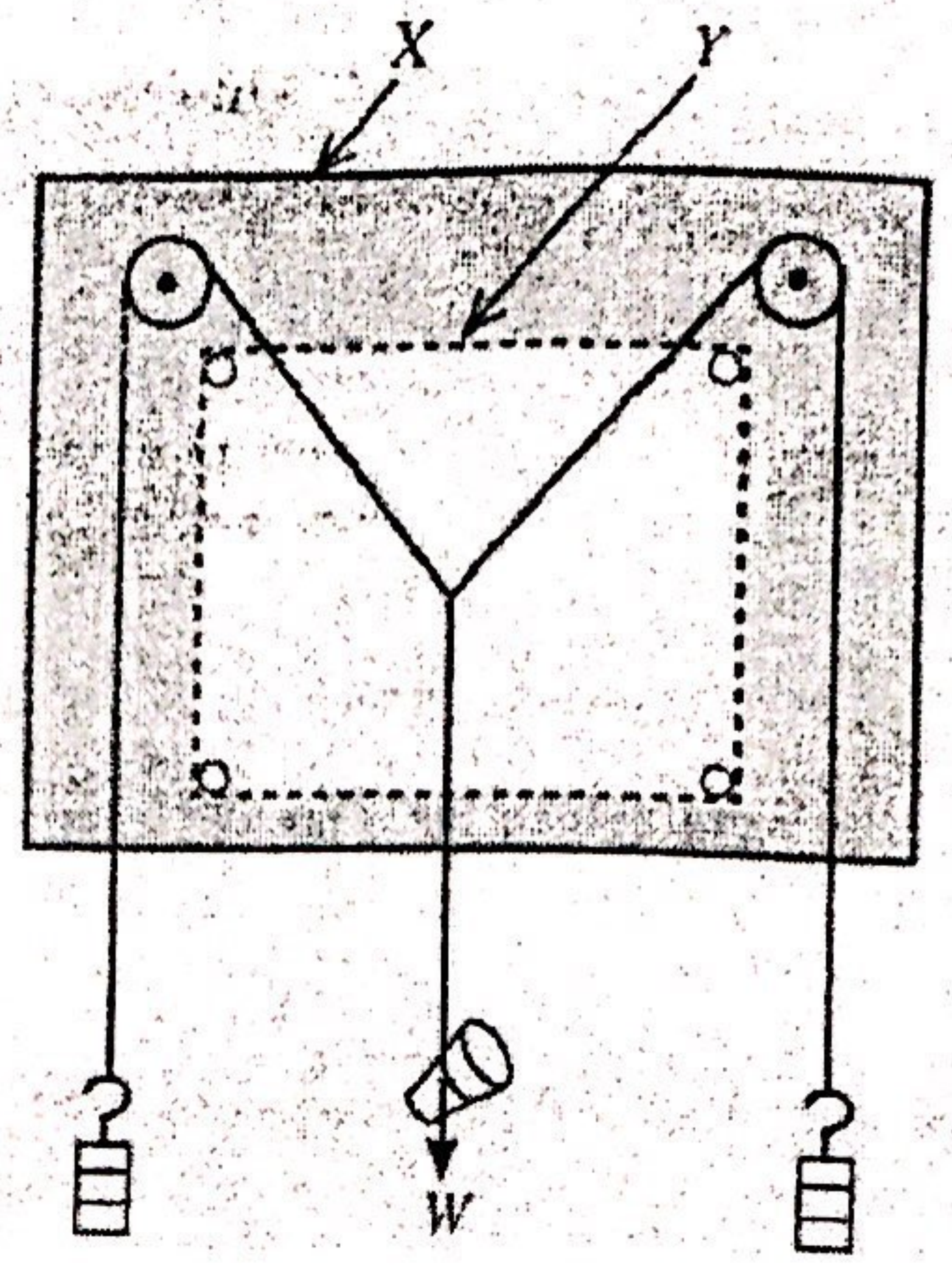
PART A — Structured Essay

Answer all four questions on this paper itself.

($g = 10\,m\,s^{-2}$)

8

1. You are asked to determine the weight (W) of a small glass stopper and hence the relative density of a liquid using the setup available in the school laboratory as shown in the figure.



(a) Name the items in the figure represented by X and Y.

X: Drawing board/Parallelogram board/Parallelogram apparatus .... (01)

Y: White paper/sheet *OR* Photocopy paper/sheet ....(01)

(No marks for just stating board and paper)

(b) (i) How do you check whether the pulleys are frictionless?

Pull down weight W/stopper and ....(01)

release to check the midpoint /intersection of strings /weight/stopper returns back to the original position ....(01)

(ii) If friction is present, how do you minimize it?

Apply a lubricant oil *OR* Engine Oil *OR* Machine oil ....(01)

(No marks for just oil *or* coconut oil *or* grease)

(c) (i) Known weights P and Q, and the glass stopper of weight W are hung using light strings as shown in the figure. How do you accurately mark the positions of relevant strings?

By placing a <u>set square</u> <u>perpendicular</u> to the board and ....(01)

<u>just touching</u> along the strings and ....(01)

marking <u>two dots</u> ....(01)

with <u>sufficient/maximum</u> separation/at least 5 cm apart ....(01)

{Alternative answer:

By placing a <u>plane mirror</u> (strip with sufficient length) <u>under the string</u> and ......(01)

look <u>perpendicularly</u> and ....(01)

mark the <u>two ends</u> of image of the string when ....(01)

the <u>string covers its image</u>. ....(01)}

(ii) After constructing the parallelogram using a suitable scale, how do you determine the weight W?

<u>Measure the length of the diagonal</u> and ....(01)

convert it into weight <u>using the selected scale</u> ....(01)

(*d*) (i) Now the setup is used to determine the relative density of the liquid. Two beakers, water and the liquid are available. Write down the experimental steps that you would follow to determine the apparent weight of the stopper either in water or liquid.

<u>Fully immerse</u> the stopper (either in water or liquid) ....(01)

Construct respective parallelograms and measure the length of the corresponding diagonal ....(01)

(ii) What are the two apparent weights of the stopper to be identified from the above measurements?

W_1: Weight of the stopper in water ....(01)

W_2: Weight of the stopper in liquid ....(01)

(Award full marks even if W_1 & W_2 are interchanged)

(iii) Write down two expressions for loss of apparent weight of the stopper in terms of W, W_1 and W_2.

Loss of apparent weight in water $= W - W_1$ ....(01)

Loss of apparent weight in liquid $= W - W_2$ ....(01)

(Award these marks according to the answers in part (ii) above)

(iv) Hence, write down an expression for relative density of the liquid using your answers in (*d*) (iii) above.

Relative density of the liquid = $\frac{W-W_2}{W-W_1}$ ....(01)

(Award this mark according to the answer in part (iii) above)

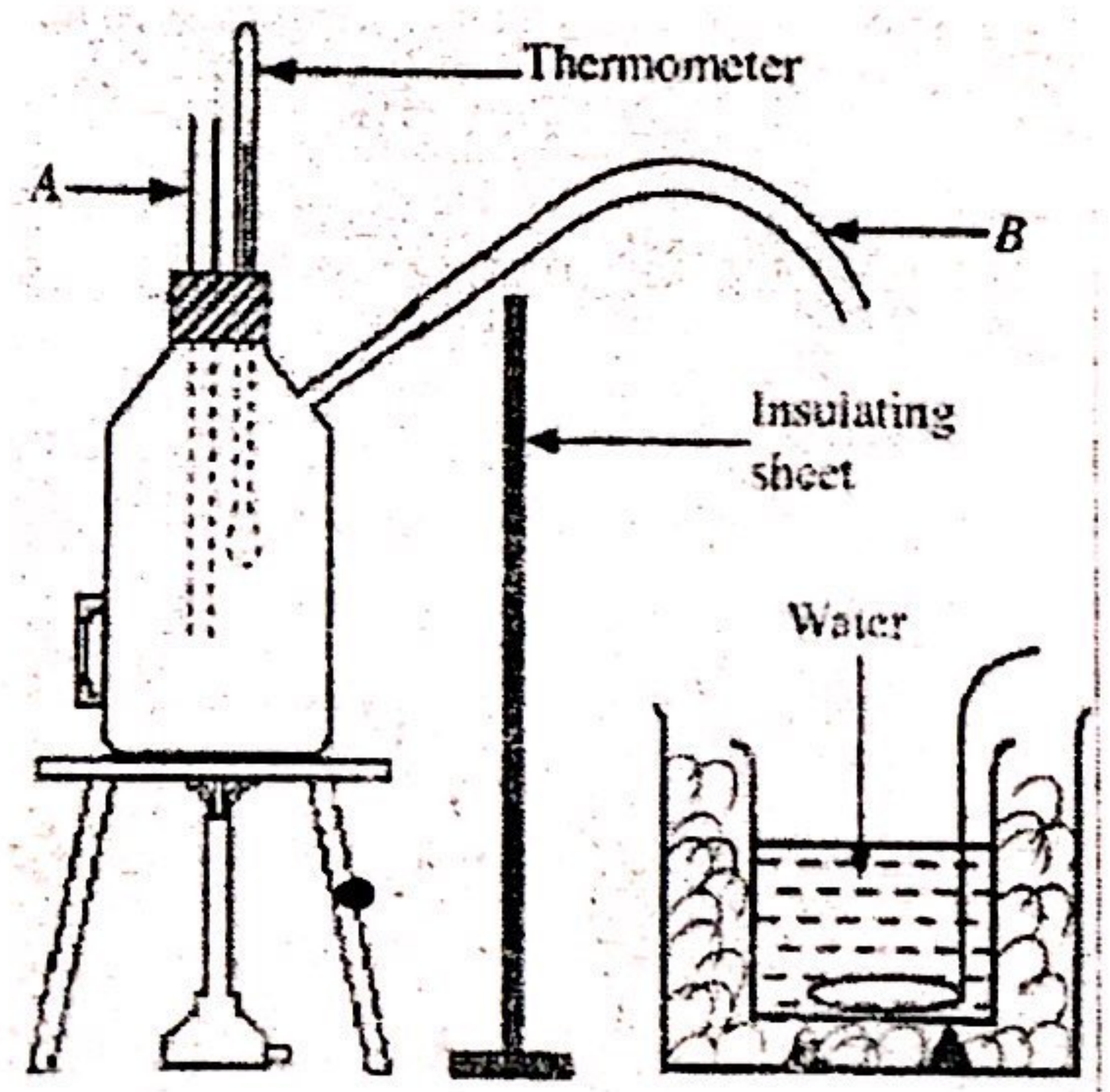
(v) Write down a possible error (not relevant to constructing the parallelogram) that could happen in the experimental procedure mentioned in (*d*)(i) above which would affect the value of relative density of the liquid.

Touching the stopper at the bottom/ side wall of the beaker

OR existence of air bubbles attached to the stopper ....(02)

(02 marks for one correct answer)

2. You are asked to determine the specific latent heat of vaporization of water using the method of mixtures. The figure shows a copper boiler used in the laboratory to generate steam. The rubber tube *B* is used to take steam out. An insulated copper calorimeter and a copper stirrer are also provided.



(*a*) (i) If the level of water in the boiler is not sufficient, how do you identify it using the tube *A*?

Steam will come out of tube *A* during heating ....(02)

(ii) After rectifying the defect in (*a*) (i) above, steam is generated in the boiler. If the rubber tube taking steam out is blocked how do you identify it?

(Hot) water will come out of tube *A* during heating ....(02)

(*b*) It is not correct to mix the steam coming out of the tube *B* directly with water in this experiment.

(i) Give the reason.

Condensed (hot) water may come from the tube with steam ....(01)

(ii) How do you correct it?

A steam trap must be connected to the end of the rubber tube ....(02)

OR a correct diagram in the figure.

(c) What are the other two measuring instruments that you need for this experiment?

Another thermometer and ....(01)

a four/three-beam balance [*OR* chemical balance *OR* (laboratory) electronic balance] ...(01)

(No marks for just stating balance)

(d) After making the correction mentioned in (b)(ii) above you pass steam into water in the calorimeter. How do you correctly keep the end of the glass tube which passes steam? Underline the correct procedure.

Just above the water level / Touching the water level / Below the water level ....(01)

(e) What are the temperature measurements that you expect to take in this experiment? Give them in order of measurements.

θ_1 : the temperature of steam (in the boiler) [no marks for 100 ^{0}C]

θ_2 : the initial temperature of water (in the calorimeter)

θ_3 : the maximum temperature of the mixture (of water and steam) ....(03)

[03 marks for all three correct answers in correct order ; 02 marks for all three correct answers but wrong order ; 01 mark for two answers in correct order]

{Interchange of the order of θ_1 and θ_2 is accepted}

(f) (i) In addition to the above temperature measurements what are the other measurements that you would take in this experiment? Give them in order of measurements.

m_1 : the mass of the (empty) calorimeter and stirrer/calorimeter with contents

m_2 : the mass of the calorimeter, stirrer and water

m_3 : the total/final mass of the system/mixture (after adding steam) ....(02)

[02 marks for all three correct answers in correct order ; 01 mark for all three correct answers but wrong order ; 01 mark for two answers in correct order]

(ii) If the specific heat capacities of copper and water are c_c and c_w respectively, write down an expression to determine the specific latent heat (L) of vaporization of water in terms of the symbols mentioned in (e) and (f) above. Assume that there is no heat exchange with the surrounding.

$$[(m_2 - m_1)c_w + m_1c_c](\theta_3 - \theta_2) = (m_3 - m_2)[L + c_w(\theta_1 - \theta_3)] \quad ....(03)$$

[01 mark for the correct L.H.S. ; 01 mark for the R.H.S; 01 for equating]

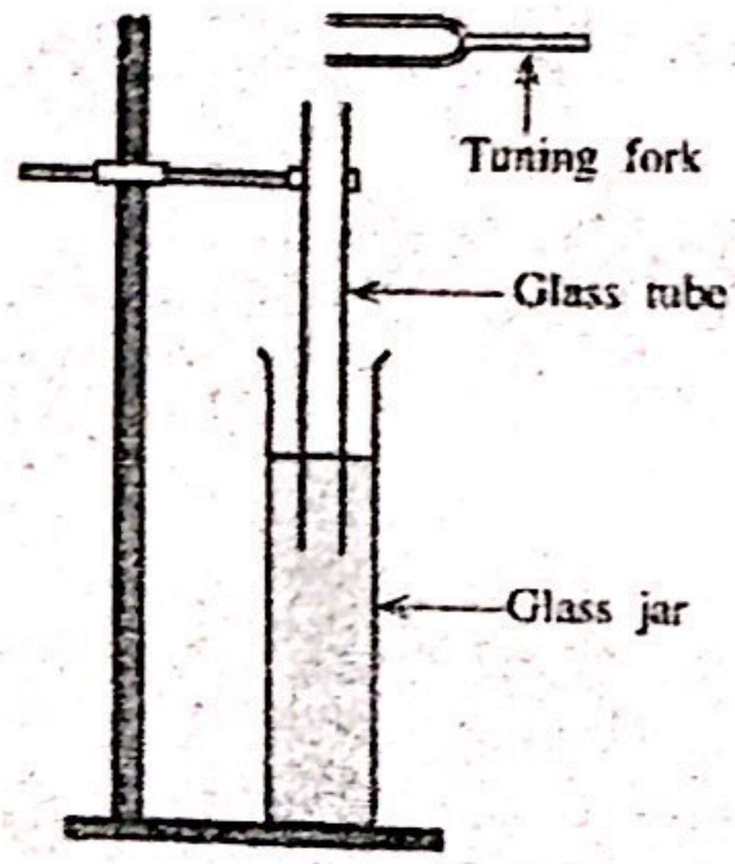
{If a student has interchanged θ_1 and θ_2, check the appropriate changes in the above expression and award full marks; taking θ_1/θ_2 as 100 is accepted in the expression}

(g) What precaution would you take in this experiment to minimize the error due to heat exchange with the surrounding?

The starting (initial) temperature of water is lowered by 5 ^{0}C (some amount) from the room temperature by (adding ice). ....(01)

and steam is passed until the maximum temperature of the mixture is 5 ^{0}C (same amount) above the room temperature. ....(01)

3. The usual experimental setup in the school laboratory to determine the speed of sound (v) in air is shown in the figure. The setup includes a glass tube opened at both ends, a tall glass jar filled with water and a set of tuning forks. The resonance method is adopted to determine the speed of sound in air.



(a) What is the other measuring instrument needed to perform this experiment?

Metre ruler ....(01)

(b) Fill in the blank with the appropriate word in the following incomplete statement.

When an object vibrating at the **natural** frequency of another object the first object makes the second object resonate. ....(01)

(c) (i) What is the type of wave produced inside the tube at resonance? Underline the correct answers.

(1) longitudinal / transverse ....(01)

(2) progressive / stationary ....(01)

(ii) How is the wave that you have selected in (c)(i) above produced?

Superposition of incident wave with reflected wave (from the water surface)...... (01)

(d) State the experimental procedure in stepwise that you would adopt in order to obtain the resonance length corresponding to the first mode of vibration (fundamental) of the tube accurately.

Immerse the tube completely/start with the shortest length of the air column. ....(01)

Keep the vibrating tuning fork above the top/open end of the tube......................(01)

Raise the tube out of the water/increase the length of the air column

until a loud/maximum sound is heard (where resonance occurs) ....(01)

(e) You have to measure the resonance lengths corresponding to the first mode of vibration and the second mode of vibration for a given tuning fork of frequency f.

(i) If the resonance length corresponding to the first mode of vibration is l_1, write down an expression for l_1 in terms of the wavelength λ of the wave and end correction e of the tube.

$l_1 = \frac{\lambda}{4} - e$ ....(01)

(ii) If the resonance length corresponding to the second mode of vibration is l_2, write down an expression for l_2 in terms of the wavelength λ of the wave and end correction e of the tube.

$l_2 = \frac{3\lambda}{4} - e$ ....(01)

{If a student has written $l_1 + e = \frac{\lambda}{4}$ and $l_2 + e = \frac{3\lambda}{4}$ award 01 mark only}

(iii) Hence write down an expression for $(l_2 - l_1)$ in terms of λ.

$l_2 - l_1 = \frac{\lambda}{2}$ ....(01)

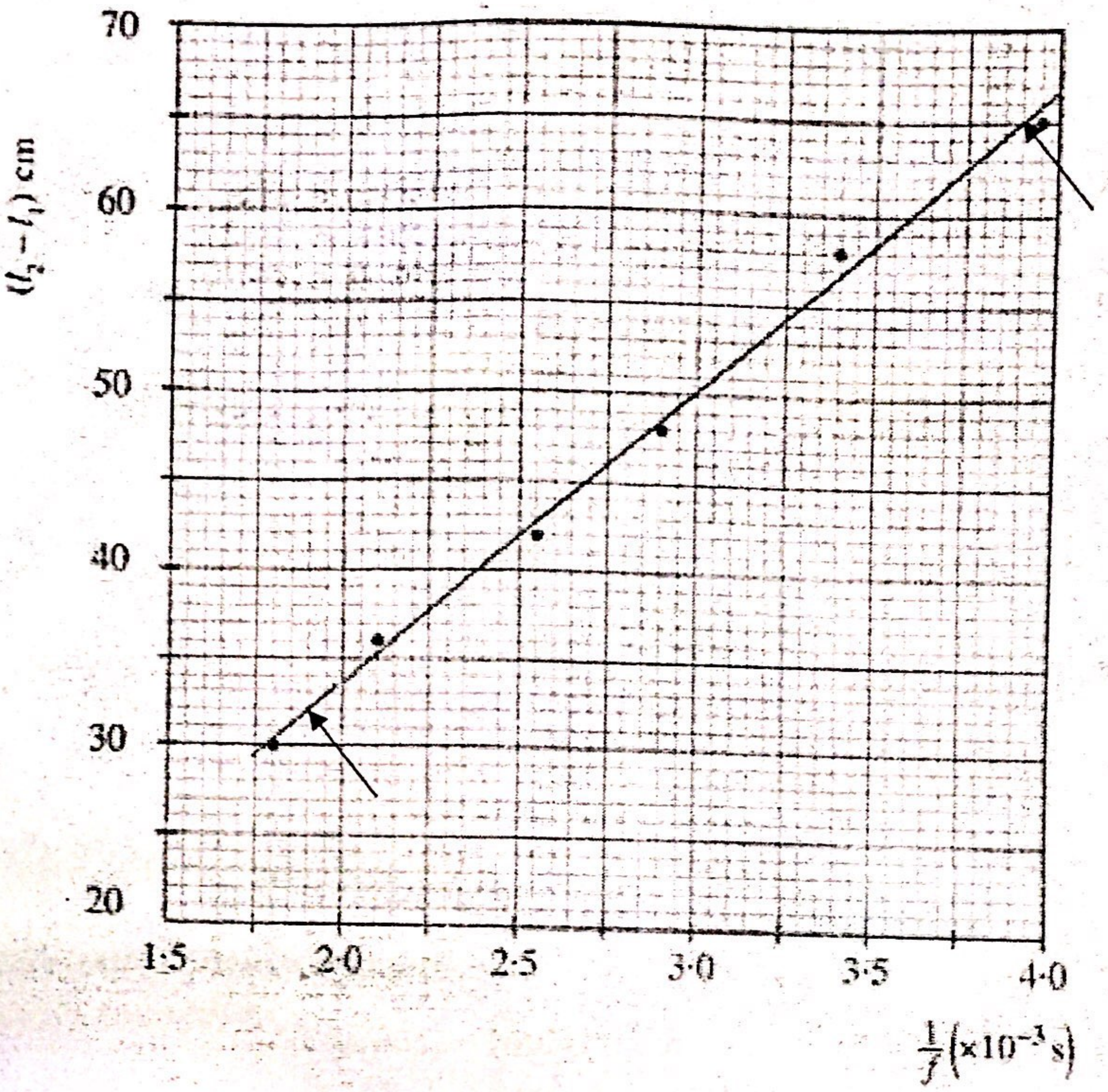
(iv) What is the advantage of obtaining $(l_2 - l_1)$?

The end correction of the tube/e is eliminated .... (01)

(v) Substitute v and f in the expression written in (e) (iii) above and rearrange it to obtain a straight line graph.

$l_2 - l_1 = \frac{v}{2f}$ ....(01)

(f) A graph of $(l_2 - l_1)$ against $\frac{1}{f}$ is shown on the grid below. Calculate the value of speed of sound in air v (in m s^{-1}) using the graph.



Identifying $\frac{v}{2}$ as the gradient .... (01)

Selecting the lower point as $(1.9, 32)$ .... (01)

Selecting the higher point as $(3.9, 65)$ .... (01)

[No marks for any other points]

Gradient $= \frac{(65-32)\times 10^{-2}}{(3.9-1.9)\times 10^{-3}}$ OR $\frac{(65-32)}{(3.9-1.9)\times 10^{-3}}$ .... (01)

(For gradient calculation)

$v = 330$ m s^{-1} ....(01)

{If a student has obtained the correct answer for v selecting different points for gradient calculation, award 03 marks only. i.e. for identification of the gradient, gradient calculation and the final answer}

(g) Instead of the instrument mentioned in (a) above suggest an alternative way to determine the resonance lengths more accurately.

Use a resonance tube with a scale attached/ or graduated resonance tube ....(01)

[No marks for travelling microscope, since heights are larger]

(h) What is the essential parameter that must be given when expressing the speed of sound in air?

(Room) Temperature (of air/lab/surrounding) ....(01)

4. A student has planned an experiment to compare the electromotive forces E_1 and E_2 of two cells using a potentiometer. Incomplete circuit diagram that can be used is shown in figure (1). Other items that can be connected to the circuit are shown separately.

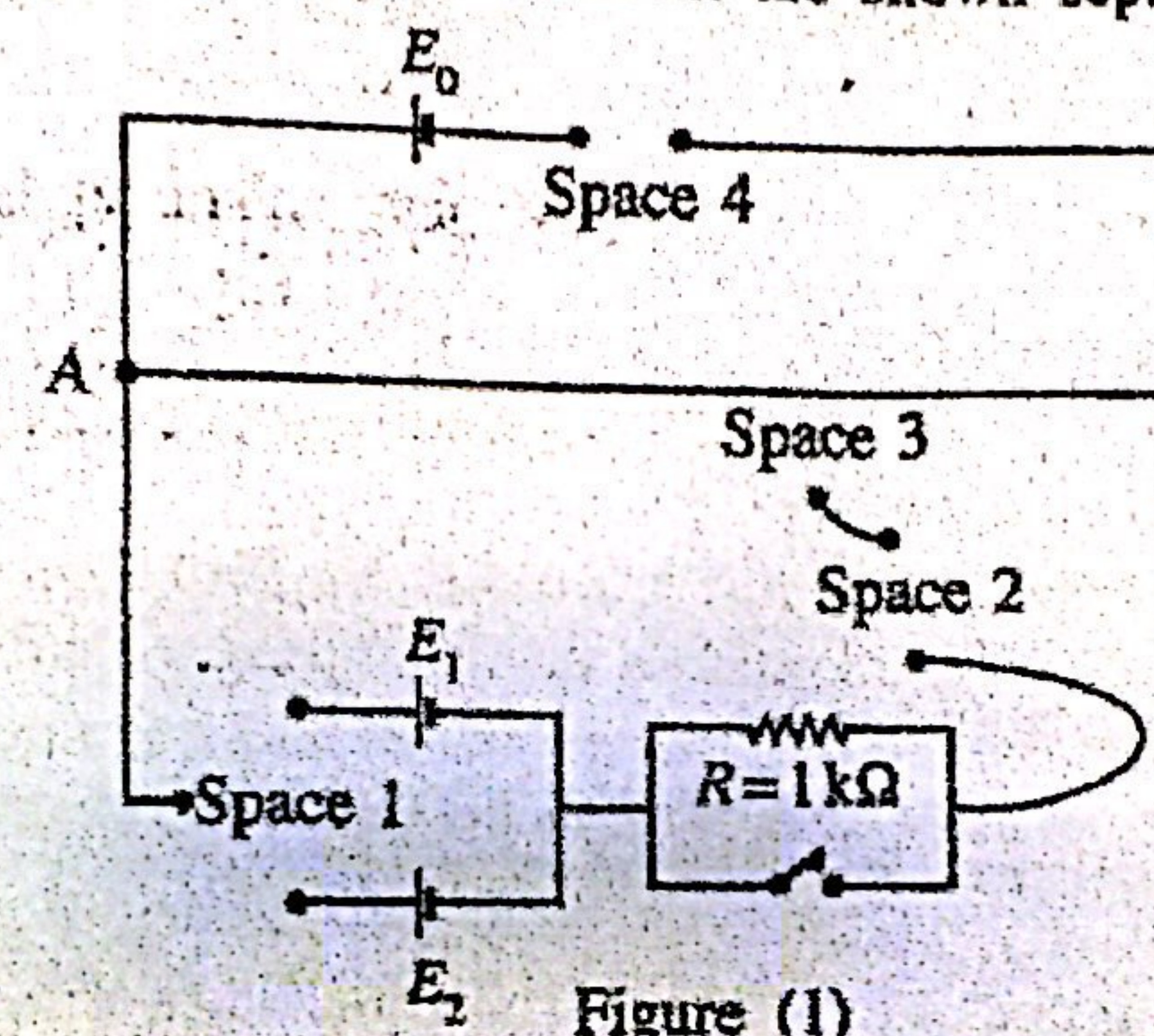
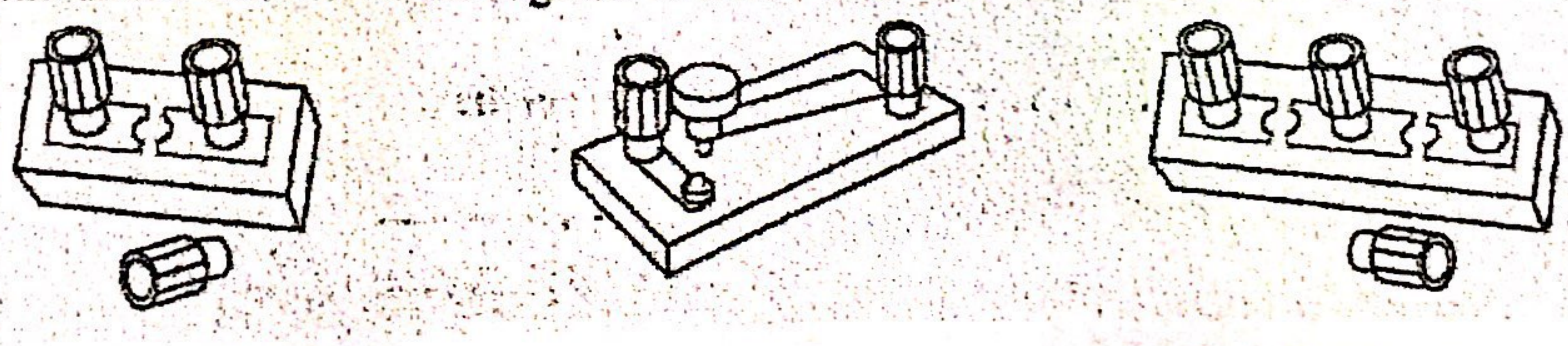
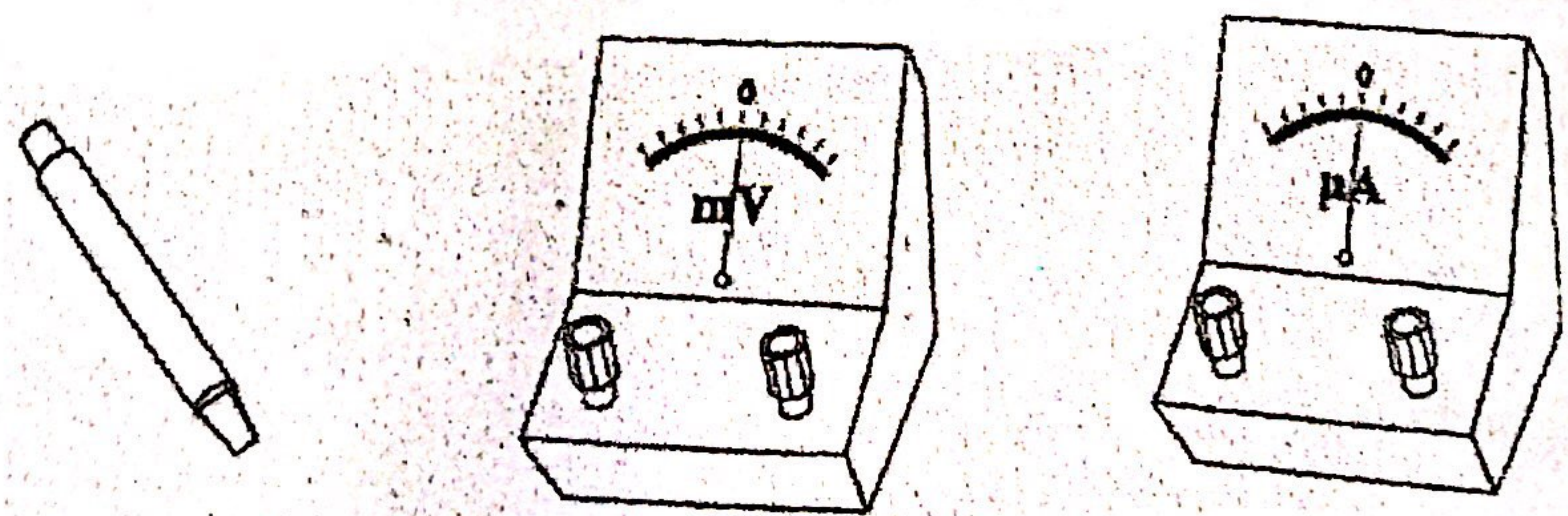


Figure (1)

(a) Name the items shown in the figures below.



A plug key B tap key C two-way switch/key



D jockey/sliding key E (center zero milli)-voltmeter F center zero galvanometer/micro-ammeter

..........................(03)

[03 marks for all correct answers]

[02 marks for any five/four correct answers]

[01 mark for any three/two correct answers]

(*b*) **Write down the correct letter corresponding to each item given in (*a*) above that has to be connected in Space 1, Space 2, Space 3 and Space 4 in figure (1).**

Space 1: C Space 2: F

Space 3: D Space 4: A ....(03)

[03 marks for all correct answers]
[02 marks for any three correct answers]
[01 mark for any two correct answers]

(*c*) **Name the type of cell which gives electromotive force (e.m.f.) E_0 and write down its value.**

A 2 V lead-acid accumulator *OR* (two) 1.2 V Ni-Cd cells (connected in series) ....(02)
[one mark for the type and one mark for the e.m.f. value]

(*d*) **Why is the internal resistance of the cell of e.m.f. E_0 must be very small compared to the resistance of the potentiometer wire?**

Potential drop across the potentiometer wire (*AB*) must be larger compared to the potential drop across the cell
OR Potential drop across the cell must be smaller compared to the potential drop across potentiometer wire (*AB*) ....(01)

(*e*) **In order to perform this experiment certain conditions have to be satisfied with regards to E_1, E_2 and E_0. What are they?**

E_1 should be less than E_0 *OR* $E_1 < E_0$ ....(01)

E_2 should be less than E_0 *OR* and $E_2 < E_0$ ....(01)

(*f*) **Compared to a non-ideal voltmeter, a potentiometer is considered as a suitable apparatus for measuring an e.m.f. accurately. What is the reason for it?**

Non-ideal voltmeter measures terminal potential difference rather than an e.m.f. ....(01)

and the potentiometer at balance does not draw any current from the cell and measures the e.m.f. *OR* potentiometer uses a null method at balance ....(01)

(g) Why should the cross-sectional area of the potentiometer wire be uniform?

To obtain a uniform/constant potential drop/ *OR* uniform/constant potential gradient along the wire ....(01)

(h) (i) Write down the experimental procedure that must be followed to determine E_1/E_2 ratio.

Connect one cell (e.m.f. E_1) only (using the two-way switch) and obtain the balance length ....(01)

Then connect the other cell (e.m.f. E_2) and obtain the relevant balance length............(01)

(ii) If the measurements taken in (h) (i) above corresponding to E_1 and E_2 are x_1 and x_2 respectively, write down an expression for the E_1/E_2 ratio.

$$\frac{E_1}{E_2} = \frac{x_1}{x_2}$$ ....(01)

(No mark for writing l instead of x)

(i) (i) Another student has planned to determine E_1/E_2 ratio by using a graphical method by changing the effective length of the potentiometer wire of length 6 m shown in the figure (2). What experimental procedure should the student adopt?

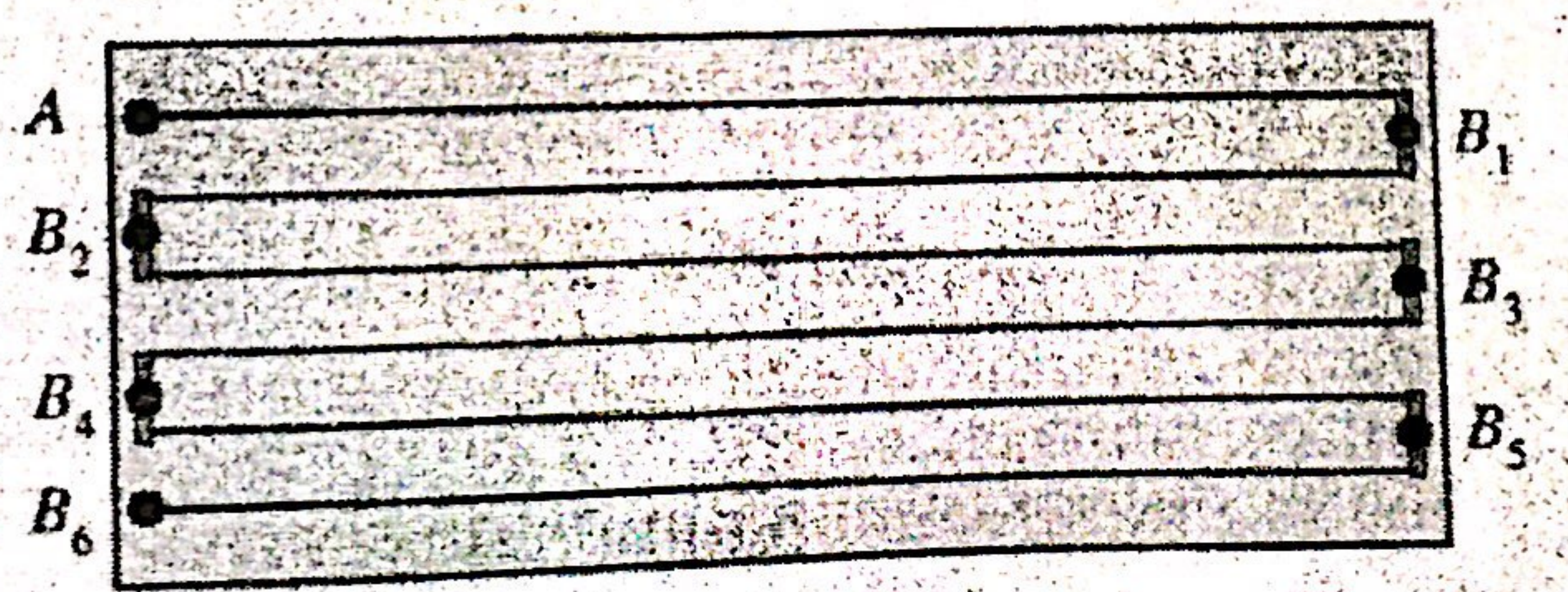


Figure (2)

Connect the terminal B to each end (B_1, B_2, B_3......,B_6) of the one meter wires ....(01)
and measure the corresponding balance lengths ....(01)

(ii) If the gradient of the graph that can be drawn in (i)(i) above is m and the value of E_1 is known, write down a relationship for E_2 in terms of m and E_1.

....(01)

$$E_2 = \frac{E_1}{m} \quad OR \quad E_2 = mE_1$$

Answer four questions only.
($g = 10\,\mathrm{m\,s^{-2}}$)

- **Note:** For an example the number 65210 can be written as $6{\cdot}52\times10^4$ in scientific notation after rounding off to two decimal places.

5. (*a*) For a steady flow of a non-viscous, incompressible fluid, the Bernoulli's equation can be written as $P + \frac{1}{2}\rho v^2 + h\rho g$ = constant. Here all symbols have their usual meaning. Identify the terms in the left hand side of the equation.

(*b*) A racing car having a rear spoiler with a curved surface at the bottom is shown in Figure (1). According to the Bernoulli's principle, when the car is moving at high speed, a downward force acts on the spoiler.

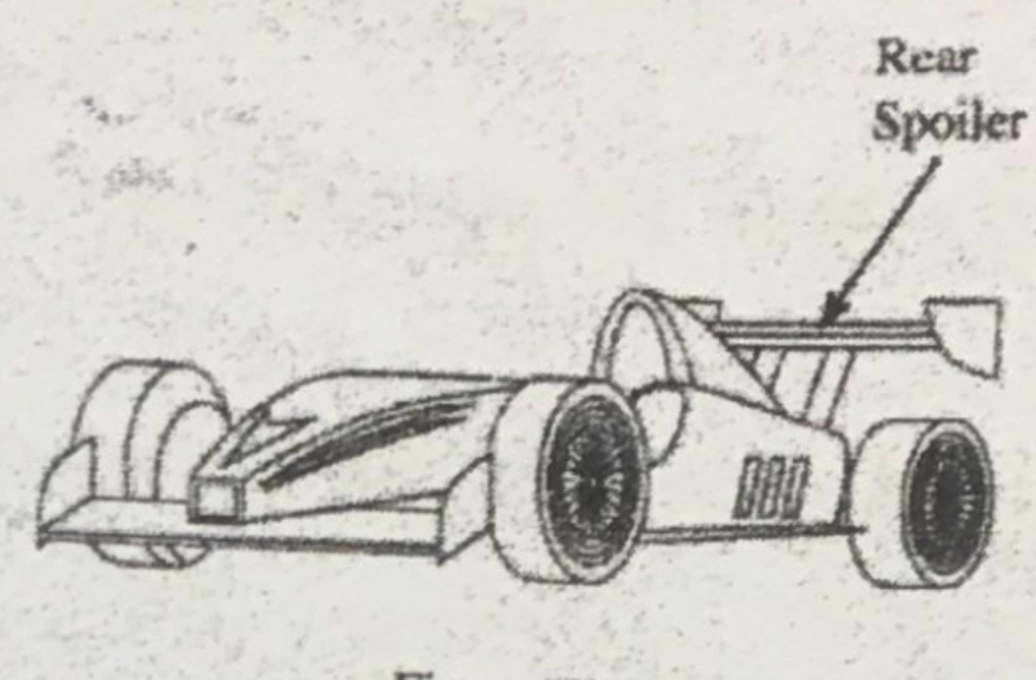


Figure (1)

A vertical cross-section of the rear spoiler of the racing car moving horizontally through air towards left with constant velocity v relative to the ground is shown in Figure (2).

(i) What is the velocity of air at point X relative to the car? Assume that the air is at rest relative to the ground.

(ii) As shown in Figure (2), the cross-sectional area of an imaginary flow tube far away from the spoiler is A_1 and the corresponding cross-sectional area of the same flow tube below the spoiler is A_2. If $\frac{A_1}{A_2} = 1{\cdot}2$, write down an expression for the speed (v_2) of air relative to the car flowing under the spoiler in terms of v.

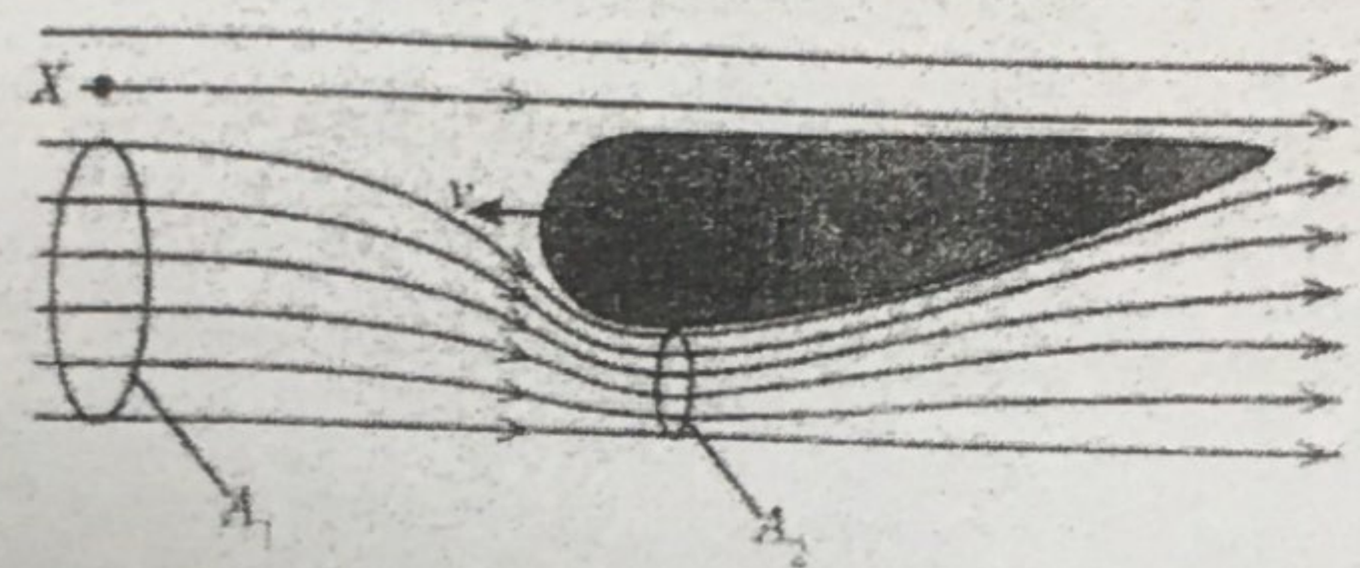


Figure (2)

(iii) If the effective horizontal cross-sectional area of the spoiler is $0{\cdot}2\,\mathrm{m}^2$, calculate the downward force acting on the spoiler. $v = 360\,\mathrm{km\,h^{-1}}$ and density of air $= 1{\cdot}2\,\mathrm{kg\,m^{-3}}$.

(iv) If wind is blowing horizontally relative to the ground from left to right with a constant velocity will the force calculated in (*b*)(iii) above increase or decrease? Give reasons to your answer without any calculations.

(*c*) When a car moves at high speed, the drag force (F_d) acting on the car due to air is given by $F_d = \frac{1}{2}C\rho A v^2$. Here C is known as the drag coefficient, ρ is the density of air, A is the effective frontal area of the car facing air and v is the speed of the car relative to air. Spoilers also change direction of airflow on vehicles and reduce the drag coefficient.

(i) Show that C is dimensionless.

(ii) Taking $C = 0{\cdot}3$, $A = 1{\cdot}4\,\mathrm{m}^2$, $\rho = 1{\cdot}2\,\mathrm{kg\,m^{-3}}$ and $v = 360\,\mathrm{km\,h^{-1}}$, calculate the drag force F_d acting on the racing car mentioned in (*b*) above. Assume that air is at rest relative to the ground.

(iii) Calculate the power (P) needed to overcome the drag force when the car is moving at constant velocity of $360\,\mathrm{km\,h^{-1}}$.

(iv) The car starts from rest and achieves the speed of $360\,\mathrm{km\,h^{-1}}$. A student argues that the average power needed to overcome air drag in this process is $\frac{P}{2}$, where P is the value calculated in (*c*)(iii) above. Giving reasons state whether you agree with the argument of the student.

(v) The power needed to overcome other frictional forces acting on the car is 48 kW. The energy released by burning one litre of petrol is $4{\cdot}0\times10^7\,\mathrm{J}$ and only 15% of this energy is used to move the car. When the car is travelling at constant speed of $360\,\mathrm{km\,h^{-1}}$ determine the fuel efficiency of the car in km per litre.

(vi) If wind is blowing horizontally relative to the ground from left to right with a constant velocity of $10\,\mathrm{m\,s^{-1}}$, calculate the power (P') needed to overcome the drag force when the car is travelling at constant velocity of $360\,\mathrm{km\,h^{-1}}$. (Give your answer to the nearest integer in kW).

(a) P - Pressure/pressure energy per unit volume .... (01)

$\frac{1}{2}\rho v^2$ – Kinetic energy per unit volume .... (01)

$h\rho g$ - (Gravitational) potential energy per unit volume (from a reference level) .... (01)

(b) (i) Velocity of air at point X relative to the car is $-v$/ OR $\vec{v}$/ OR v from left to right. .... (01)

$[\, v_{A,C} = v_{A,G} + v_{G,C} = 0 - v]$

(ii) $A_2v_2 = A_1v$ OR $A_2v_2 = 1.2A_2v$ .... (01)

$v_2 = 1.2v$ .... (01)

(iii) If pressures of air above and below the spoiler are P_1 and P_2 respectively, then applying Bernoulli's equation

$P_1 + \frac{1}{2}\rho v^2 = P_2 + \frac{1}{2}\rho v_2^2$ OR $P_1 + \frac{1}{2}\rho v^2 = P_2 + \frac{1}{2}\rho (1.2v)^2$ .... (01)

$P_1 - P_2 = \frac{1}{2}\rho[(1.2v)^2 - v^2]$

$v = \frac{360\times10^3}{60\times60}$ (Converting km hr^{-1} into m s^{-1}) .... (01)

$v = 100$ m s^{-1}

$P_1 - P_2 = \frac{1}{2}\times 1.2 \times 100^2(1.44 - 1)$

Downward force acting on the car due to the spoiler $= (P_1 - P_2)\times 0.2$ .... (01)

(for multiplication pressure difference and area)

$= \frac{1}{2}\times 1.2 \times 100^2 \times 0.44 \times 0.2$ .... (01)

(For correct substitution)

$= 528$ N .... (01)

(iv) Will increase .... (01)

Velocity of air relative the car will increase OR v/ v_2 will increase. .... (01)

OR $v_{A,C} = v_{A,G} + v_{G,C} = -v'' - v$, where v'' is the velocity of wind relative to the ground

(c) (i) Dimension of force (LHS) = MLT^{-2} .... (01)

Dimensions of $\rho A v^2$ = ML^{-3}L^{2}L^{2}T^{-2} .... (01)

= MLT^{-2}

$\therefore$ C is dimensionless.

(ii) Drag force $F_d = \frac{1}{2}C\rho Av^2 = \frac{1}{2} \times 0.3 \times 1.2 \times 1.4 \times 100^2$ .... (01)
(For correct substitution)
$F_d = 2520$ N .... (01)

(iii) Power (P) needed to overcome the drag force $= F_d v$ .... (01)
$= 2520 \times 100$

$= 252$ kW (252000 W) .... (01)

(iv) Do not agree .... (01)

Power (P) does not vary with v linearly *OR* power is proportional to v^3 not to v. .... (01)

(v) The energy used to move the car by burning one liter of petrol $= \frac{4.0\times10^7}{100} \times 15$ ..... (01)

$= 6 \times 10^6$ J per litre

Total power needed = 252 + 48 = 300 kW .... (01)

(for the addition)

Time that the car can travel by burning 1 liter of petrol $= \frac{6\times10^6}{300\times10^3}$ .... (01)

(for the division)

$\therefore$ distance that the car travel by burning 1 liter of petrol $= \frac{6\times10^6}{300\times10^3} \times 100$ .... (01)

(for the multiplication by 100 *or* 100×10^{-3})

Fuel efficiency of the car in km per liter = 2 km per litre .... (01)

{Alternative method:

Time taken by car to travel 1 km (in seconds) $= \frac{360}{60\times60}$ ....(01)

$\therefore$ distance that the car travel by burning 1 liter of petrol $= \frac{6\times10^6}{300\times10^3} \times \frac{360}{60\times60}$ .....(01)

= 2 km per litre.........(01)}

(vi) Speed of the car relative to air = 100 + 10 (for the addition) .... (01)

New drag force $F_d = \frac{1}{2} \times 0.3 \times 1.2 \times 1.4 \times 110^2$ .... (01)
(For correct substitution)
Power needed to overcome the drag force
$P' = \frac{1}{2} \times 0.3 \times 1.2 \times 1.4 \times 110^2 \times 100$ .... (01)
(Multiplying drag force by 100)

$= 305$ kW .... (01)

6. (a) (i) Define angular magnification (m) of an astronomical (optical) telescope.

(ii) Why is angular magnification a better measure compared to linear magnification for an optical instrument?

(b) An astronomical telescope is made with an objective lens L_o of focal length f_o and an eyepiece L_e of focal length f_e.

(i) What is meant by the normal adjustment of a telescope?

(ii) Draw a clearly labelled ray diagram for the telescope when it is in normal adjustment.

(iii) Using the ray diagram obtain an expression for the angular magnification of the telescope. For very small values of α (in radians) $\tan(\alpha) = \alpha$.

(c) (i) An astronomical telescope having $f_o = 100$ cm and $f_e = 10$ cm is adjusted to form the final image of the moon at the least distance of distinct vision of the eye, $D = 25$ cm. The moon subtends an angle $0{\cdot}5°$ at the unaided eye. Calculate the angle (in degrees) subtended by the image of the moon through the telescope at the eye and the angular magnification in this adjustment. Assume that the distance between the eye and eyepiece is negligible. You may use $1° = 0{\cdot}018$ radians.

(ii) With a suitable modification the above telescope is used to take a real image of the moon on a screen. Draw the ray diagram for this situation clearly labelling the focal points and distances.

(iii) After the modification mentioned in (c) (ii) above, if the real image is formed on the screen placed at 30 cm from the eye piece, calculate the size of the image (diameter) of the moon on the screen.

(iv) Yerkes Observatory in Wisconsin, USA has the largest and the oldest refracting astronomical telescope functioning from 1897 to date. The observatory was the birthplace of modern astrophysics and collected over 170 000 photographic plates of astronomical objects.

The focal length of the objective lens of Yerkes telescope is 19·0 m. It gives a real image of the moon of diameter 17·1 cm on a photographic plate placed 30 cm behind the eyepiece. Calculate the focal length of the eyepiece of the Yerkes telescope and the angular magnification in this situation. (Give the angular magnification to the nearest integer.)

6. (a) (i) Angular magnification $m = \dfrac{\alpha'}{\alpha}$ ——————— (equation1) ....(01)

where α' is the angle subtended at the eye by the rays from the final image and

α is the angle subtended at the unaided eye by the rays from the object

(both correct) ....(01)

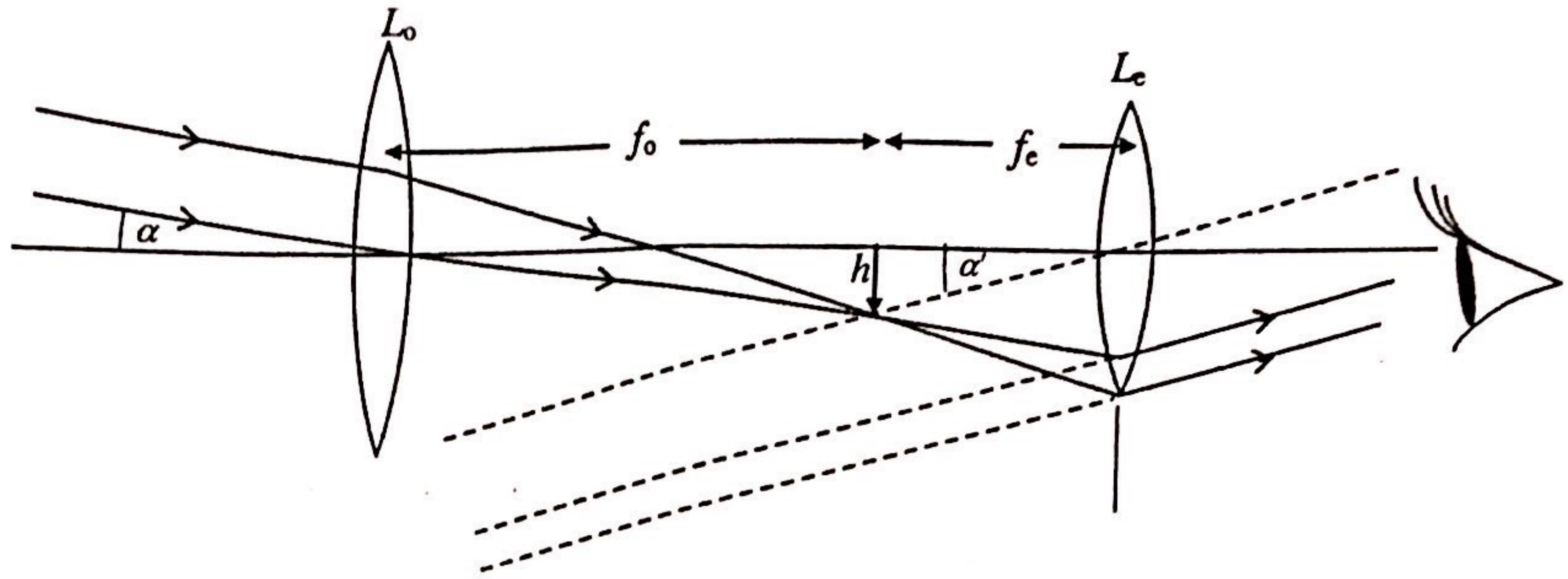
(ii) The linear magnification of an image depends on <u>both the size</u> and <u>the distance</u> to the object. ....(01)

However, the size of the image formed <u>on the retina</u> (of the eye) depends only on the <u>angle subtended</u> at the eye by the rays coming from the image. ....(01)

Therefore, angular magnification is a better measure compared to linear magnification.

(b) (i) When the final image is formed at infinity *OR* When the eye is relaxed ....(01)

(ii)



Two parallel rays passing through L_o and correct ray diagram up to the image ....(01)

Construction of parallel rays after passing through L_e to the eye ....(01)

Correct marking of f_o and f_e ....(01)

(deduct 01 mark if arrow heads are missing)

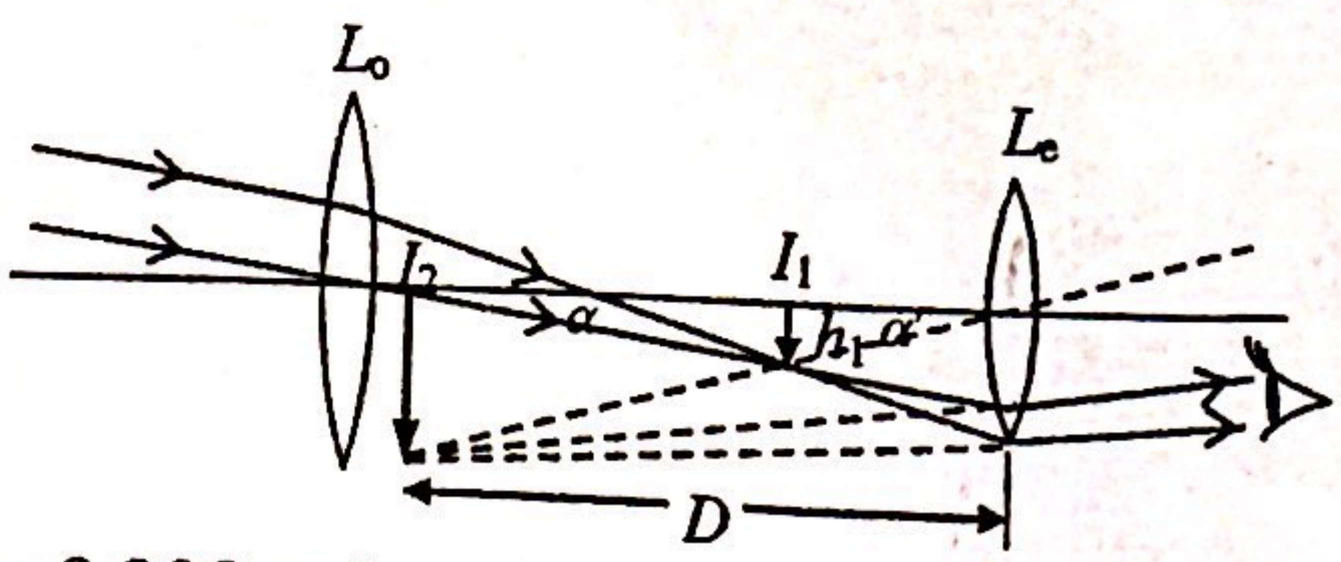
(iii) According to the diagram $\alpha = \frac{h}{f_o}$ ———————— (2) and

$\alpha' = \frac{h}{f_e}$ ———————— (3)

For any one equation ....(01)

Substuting in eq. (1) $m = \frac{f_o}{f_e}$ ....(01)

(c) (i)



$\alpha = 0.5° = 0.009$ rad

For the first image I_1, substituting in eq. (2) $0.009 = \frac{h_1}{100}$ ....(01)

$h_1 = 0.9$ cm

For the eye lens (F_e), using <u>Cartesian Sign Convention</u>

$v = +25$ cm, $f = -10$ cm

Applying the lens formula $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ ....(01)

$\frac{1}{+25}-\frac{1}{u}=\frac{1}{-10}$ ....(01)

(for correct substitution)

$\frac{1}{u}=\frac{1}{25}+\frac{1}{10}=\frac{7}{50}$

Consider the second image I_2, $\alpha'=\frac{h_1}{u}$ ....(01)

$\alpha'=\frac{0.9\times7}{50}$ rad ....(01)

(for substitution)

$\alpha'=\frac{0.9\times7}{50}\times\frac{1°}{0.018}$

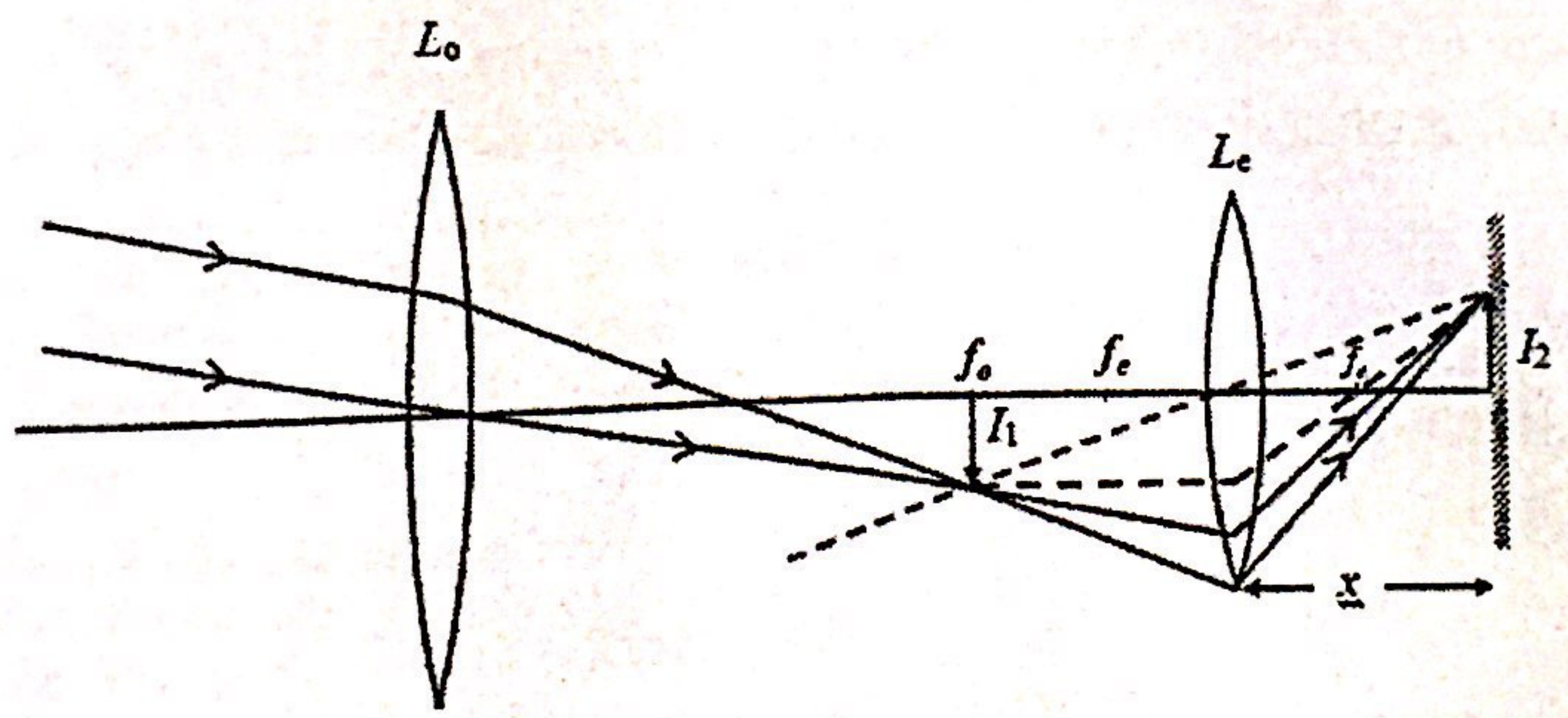
$\alpha'=7°(6.9°-7°)$..(01)

Substitute in equation (1), angular magnification (m) $m=\frac{7°}{0.5°}$ ....(01)

(for substitution)

$=14(13.8-14)$ ....(01)

(ii)



Locating the real image by drawing the two broken lines ....(01)

Marking the focal point f_e on the right side of L_e ....(01)

(iii) $v = -30$ cm, $f = -10$ cm

Applying lens formula for eye lens for this situation $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\frac{1}{-30} - \frac{1}{u} = \frac{1}{-10}$ ....(01)

(for correct substitution)

$\frac{1}{u} = \frac{1}{10} - \frac{1}{30} = \frac{3-1}{30}$

$u = 15$ cm

Linear magnification of the eye lens $M = \frac{v}{u}$ .... (01)

$M = \frac{30}{15} = 2$

Since $h_1 = 0.9$ cm and $M = \frac{h_2}{h_1}$ ....(01)

$h_2 = 2 \times 0.9$

$= 1.8$ cm .... (01)

(iv) For the objective lens of Yerkes telescope apply $\alpha = \frac{h}{f_o}$

$0.009 = \frac{h_1}{19.0}$ ....(01)

$h_1 = 17.1$ cm

Since the size of the first image and the size of the second real image <u>are equal</u> *OR* the linear <u>magnification is 1</u> *OR* identification that object distance and image distance are equal ....(01)

Therefore, image distance = object distance = $2f_e$

OR $-\frac{1}{30} - \frac{1}{30} = \frac{1}{f_e}$ ....(01)

$2f_e = 30$ cm

$f_e = 15$ cm (0.15 m) ....(01)

Using eq. (1), The angular magnification $m = \frac{h_1}{0.3} \times \frac{19}{h_1} (OR \frac{19}{0.3})$ ....(01)

$m = 63$ (OR 63.3) ....(01)

. (a) Young's modulus of a material is defined by $\frac{F/A}{e/l}$, where all symbols have their usual meaning. Name the terms $\frac{F}{A}$ and $\frac{e}{l}$.

(b) As shown in Figure (1) a Karate player tried to break a wooden plank by a single kick from his heel When the player hits the plank, the heel of the player comes to rest from an initial speed of 24 m s^{-1} in 4·0 ms without breaking the plank. The leg has an effective mass of 16·0 kg and the effective cross-sectional area of the smallest part of the leg bone is $3{\cdot}0 \times 10^{-4} \text{ m}^2$. The bone material of the leg can withstand a maximum compressive stress of $1{\cdot}8 \times 10^7 \text{ N m}^{-2}$. Assume that the stress is uniformly distributed along the bone.

(i) Determine the mean force acting on the leg during the heel of the player comes to rest from 24 m s^{-1}.

(ii) What is the maximum compressive stress on the bone of the leg?

(iii) Is there a possibility to fracture the bone? Give reasons for your answer.

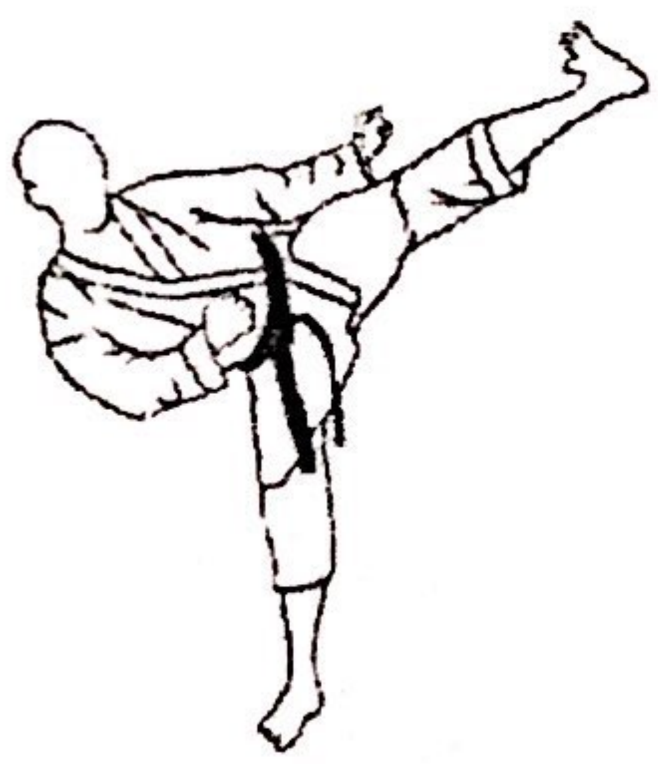


Figure (1)

(c) The leg bone of the Karate player is fractured in the process of kicking as mentioned in (b) above. He uses crutches made of single tube for walking until recovery as shown in Figure (2). The mass of player is 90 kg. Half of the player's weight is supported by crutches and other half is supported by his other leg. While he is standing each crutch makes an angle of 22° with the vertical. Each crutch is made of hollow aluminium tube with inner radius $1{\cdot}0 \times 10^{-2}$ m and outer radius $2{\cdot}0 \times 10^{-2}$ m respectively. Young's modulus of aluminium is $7{\cdot}0 \times 10^{10} \text{ N m}^{-2}$.

(i) What should be the minimum coefficient of static friction between the end of a crutch and ground in order for him to stand still without slipping? Take $\tan(22°) = 0{\cdot}4$

(ii) Determine the magnitude of the compressive force acting on each crutch. Take $\cos(22°) = 0{\cdot}9$

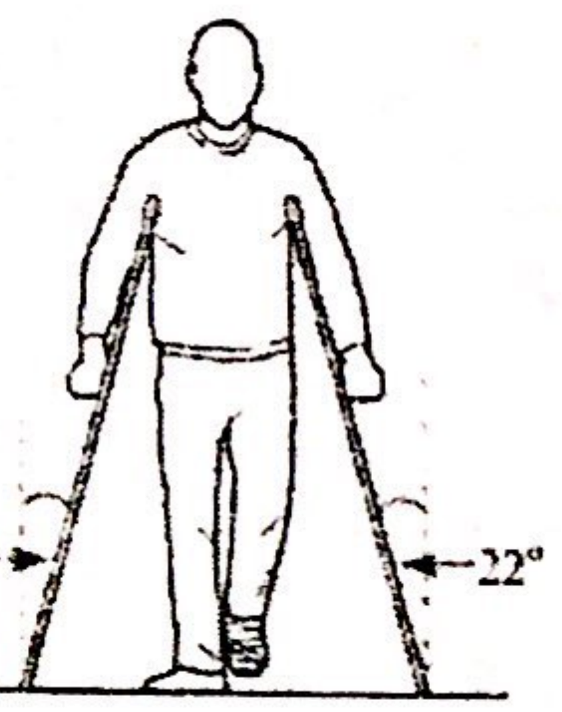


Figure (2)

- Round off your answers for (c) (iii), (c) (iv) and (d) (ii) below to two decimal places in scientific notation. See the note given before the question 5.

(iii) Calculate the compressive stress and the compressive strain on a crutch. Take $\pi = 3$.

(iv) If the length of a crutch is 125 cm what is the change in length of a crutch?

(d) Suppose instead of the crutches mentioned in (c) above crutches made of two coaxial hollow tubes are used by the player. Inner tube of the cylindrical crutch is made of aluminium having Young's modulus E_1 and the outer tube is made of stainless steel having Young's modulus E_2. Respective cross-sectional areas of aluminium and stainless steel tubes are A_1 and A_2. The cross-section of the composite tube is shown in Figure (3).

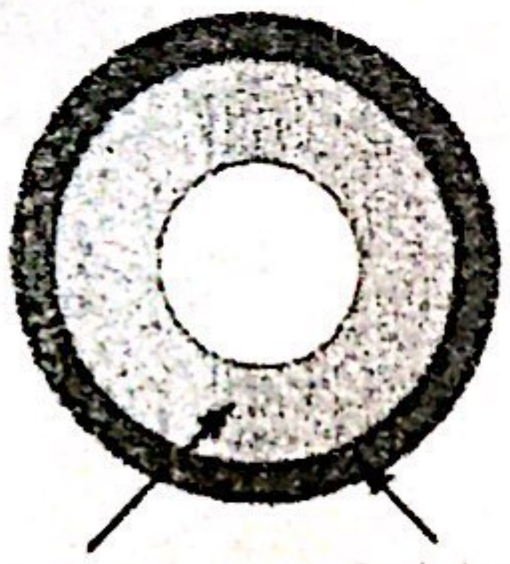


Figure (3)

(i) Show that the effective Young's modulus E of the composite tube is given by $E = \frac{E_1A_1 + E_2A_2}{(A_1 + A_2)}$.

(ii) Let $E_1 = 8{\cdot}0 \times 10^{10} \text{ N m}^{-2}$, $A_1 = 10{\cdot}0 \times 10^{-4} \text{ m}^2$, $E_2 = 2{\cdot}0 \times 10^{11} \text{ N m}^{-2}$, $A_2 = 6{\cdot}0 \times 10^{-4} \text{ m}^2$. The length of each crutch is 125 cm. Determine the change in the length of the composite tube when the force in (c)(ii) is applied to the crutch.

(e) Normally rubber caps are fixed to the lower ends of the aluminium crutches. Use physics principles to state advantages that would occur for a person walking using these crutches with rubber caps

(a) $\frac{F}{A}$ = stress ....(01)

$\frac{e}{l}$ = strain ....(01)

(b) (i) $F = m(v-u)/t$ ....(01)

$F = 16 \times \left(\frac{24-0}{4\times10^{-3}}\right)$ (for correct substitution) ....(01)

$F = 9.6 \times 10^{4}$ N ....(01)

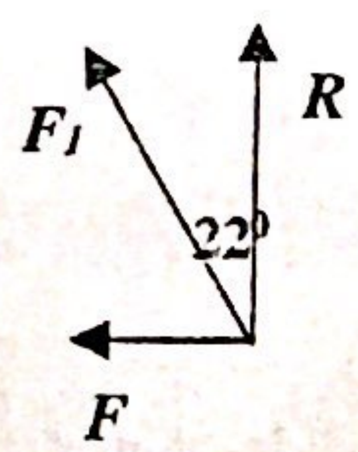
(ii) $\frac{F}{A} = \frac{9.6\times10^{4}}{3\times10^{-4}}$ (for substitution) ....(01)

$\frac{F}{A} = 3.2 \times 10^{8}$ N m^{-2} ....(01)

(iii) Yes (No mark for incorrect values) .... (01)

Maximum compressional stress of 1.8×10^{7} N m^{-2} < 3.2×10^{8} N m^{-2} ....(01)

(c)



(i) If the force along a single crutch is F_1,

then the frictional force $F = F_1 \sin(22^0)$ ....(01)

The normal reaction force $R = F_1 \cos(22^0)$ ....(01)

Since coefficient of friction $\mu = \frac{F}{R}$; ....(01)

$\mu = \tan(22°)$

$\mu = 0.4$ ....(01)

(ii)Normal reaction force on a crutch $F_1 \cos(22°) = \frac{900-450}{2}$

$F_1 = \frac{225}{\cos(22°)}$ *OR* $\frac{225}{0.9}$ ....(01)

$F_1 =$ 250 N (234 - 250) N ....(01)

{Alternative methods:

$R = \frac{900-450}{2} = 225$ N ; $F = \mu R = 0.4 \times 225 = 90$ N

$F_1 = 225 \times \cos(22^0) + 90 \times \sin(22^0)$ .... (01)

$= 225 \times \cos(22^0) + 90 \times \tan(22^0) \times \cos(22^0) = 225 \times 0.9 + 90 \times 0.4 \times 0.9$

$= 234.9$ N (235 N) ....(01)

OR $F_1^2 = 225^2 + 90^2$ ....(01)

$= 242$ N ....(01) }

[Due to these alternative methods a wider range has to be assigned for final answers]

(iii) Effective area $= \pi(2^2 - 1^2) \times 10^{-4}$ .... (01)

Compressional stress $= \frac{250}{\pi(2^2-1^2)\times 10^{-4}}$ (for dividing by the area) .... (01)

$= \frac{250\times 10^4}{3\times\pi}$

$= 2.78 \times 10^5$ N m^{-2} ....(01)

$(2.48 - 2.78) \times 10^5$ N m^{-2}

Compressional strain $= \frac{2.78\times 10^5}{7.0 \times 10^{10}}$ (for dividing by the Young's modulus) ....(01)

$= 3.97 \times 10^{-6}$ ....(01)

$(3.54 - 3.97) \times 10^{-6}$

(iv) Change in length of a crutch $= 3.97 \times 10^{-6} \times 125 \times 10^{-2}$ ....(01)

(for multiplication of strain by the length)

$= 4.96 \times 10^{-3}$ mm $(4.96 \times 10^{-6}$ m) ....(01)

$(4.42 - 4.96) \times 10^{-3}$ mm

(*d*) (i) If the forces applied to the aluminum and stainless steel tubes are F_1 and F_2 , then ....(01)

the total force $F_{total} = F_1 + F_2$

$\frac{Ee(A_1+A_2)}{l} = \frac{E_1 e A_1}{l} + \frac{E_2 e A_2}{l}$ ....(01)

$E = \frac{E_1 A_1 + E_2 A_2}{(A_1+A_2)}$

(ii) If the change in the length of the composite tube is e

$$F = \frac{E_1A_1+E_2A_2}{(A_1+A_2)} \times e \times \frac{(A_1+A_2)}{l}$$

$$e = \frac{F \times l}{E_1A_1+E_2A_2}$$

$$e = \frac{250 \times 125 \times 10^{-2}}{8.0 \times 10^{10} \times 10.0 \times 10^{-6} + 20.0 \times 10^{10} \times 6.0 \times 10^{-6}}$$ ....(03)

[One mark for substitution in $F \times l$ term; one mark for correct substitution in E_1A_1 term; one mark for correct substitution in E_2A_2 term]

$e = 1.56 \times 10^{-3}$ mm (1.56×10^{-6} m) ....(01)

$(1.46 - 1.56) \times 10^{-3}$ mm

(*e*) The (maximum) force felt by the person will be lower (as it increases the time of contact)

OR the (maximum) impulse felt by the person will be lower (as it increases the time of contact)

OR energy will be stored as elastic potential energy of rubber which provides cushioning effect

OR provides good adhesion to the floor

OR coefficient of friction is increased/friction is increased/reduce slipping

..............................(02)

[Two marks for two valid reasons]

Read the following passage and answer the questions.

Black holes are one of the most curious objects in the universe. They have enormous amount of matter packed into a minimal volume resulting a very strong gravitational field. Because no light can escape from a black hole, they are invisible.

The escape velocity (v_e) from the surface of a spherical object of mass M with uniform density and radius R is given by $\sqrt{\frac{2GM}{R}}$, where G is the universal gravitational constant. This expression for escape velocity suggests that a body of mass M will act as a black hole if its radius R is less than or equal to a certain critical value. This critical radius is known as the Schwarzschild radius R_S, and the surface of the sphere with this radius surrounding a black hole is called the event horizon. Since light cannot escape from within this sphere, we cannot detect events occurring inside it.

If light cannot escape from a black hole, how can we know the existence of such objects? Any gas or dust near a black hole tends to swirl around and pull into the black hole. This causes heating of the dust/gas, just as air compressed in a pump gets hotter. Temperatures of dust/gas in excess of 10^6 K can occur, so it emits not only visible light but also X-rays. Astronomers look for these X-rays emitted by the dust/gas before they cross the event horizon to detect the presence of a black hole.

There are also strong evidences for the existence of much larger supermassive black holes. One such black hole is found to exist at the center of our Milky Way galaxy, 26 000 light-years from Earth in the direction of the constellation Sagittarius. Astrophysicists have discovered a star designated by S4716 revolving around this black hole. This star completes one revolution around the supermassive black hole in a short period of time like four years. This means that the star is travelling at very high speed $8{\cdot}0 \times 10^6$ m s^{-1} around this black hole. By analyzing this motion, the mass of the unseen supermassive black hole can be calculated. You may take $G = 6{\cdot}0 \times 10^{-11}$ N m^2 kg^{-2} and speed of light $c = 3{\cdot}0 \times 10^8$ m s^{-1}.

(a) What is a black hole?

(b) (i) Starting from first principles derive the expression for the escape velocity $v_e = \sqrt{\frac{2GM}{R}}$

(ii) For a spherical object with uniform density ρ, show that v_e is directly proportional to the radius R of the object.

(iii) Letting $v_e = c$ in the expression derived in (b) (i) above, obtain an expression for the Schwarzschild radius (R_S) for a spherical object of mass M in terms of G, M and c.

(c) What is the reason for defining an event horizon?

(d) Can a black hole emit X-rays? Give reasons for your answer.

(e) Determine the peak wavelength (λ_m) of radiation emitted by gas or dust with temperature 10^6 K swirling into a black hole. (Wein's displacement constant = 2900 μm K).

- Round off your answers for (f) (i) to two decimal places and (f) (ii) to one decimal place respectively in scientific notation. See the note given before the question 5.

(f) Assume that the star S4716 revolves around the supermassive black hole in a circular path of radius r. Further assume that the star and the supermassive black hole are spherical in shape with uniform density.

(i) Using the data given in the paragraph determine the value of r. (Take $\pi = 3$)

(ii) Hence calculate the mass, M_B, of the supermassive black hole.

(iii) Calculate the Schwarzschild radius, R_S, of the supermassive black hole.

(g) Suppose hypothetically, the sun suddenly becomes a black hole with the same mass as it has today.

(i) Would the earth continue to revolve around the sun along the same orbit as of today? Give reasons for your answer.

(ii) Could life on earth get affected due to this? Give the main reason for your answer.

(iii) Show that the sun would become a black hole if its mass could shrink to a sphere of 2.4 km in radius. Take the mass of the sun as $1{\cdot}8 \times 10^{30}$ kg.

(*a*) Black holes are objects with

enormous amount of matter .... (01)

packed into a minimal volume .... (01)

(*b*) (i) Let the mass of the escaping body be m.

Kinetic energy of mass $m = \frac{1}{2}mv_e^2$ .... (01)

Gravitational potential energy of mass $m = -\frac{GMm}{R}$ .... (01)

From energy conservation,

$\frac{1}{2}mv_e^2 - \frac{GMm}{R} = 0$ (*OR* any other correct form) .... (01)

$$v_e = \sqrt{\frac{2GM}{R}}$$

(ii) $\rho = \frac{M}{\frac{4}{3}\pi R^3}$ .... (01)

Substituting for M ,⇒ $v_e = \sqrt{\frac{2G\,{}^4/_3\pi\rho R^3}{R}}$ .... (01)

$v_e = \sqrt{\frac{8G\pi\rho}{3}}\,R$ .... (01)

∴ v_e is directly proportional to the radius R of the object.

(iii) $c = \sqrt{\frac{2GM}{R_S}}$ .... (01)

$R_S = \frac{2GM}{c^2}$ .... (01)

(*c*) Cannot detect events occurring inside it. .... (01)

(*d*) No. .... (01)

X - rays are also electromagnetic waves like light .... (01)

(*e*) $\lambda_m T$ = constant *OR* $\lambda_m T = 2900$ .... (01)

$\lambda_m = \frac{2900}{10^6}$ (For substitution) .... (01)

$\lambda_m = 2.9 \times 10^{-3}$ μm .... (01)

(i) $T = \frac{2\pi r}{v}$ .... (01)

$4 \times 365 \times 24 \times 60 \times 60 = \frac{2\times 3\times r}{8.0\times 10^6}$ (For substitution) .... (01)

$r = 1.68 \times 10^{14}$ m $(1.680 - 1.682) \times 10^{14}$ m .... (01)

(ii) Let m be the mass of the star,

$\frac{GM_B m}{r^2} = \frac{mv^2}{r}$ .... (01)

$M_B = \frac{v^2 r}{G}$ .... (01)

$M_B = \frac{(8.0\times 10^6)^2 \times 1.68\times 10^{14}}{6.0\times 10^{-11}}$ (For substitution) .... (01)

$M_B = 1.8 \times 10^{38}$ kg$(1.79 - 1.80) \times 10^{38}$ kg .... (01)

(iii) $R_S = \frac{2GM}{c^2}$

$R_S = \frac{2\times 6.0\times 10^{-11}\times 1.8\times 10^{38}}{9\times 10^{16}}$ .... (01)

(For substitution)

$R_S = 2.4 \times 10^{11}$ m .... (01)

$(2.38 - 2.40) \times 10^{11}$ m

(g) (i) Yes .... (01)

There will be no change in the gravitational field/force experienced by the earth *OR* solar black hole would exert no more gravitational pull than our Sun. .... (01)

(ii) Yes .... (01)

No light/heat will reach the earth. .... (01)

(iii) R_S value for the sun:

$R_S = \frac{2\times 6.0\times 10^{-11}\times 1.8\times 10^{30}}{9\times 10^{16}}$ .... (01)

(For substitution)

$R_S = 2.4 \times 10^3$ m (2.4 km)

9. Answer either part (A) or part (B) only.

Part (A)

(*a*) The capacity of a cell is defined as the maximum constant current that can be drawn in one hour and its unit is given by ampere-hour (Ah). Two identical cells each of capacity 6 Ah and e.m.f. 5·0 V are connected to form a battery.

Calculate the capacity (in Ah) and the e.m.f. (in V) of the battery if the two cells are connected

(i) in series and

(ii) in parallel.

(*b*) An electric car battery is made using identical 192 cells each with e.m.f. 4·0 V. Eight cells are connected as shown in Figure (1) to form a battery module. A total of 24 such modules are connected in series to form a 24 kWh electric car battery.

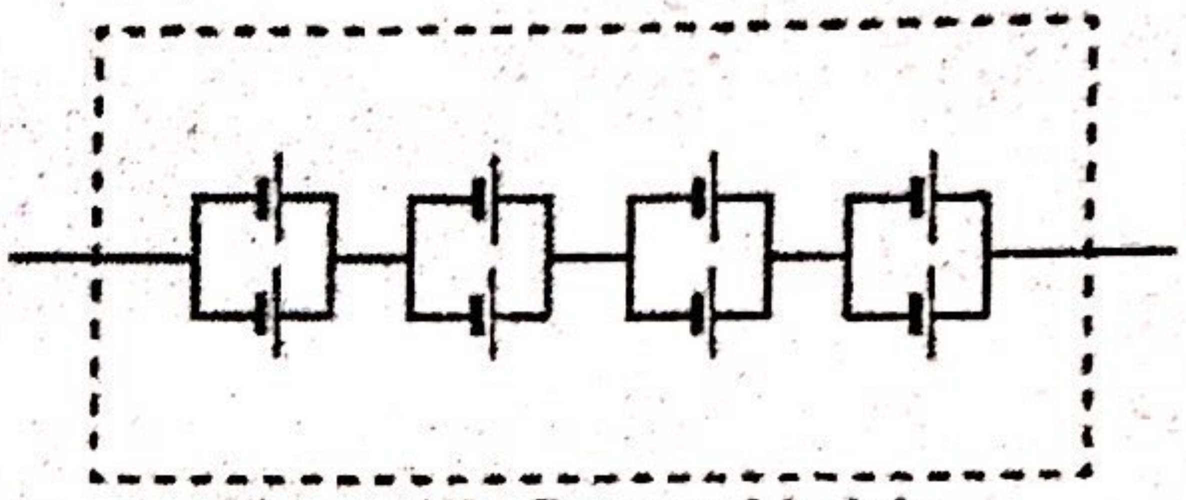


Figure (1): Battery Module

(i) Calculate the e.m.f. (in V) and the capacity (in Ah) of a single module. (You may use $1\ \text{kWh} = 10^3\ \text{V Ah}$)

(ii) Calculate the capacity (in Ah) and the e.m.f. (in V) of the 24 kWh electric car battery.

(*c*) The above electric car travelling at a constant speed of $36\ \text{km h}^{-1}$ on a horizontal road experiences a total resistive force of 480 N against its motion. The power consumption of the air conditioner (A/C) of the car is 1·2 kW. Calculate the maximum distance that the car can travel consuming only 50% of the full stored energy (in kWh) of the battery,

(i) with A/C on for the entire journey. (Assume that the power consumption of the A/C is constant during the entire journey.)

(ii) with A/C off for the entire journey.

(*d*) The electrical circuit used for heating the interior of the above car is shown in Figure (2). When the interior of the car needs to be heated during cold weather, the driver can set a switch to pass a current through the resistors R_1 or R_2 ($R_1 < R_2$). The current passing through the resistors R_1 and R_2 dissipates power and heats up the interior. Therefore resistors act as heaters. Suppose the battery develops an internal resistance over time. An ammeter with internal resistance of 10 Ω and an ideal voltmeter are connected to test the circuit.

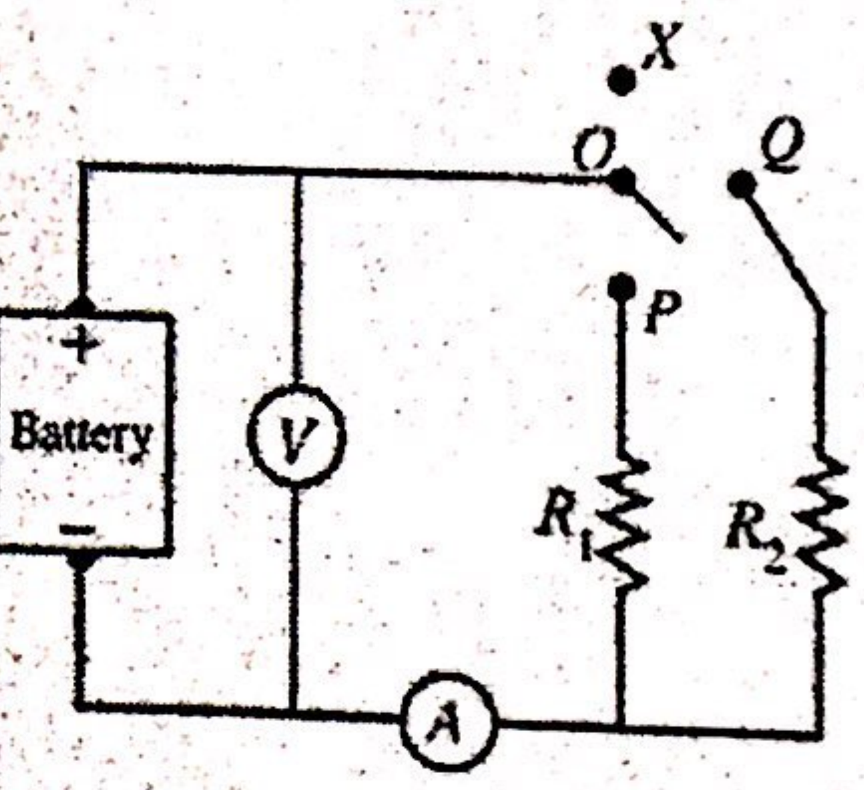


Figure (2)

(i) The driver can complete the circuit by connecting *OP* or *OQ*. Identify and write down the appropriate connections to get a low and a high rates of power dissipation. For example, making the connection *OX* switch off the heaters and disconnects R_1 and R_2 from the circuit.

(ii) The voltmeter reading is 255 V when the heaters are off. The voltmeter reading drops to 250 V and the ammeter reads 5·0 A when the circuit is connected to R_1. Calculate the e.m.f. of the battery, the internal resistance of the battery, and the value of resistance of the resistor R_1.

(iii) Calculate the power dissipation of the heater operating in the power mode mentioned in (*d*) (ii) above.

...) (i) In series combination, the current does not change.

Therefore, the capacity $= 6$ Ah .... (01)

The e.m.fs are added. .... (01)

Therefore, e.m.f. $= 5.0 + 5.0$

$= 10.0$ V ....(01)

(ii) In parallel combination, the currents are added .... (01)

Therefore, the capacity $= 6 + 6$

$= 12$ Ah .... (01)

The e.m.fs are not changed

Therefore, e.m.f. $= 5.0$ V .... (01)

(*b*) (i) e.m.f. $= 4.0 \times 4$

$= 16.0$ V .... (01)

Energy stored in the full battery (24 modules in series) = 24 kWh

Energy stored in a module $= 24/24$ .... (01)

$= 1$ kWh

Capacity of a module $= 1000/16$

$= 62.5$ Ah .... (01)

(ii) Capacity of the full battery = Capacity of a single module

$= 62.5$ Ah .... (01)

e.m.f. of the full battery $= 24000/62.5$

$= 384$ V ....(01)

{Alternative method

e.m.f. of the full battery $= 16 \times 24$

$= 384$ V .... (01)}

(*c*) (i) Usable energy = $24 \times 50\%$ (multiplying by 50%) .... (01)

$= 12$ kWh

Speed of the car = 36 km/h = 10 m/s (converting km/h to m/s) ....(01)

Total power consumption in 1 hr = $10 \times 480 + 1200$ (for addition) .... (01)

$= 6$ kW

Total travel time = 12 / 6 (for division) .... (01)

$= 2$ hr

Range $= 36 \times 2$

$= 72$ km (*OR* 72000 m).. (01)

(ii) Total power per 1 hr = $10 \times 480 = 4.8$ kW

Total travel time = 12 / 4.8 (for division) .... (01)

$= 2.5$ hr

Range $= 36 \times 2.5$

$= 90$ km (OR 90000 m) .... (01)

(*d*) (i) For high power dissipation – *OP* .... (01)

For low power dissipation – *OQ* .... (01)

(ii) e.m.f. = 255 V .... (01)

Let r be the internal resistance of the battery. Then Applying Kirchhoff's law,

$255 - 5 \times r = 250$ .... (02)

(01 mark for the L.H.S.; 01 mark for equating L.H.S. to 250)

$r = 1\ \Omega$ .... (01)

$250 - 5R_1 - 5\times10 = 0$ .... (02)

(01 mark for the $5R_1$ term.; 01 mark for rest of the equation)

$R_1 = 40\ \Omega$ .... (01)

(iii)

$P = I^2R_1$ .... (01)

$= 5 \times 5 \times 40$ (for substitution) .... (01)

$= 1$ kW (1000 W) .... (01)

Part(B)

(a) The circuit shown in figure (1) is used to obtain an appropriate output voltage V_{out} from a variable input voltage V_{in} using a Zener diode and a transistor arrangement. The circuit uses a Zener diode with minimum current of 10 mA and a silicon transistor. Let resistance $R_S = 70\ \Omega$, the load resistance $R_L = 90\ \Omega$ and the Zener voltage $V_Z = 8{\cdot}3$ V. Suppose $V_{in} = 23$ V.

Calculate the following,

(i) V_{out} (Take $V_{BE} = 0{\cdot}7$ V)
(ii) Current I_L
(iii) Current I_S and
(iv) I_C corresponding to the minimum Zener current.

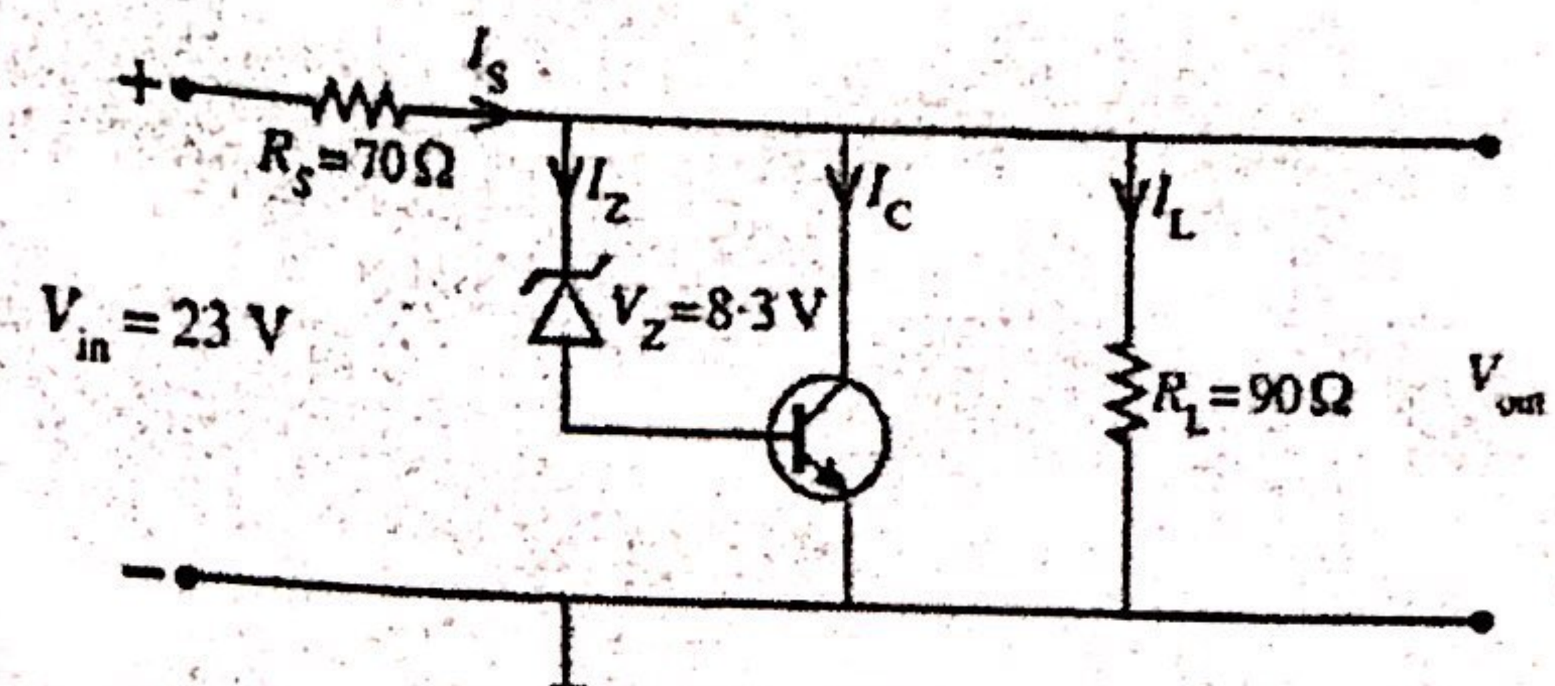


Figure (1)

(b) The circuit in Figure (1) can regulate a voltage variation in the input to maintain a constant V_{out}.

(i) Calculate the amount of power dissipated across the R_S resistor when V_{in} = 23 V and 30 V.
(ii) Using your calculations in (b)(i) above, briefly explain how a change in the input voltage is regulated by the circuit.

(c) The circuit in Figure (1) can also regulate a voltage variation in V_{out} due to an increase in output load resistance.

(i) If the load resistance is increased, what will happen to Zener current I_Z and I_C? Explain your answer.
(ii) Briefly explain how the Zener diode and transistor combination regulate the output voltage when the load resistance is increased.

(d) The circuit shown in Figure (2) is used to charge a battery from a solar panel with an internal resistance (r) that can generate up to 15 V. The output voltage of the circuit should not exceed 14 V.

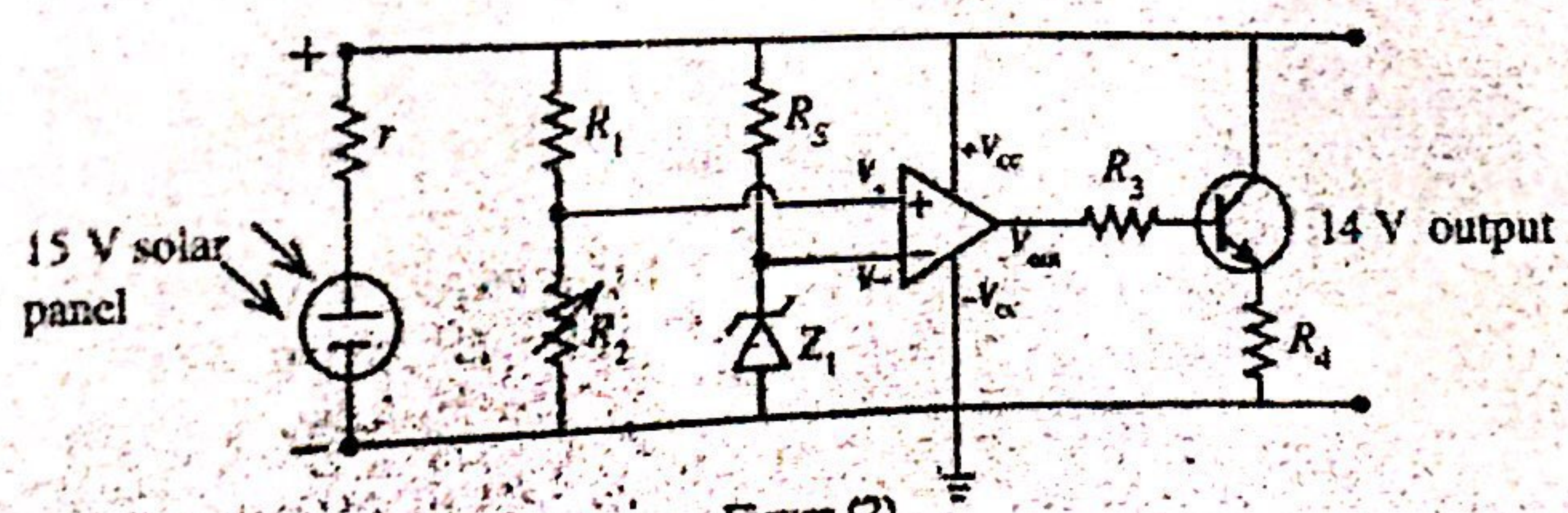


Figure (2)

(i) Write down the operational mode of the operational amplifier in the above circuit from the given choices. (inverting amplifier, non-inverting amplifier, comparator)
(ii) Under bright sunlight R_2 is adjusted to produce 14 V output voltage. When $R_1 = 9$ kΩ and $R_2 = 5$ kΩ for the output of the op-amp to be positively saturated, calculate the most appropriate maximum voltage V_{Z_1} for that Zener diode Z_1 should have.
(iii) If the output of the op-amp saturates for 100 μV voltage difference between the non inverting input and the inverting input, calculate the open-loop voltage gain of the op-amp when the output voltage of the circuit is 14 V? Assume that the output saturation voltage of the op-amp is 2 V below the supply voltage.
(iv) Briefly explain the action of the op-amp and the transistor in this circuit when the solar panel produces less than 14 V under weak sunlight.

(a)(i) $V_{out} = V_z + V_{BE}$ .... (01)

$= 8.3 + 0.7$ (for addition) .... (01)

$V_{out} = 9\text{ V}$ .... (01)

(ii) $I_L = V_{out} / R_L$ .... (01)

$I_L = 9/90$ (for division) .... (01)

$I_L = 0.1\text{ A}$ .... (01)

(iii) $I_S = (V_{in} - V_{out}) / R_s$ .... (01)

$I_S = (23 - 9)/70$ (for division) .... (01)

$I_S = 0.2\text{ A}$ .... (01)

(iv) $I_S = I_Z + I_C + I_L$ .... (01)

$I_C = I_S - I_L - I_Z$

$I_C = 0.2 - 0.1 - 0.01$ (for subtractions) .... (01)

$I_C = 0.09\text{ A}$ .... (01)

(b) (i) $P = V^2/R$ (OR I^2R)

$P = 2.8\text{ W}$ .... (01)

$P = 6.3\text{ W}$ .... (01)

(ii) The output voltage is constant across the Zener and Transistor. .... (01)

When the input voltage is changed, the excess power is dissipated through the resistor R_s (as heat) .... (01)

(c) (i) $I_S = I_Z + I_C + I_L$

If the load resistance is increased, the load current decreases.

Therefore, I_z should increase .... (01)

Increase of I_z allow additional amount of current pass through the transistor

Therefore, I_c will increase .... (01)

(ii) The output voltage is constant across the Zener and Transistor.

When the load resistance is increased, I_z increases to allow more current to pass through the transistor keeping I_S constant.

The excess power is dissipated through the transistor to provide a constant output voltage.

.... (01)

(*d*)

(i) Comparator

.... (02)

(ii) $V_+ = V_{max} \times [R_2 / (R_1 + R_2)]$

$= 14 \times 5/(5+9)$ (substitution) .... (01)

$V_z = 5$ V .... (01)

(iii) $A = V_{out}/(V_+ - V_-)$ .... (01)

$A = (14-2)/(100 \times 10^{-6})$ .... (01)

$A = 120{,}000$ .... (01)

(iv) When there is less sunlight, the voltage at the positive terminal of the op-amp decreases below 5 V. .... (01)

This causes the output of the op-amp to goes to zero volt .... (01)

This causes the transistor to go to the cutoff mode (transistor is off). .... (01)

Therefore, the voltage appearing across the solar panel will be the same as the voltage across the battery. .... (01)

10. Answer either part (A) or part (B) only.

Part (A)

(*a*) Clearly identifying the symbols used, write down an expression for the volume expansivity (γ) of a liquid.

(*b*) In the tank of a filling station at Nuwaraeliya, the temperature of petrol in a certain day is 7 °C in the morning and 27 °C in the afternoon. The average volume expansivity of petrol is $9{\cdot}6 \times 10^{-4}$ °C^{-1} and the density of petrol at 7 °C is 730 kg m^{-3}. A car is going to be filled 20 litres of petrol from the filling station.

(i) What is the mass of 20 litres of petrol at 7 °C? (1 m^3 = 1000 litres)

(ii) If the temperature of 1 m^3 of petrol at 7 °C increases to 27 °C, calculate its new volume. (Round off your answer to three decimal places in m^3.)

(iii) What is the density of petrol at 27 °C? [Take $\frac{7{\cdot}3}{1{\cdot}019} = 7{\cdot}164$. Give your answer to the nearest integer in kg m^{-3}.]

(iv) Calculate the mass of 20 litres of petrol at 27 °C.

(v) How many extra kilograms of petrol would the car get if 20 litres of petrol is filled at 7 °C instead of at 27 °C from the filling station.

(*c*) A tank of a petrol bowser is made of metal and the internal volume of the tank is 25 000 litres at 7 °C. In a hot day, the temperature of petrol and the tank became 27 °C and the tank was completely filled by the petrol due to the expansion. The average volume expansivity of petrol is $9{\cdot}6 \times 10^{-4}$ °C^{-1} and linear expansivity of metal is $2{\cdot}4 \times 10^{-5}$ °C^{-1}.

- Round off your answers for (c) (i), (c) (iii) and (c) (iv) below to two decimal places in scientific notation. See the note given before the question 5.

(i) Calculate the apparent volume expansivity of petrol in the tank.

(ii) Hence calculate the volume of petrol (in litres) at 7 °C. [Take $\frac{1}{1+1{\cdot}776\times10^{-2}} = 0{\cdot}98$]

(iii) How much heat is absorbed by the tank and petrol from outside to increase the temperature from 7 °C to 27 °C? Mass of the metal of the empty tank is $2{\cdot}0 \times 10^3$ kg. Specific heat capacities of metal and petrol are $5{\cdot}0 \times 10^2$ J kg^{-1} K^{-1} and $2{\cdot}2 \times 10^3$ J kg^{-1} K^{-1} respectively.

(iv) Suppose at 7 °C the tank is half filled with petrol and the rest with air at atmospheric pressure of $1{\cdot}0 \times 10^5$ Pa and sealed. Saturated vapour pressure of petrol at 27 °C is $7{\cdot}47 \times 10^4$ Pa. Determine the total pressure inside the tank at 27 °C. Neglect the volume expansion of metal and petrol for this calculation.

(v) How many moles of petrol vapour present inside the bowser at 27 °C in the situation (c) (iv) above? Universal gas constant $R = 8{\cdot}3$ J mol^{-1} K^{-1}. Assume that the petrol vapour behaves as an ideal gas.

a) $\gamma = \frac{V_2 - V_1}{V_1(\theta_2 - \theta_1)}$ ....(01)

V_1 is the volume at temperature θ_1 and V_2 is the volume at temperature θ_2 .... (01)

b) (i) Mass of 20 l of petrol $= 20 \times 10^{-3} \times 730$ .... (01)

$= 14.6$ kg .... (01)

(ii) $V_{27} = V_7[1 + \gamma(27 - 7)]$ .... (01)

$V_{27} = 1[1 + 9.6 \times 10^{-4} \times 20]$ .... (01)

$V_{27} = 1.019\ \text{m}^3$ .... (01)

(iii) Density of petrol at 27 °C $= \frac{1 \times 730}{1.019}$ (for substitution) .... (01)

{Alternative method: $\rho_{27} = \frac{\rho_7}{1 + \gamma(27-7)}$

$= \frac{730}{1 + 9.6 \times 10^{-4} \times 20}$ ......(01)}

$= 716\ \text{kg m}^{-3}$ .... (01)

(iv) Mass of 20 l petrol at 27 °C $= 716 \times 20 \times 10^{-3}$ (for substitution) .... (01)

$= 14.3$ kg (14.32 kg) .... (01)

(v) Extra mass $= 14.6 - 14.3\ (14.32)$ (for substitution) .... (01)

$= 0.3$ kg (*OR* 0.28 kg) .... (01)

.... (01)

(c) (i) $\gamma_r = \gamma_a + 3\alpha$ .... (01)

Apparent volume expansivity $= 9.60 \times 10^{-4} - 3 \times 2.4 \times 10^{-5}$ (for substitution) .... (01)

$= 8.88 \times 10^{-4}\ °\text{C}^{-1}$

(ii) $V_{27} = V_7[1 + 8.88 \times 10^{-4} \times (27 - 7)]$ .... (01)

$25{,}000 = V_7[1 + 1.776 \times 10^{-2}]$

$V_7 = \frac{25{,}000}{1+1.776\times10^{-2}}$ .... (01)

$V_7 = 24{,}500$ litres .... (01)

(iii) $Q = mc\Delta\theta$ .... (01)

$Q = (mc\Delta\theta)_{petrol} + (m'c'\Delta\theta)_{metal}$

$Q = 24.5 \times 730 \times 2.2 \times 10^3 \times (27 - 7) + 2 \times 10^3 \times 5 \times 10^2 \times (27 - 7)$......... (02)

(01 mark for each substitution)

$= 8.07 \times 10^8$ J .... (01)

$(8.00 - 8.07) \times 10^8$ J

(iv) If the air pressure at 27°C is P_{27air}

$\frac{1.0\times10^5}{273+7} = \frac{P_{27air}}{273+27}$ .... (01)

$P_{27air} = 1.07 \times 10^5$ Pa

Total pressure $= P_{27gas} + P_{27air}$ .... (01)

$= 7.47 \times 10^4 + 1.07 \times 10^5$ (for addition) .... (01)

$P_{27\,total} = 1.82 \times 10^5$ Pa
$(1.81 - 1.82) \times 10^5$ .... (01)

(v) Applying $PV = nRT$ for petrol vapour .... (01)

$n_{vapour} = \frac{7.47\times10^4\times12.5}{8.3\times(273+27)}$ (for substitution) .... (01)

$n_{vapour} = 375$ mol .... (01)

[illegible]

[illegible] is an [illegible] used to [illegible] radiation exposure. It can be used to measure the [illegible] of radiation [illegible] to the human body which is an essential measure for the safety. There are two [illegible] active and passive. The active dosimeter can be used to obtain a real-time exposure [illegible] dosimeter measures the amount of radiation a person absorbs over a set period of time. The [illegible] passive dosimeter is Thermoluminescent dosimeter (TLD).

When a thermoluminescent crystal is exposed to ionizing radiation, it absorbs and traps the energy of the radiation in its crystal lattice. When heated, the crystal releases the trapped energy in the form of visible light. The intensity of light is proportional to the intensity of the ionizing radiation the crystal was exposed to. The emitted light is allowed to incident on a photo sensitive surface and thereby produce a weak current. Finally the current is amplified and measured.

A Geiger-Müller counter can be used to detect ionizing radiation. Absorber plates made of different materials and various thicknesses can be used to determine the type of radiation incident on a GM counter.

(a) Write down three types of radiations that can ionize air.

(b) Write down an advantage of an active dosimeter over a passive dosimeter.

(c) The activity of a radioactive material of half-life 1 hour is measured by a Geiger-Müller counter. If the initial count rate is 64 counts per second, calculate the count rate after 3 hours.

(d) How is it possible to detect the type of ionizing radiation incident on a Geiger-Müller counter using different absorber plates?

(e) A TLD dosimeter emits blue light of wavelength 400 nm with an intensity of 198 nW. Assume that the emitted light is incident normally on a photo sensitive surface made of cesium with a work function of 2.0 eV. (Planck constant = 6.6×10^{-34} J s, speed of light = 3.0×10^{8} m s^{-1}, electron charge = 1.6×10^{-19} C, 1 eV = 1.6×10^{-19} J)

(i) Determine the number of photons of blue light incident on the photo sensitive surface per second.

(ii) If 10 electrons are ejected for each 100 photons incident on the photo sensitive surface, determine the current produced by the photo sensitive surface.

(iii) Calculate the maximum kinetic energy (in J) of the ejected photoelectrons from the photo sensitive surface.

(f) A CT scanner takes a series of X-ray images from different angles around a human body. The CT scanner in a medical laboratory operates full-time for a research purpose. A TLD dosimeter placed near the CT scanner has recorded radiation dose of 250 mSv/year.

(i) A radiation scientist in the operator room of the CT scanner can be exposed to 10% of radiation during the operation. Calculate the maximum dose in mSv/year that the scientist could be exposed to.

(ii) The occupational dose limit for a radiation worker is 20 mSv/year. If the scientist works 6 hours a day for 146 days in a year prove that the radiation exposure he receives does not exceed the occupation dose limit.

(iii) If the mass of the scientist is 75 kg, how much radiation energy (in J) does he expose in a year?

[For X-rays, dose in Sv = dose in Gy, 1 Gy = 1 J kg^{-1}]

(*a*) Alpha/α, Beta/β and Gamma/γ radiations, X ray ....(02)

(Any three of above; one mark for two correct answers)

(*b*) Active dosimeter can be used to get a (real-time) exposure value

OR

Active dosimeters measure radiation levels in (real time) .... (02)

(*c*)

$$\frac{A}{A_0} = \frac{1}{2^n}$$ .... (02)

$$\frac{A}{64} = \frac{1}{2^3}$$ .... (01)

$$A = 8$$ .... (01)

(award full marks for the correct answer)

(*d*) Due to different penetration ability/power of radiation

OR

Depending on the type of radiation absorber plates can be used to stop/absorb radiation

.... (02)

(*e*)

(i). If *n* is the number of photons incident per second

$$\frac{n \times 6.6 \times 10^{-34} \times 3 \times 10^{8}}{400 \times 10^{-9}} = 198 \times 10^{-9}$$ .... (02)

(01 mark for L.H.S.; 01 mark for equating)

$n = 4 \times 10^{11}$ photons/ second

.... (02)

(ii). No of electrons emitted per sec. $= \frac{10}{100} \times 4 \times 10^{11}$ .... (01)

(for taking 10%)

$= 4 \times 10^{10}$ electrons/ seconds

~~(award full marks for the correct answer)~~

Current produced $I = 4 \times 10^{10} \times 1.6 \times 10^{-19}$ ....(01)

(for substitution)

$I = 6.4 \times 10^{-9} A$ ....(01)

(award full marks for the correct answer)

(iii) Applying $K_{max} = hf - \phi \left(OR\ \frac{hc}{\lambda} - \phi \right)$ .... (01)

$$K_{max} = \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{400 \times 10^{-9}} - 2.0 \times 1.6 \times 10^{-19}$$ .... (02)

(01 mark for the substitution in the first term; 01 mark for the second term)

$= 1.75 \times 10^{-19}$ J .... (01)

(*f*)

(i). Maximum dose $= 250 \times \frac{10}{100}$ ....(01)

(for taking 10%)

$= 25$ mSv/year .... (01)

(ii).

Radiation exposure $= 25 \times \frac{146}{365} \times \frac{6}{24}$ .... (03)

(01 mark for $\frac{146}{365}$ fraction; 01 mark for $\frac{6}{24}$ fraction; 01 mark for the multiplication)

$= 2.5$ mSv/year .... (01)

This value is less than 20 mSv/year .... (01)

(iii).

Radiation energy exposed by the scientist

$= 75 \times 2.5 \times 10^{-3}$ (for multiplication) .... (01)

$= 0.1875$ J .... (01)

$(1.87 - 1.88) \times 10^{-1}$ J