11 ප්‍රශ්නය සිරස සමඟ සාදන $\angle \theta$ නම්

$$\tan\theta = \frac{\sqrt{4a^2-b^2}}{3b}$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{4a^2-b^2}}{3b}\right) \quad \text{—— } 05$$

$$S = \frac{T\sin\alpha}{\sin\theta}$$

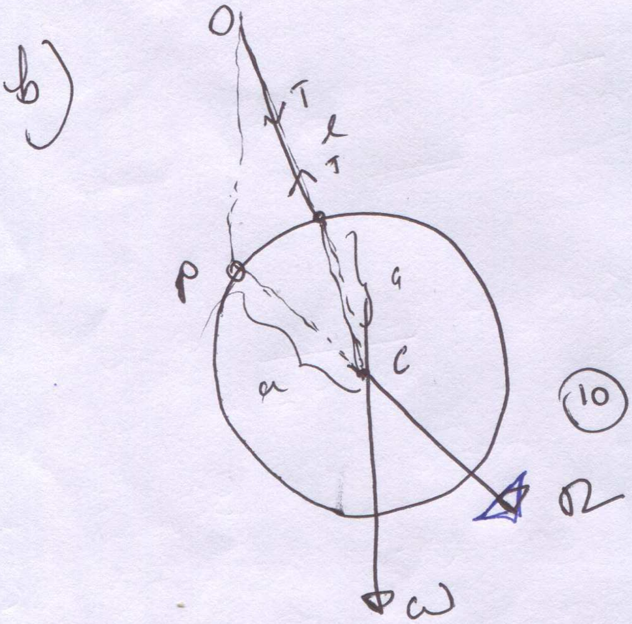
$$= T\,\frac{\cancel{\sqrt{4a^2-b^2}}}{2a} \times \frac{\cancel{\sqrt{4a^2-b^2}}}{\sqrt{a^2+2b^2}}$$

$$= \frac{aw}{b} \times \frac{\sqrt{a^2+2b^2}}{a}$$

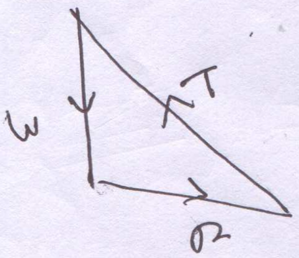
$$= \frac{\sqrt{a^2+2b^2}\cdot w}{b} \quad \text{—— } 10$$

$\boxed{75}$

b)



10



15

වස්තුව මත 3 බලයක් සමතුලිත නිසා,

$$\frac{T}{CO} = \frac{R}{PC} = \frac{w}{OP} \quad \text{—— } 15$$

$$\frac{T}{a+l} = \frac{R}{a} = \frac{w}{OP} \quad \text{—— } 10$$

$$T = w\left(\frac{a+l}{OP}\right) \quad \text{—— } 05$$

$$R = \frac{aw}{OP} \quad \text{—— } 05$$

$\boxed{60}$

17

(03) $\dfrac{4}{(x^2-x-2)(x+1)} = \dfrac{4}{(x-2)(x+1)(x+1)}$

$$\equiv \frac{4}{(x-2)(x+1)^2}$$

$$\equiv \frac{A}{(x-2)} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2} \quad 05$$

$$\equiv \frac{A(x+1)^2 + B(x+1)(x-2) + C(x-2)}{(x-2)(x+1)^2}$$

x^2 සංගුණක සමාන කි. $\quad 0 = A + B$ —— (1)

x සං. ස. කි. $\quad 0 = 2A - B + C$ —— (2) $\quad$ 05

නියත ස. කි. $\quad 4 = A - 2B - 2C$ —— (3)

(1) + (2) $\quad 0 = 3A + C$ —— (4)

(2) × 2 − (3) $\quad -4 = 3A + 4C$ —— (5) $\quad$ 05

(4) − (5) $\quad 4 = -3C$

$C = -4/3$ $\quad$ 05 $\qquad A = -C/3 = 4/9 \qquad B = -4/9$

$$\frac{4}{(x^2-x-2)(x+1)} = \frac{4}{9(x-2)} - \frac{4}{9(x+1)} - \frac{4}{3(x+1)^2} \quad 05$$

---

(4) $\dfrac{b+c}{5} = \dfrac{c+a}{6} = \dfrac{a+b}{7} = k$

$b + c = 5k$ —— (1) $\quad$ 05

$c + a = 6k$ —— (2) $\quad$ 05

$a + b = 7k$ —— (3) $\quad$ 05

$a - b = k$

$a + b = 7k$

$2a = 8k$

$a = 4k$

$b = 3k$

$c = 2k$

$\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$ $\quad$ 05

$\dfrac{\sin A}{4k} = \dfrac{\sin B}{3k} = \dfrac{\sin C}{2k}$ $\quad$ 05

$\dfrac{\sin A}{4} = \dfrac{\sin B}{3} = \dfrac{\sin C}{2}$

A කොටස   සංයුක්ත ගණිතය I   1

12 ශ්‍රේණිය

(01) $x^2+1 = a(x-1)^2$

$x^2+1 = a(x^2-2x+1)$

$(a-1)x^2 - 2ax + a - 1 = 0$ 05

x තාත්වික වීමට $\Delta x \geq 0$ විය යුතුය.   05

$b^2 - 4ac \geq 0$

$4a^2 - 4(a-1)(a-1) \geq 0$ 05

$a^2 - (a-1)^2 \geq 0$

$[a-(a-1)][a+a-1] \geq 0$ 05

$(2a-1) \geq 0$

$a \geq 1/2$ $\frac{1}{2} \leq a < \infty$ 05

---

(02) $\sin\theta + \sin 2\theta + \sin 3\theta + \sin 4\theta + \sin 5\theta = 0$

$(\sin\theta + \sin 5\theta) + (\sin 2\theta + \sin 4\theta) + \sin 3\theta = 0$

$2\sin 3\theta \cdot \cos 2\theta + 2\sin 3\theta \cdot \cos\theta + \sin 3\theta = 0$ 05

$\sin 3\theta \left(2\cos 2\theta + 2\cos\theta + 1\right) = 0$

$\sin 3\theta \left[4\cos^2\theta - 2 + 2\cos\theta + 1\right] = 0$

$\sin 3\theta \left(4\cos^2\theta + 2\cos\theta - 1\right) = 0$ 05

$\sin 3\theta = 0$ හෝ $4\cos^2\theta + 2\cos\theta - 1 = 0$

$3\theta = n\pi + (-1)^n 0$ $\cos\theta = \frac{-2 \pm 2\sqrt{5}}{2\times 4}$

$\theta = \frac{n\pi}{3}$; $n \in Z^+$ 05

$\cos\theta = \frac{-1-\sqrt{5}}{4}$ $\cos\theta = \frac{-1+\sqrt{5}}{4}$

$\theta = 2n\pi \pm \alpha$ 05 $\theta = 2n\pi \pm$ [illegible]

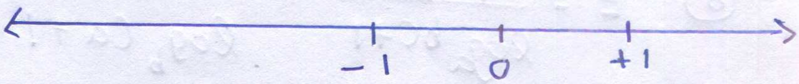
($\cos\theta = \cos\alpha$ වේ)   ($\cos\theta = \cos$[illegible] වේ)

$n \in Z$ 05

(5) $x - \frac{1}{x} \leq 0$

$\frac{x^2-1}{x} \leq 0$

$\frac{(x+1)(x-1)}{x} \leq 0$ 05



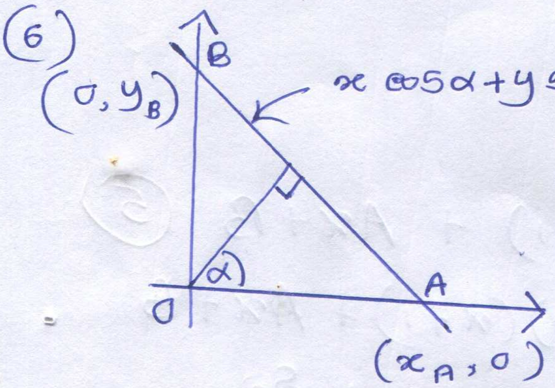
	$x<-1$	$-1<x<0$	$0<x<1$	$x>1$
$x+1$	$-$	$+$	$+$	$+$
$x-1$	$-$	$-$	$-$	$+$
x	$-$	$-$	$+$	$+$
$\frac{(x+1)(x-1)}{x}$	$-$	$+$	$-$	$+$
	05		05	

05 05

$\left\{\forall x \in \mathbb{R};\ (-\infty, -1] \cup (0, 1] : x - \frac{1}{x} \leq 0\right\}$

---

(6)



$x=0 \ ; \ y_B = \frac{p}{\sin\alpha}$

$y=0 \ ; \ x_A = \frac{p}{\cos\alpha}$ 05

$OAB\ \Delta = 2p^2$

$\frac{1}{2} x_A \cdot y_B = 2p^2$

$\frac{1}{2} \frac{p}{\cos\alpha} \cdot \frac{p}{\sin\alpha} = 2p^2$ 05

$\sin 2\alpha = \frac{1}{2} = \sin\frac{\pi}{6}$ 05

$2\alpha = n\pi + (-1)^n \pi/6$

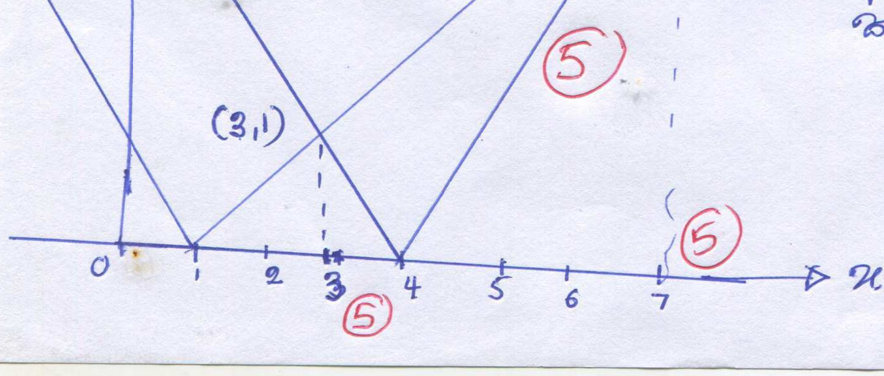
$\alpha = \frac{n\pi}{2} + (-1)^n \frac{\pi}{12} \ ; \ n \in \mathbb{Z}$ 5

$\alpha = \left\{\frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}\right\}$

05

(7) $y = \frac{1}{2}|x-1| \qquad y = |x-4|$

$y = \begin{cases} \frac{1}{2}(x-1) & x \geq 1 \\ -\frac{1}{2}(x-1) & x<1 \end{cases} \qquad y = \begin{cases} x-4 & x \geq 4 \\ -(x-4) & x<4 \end{cases}$



අසමානතාව තෘප්ත කරන x හි අගය පරාසය 5

$\{x \in \mathbb{R} \,/\, 3 < x < 7$

8. $E = \dfrac{1}{\log_a bc + 1} + \dfrac{1}{\log_b ca + 1} + \dfrac{1}{\log_c ab + 1}$

$= \dfrac{1}{\log_a bc + \log_a a} + \dfrac{1}{\log_b ca + \log_b b} + \dfrac{1}{\log_c ab + \log_c c}$ 5

$= \dfrac{1}{\log_a abc} + \dfrac{1}{\log_b abc} + \dfrac{1}{\log_c abc}$ 5

$= \cancel{\dfrac{1}{\log_{abc} a}} \log_{abc} a + \log_{abc} b + \log_{abc} c$ 5

$= \log_{abc} (abc)$ 5

$= \underline{\underline{1}}$ 5

9. $x^n + 2 = \phi(x)(x^2 - 1) + Ax + B$ 5

$x^n + 2 = \phi(x)(x-1)(x+1) + Ax + B$

$x = 1$ විට, $3 = A + B$ — ① 5 $x = -1$ විට $1 = B - A$ — ②

① + ② $\Longrightarrow$ $4 = 2B$

$\underline{\underline{B = 2}}$ 5

① − ② $\Longrightarrow$ $2 = 2A$

$\underline{\underline{A = 1}}$ 5

$Ax + B = \underline{\underline{x + 2}}$ 5

(10)

$$a(b\cos C - c\cos B) = ab\cos C - ac\cos B$$

$$= \cancel{ab}\left[\frac{a^2+b^2-c^2}{2\cancel{ab}}\right] - \cancel{ac}\left[\frac{a^2+c^2-b}{2\cancel{ac}}\right.$$ 5 5

$$= \frac{b^2+a^2-c^2-a^2-c^2+b^2}{2}$$ 5

$$= b^2-c^2$$ 5

$$b^2-c^2 = \frac{a^2\sin(B-C)}{\sin(B+C)}$$ 5

$$a(b\cos C - c\cos B) = \frac{a^2\sin(B-C)}{\sin(B+C)}$$

11 (a) $f(x) = 1 + x + ax^2 + bx^3 + cx^4 + \cdots$

$f(y) = 1 + y + ay^2 + by^3 + cy^4 + \cdots$ 5

$f(x) \cdot f(y) = f(x+y)$

$= (1 + x + ax^2 + bx^3 + cx^4 + \cdots)(1 + y + ay^2 + by^3 + cy^4 \cdots)$

$= 1 + (x+y) + a(x+y)^2 + b(x+y)^3 + c(x+y)^4 \cdots$ 5

$= 1 + x + y + a(x^2 + 2xy + y^2) + b(x^3 + 3x^2y + 3xy^2 + y^3)$ 5 5

$+ c(x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4) + \cdots$ 5

25

xy සංගුණක සැසඳීමෙන්,

$1 = 2a$

$\therefore a = \frac{1}{2}$ 10

x^2y හි සංගුණක සැසඳීමෙන්,

$a = 3b$

$\frac{1}{2} = 3b$

$b = \frac{1}{6}$ 10

x^3y හි සංගුණක සැසඳීමෙන්,

$b = 4c$

$\frac{1}{6} = 4c$

$c = \frac{1}{24}$ 10

30

වසම $= [-1, 1]$ 5

පරාසය $= [0, 1]$

වසම $= (-\infty, +\infty)$

පරාසය $= (-\infty, 9]$

III). $y = x^2 - 12x + 36$

$= (x-6)^2$

වසම $= (-\infty, +\infty)$ 5

පරාසය $= [0, \infty)$

(IV) $y = x^2 - 4x + 1$

$= (x-2)^2 - 3$

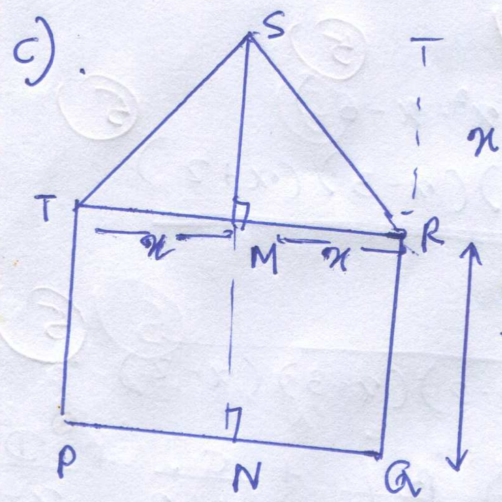
වසම $(-\infty, +\infty)$

පරාසය $= [-3, +\infty$

5

20

c).



$2(NQ + QR + RS) = 30$ 5

$MR + QR + SM = 15$

$x + y + \sqrt{MR^2 + SM^2} = 15$

$x + y + \sqrt{(\frac{3}{4}x)^2 + x^2} = 15$

$x + y + \frac{5}{4}x = 15$ 5

20

$\underline{y = 15 - \frac{9}{4}x}$ 5

$A = \cancel{2}\frac{(QR + SN)}{\cancel{2}} NQ$ 5

$= \left(y + y + \frac{3x}{4}\right)x$ 5

$= \left\{2\left(15 - \frac{9}{4}x\right) + \frac{3}{4}x\right\}x$ 5

$= \left(30 - \frac{9}{2}x + \frac{3}{4}x\right)x$

20

$= 30x - \frac{15x^2}{4}$ 5

$A = -\frac{15}{4}(x^2 - 8x)$

5

$= \frac{16 \times 15}{4} - \frac{15}{4}(x^2 - 8$

$= 60 - \frac{15}{4}(x-4)^2$ 5

A උපරිම $= 60 - \frac{15}{4}(x-4$

5

$= 60 - 0 \; ; \; x = 4$

A උපරිම විට SN 5

$S = y + \frac{3}{4}x$ 3

$= 15 - \frac{9}{4}x + \frac{3}{4}x$

$= 15 - \frac{3}{2}x$ 3

10

$= 15 - \frac{3}{2} \times 4$

$= \underline{9cm}$ 5

$\underline{A = 60cm^2}$ 5

25

$\underline{x = 4c}$

5

(8)

(12) $g(x) = (x^2+px-6)(x^2+Qx-6) \equiv x^4-13x^2+36$

x^2 හි සං: $PQ-12 = -13$

$PQ = -1$ — (1) (5)

x හි සං: $-6(P+Q) = 0$

$P+Q = 0$ — (2) (5)

$Q = -P$

(1) හා (2) න් $P(-P) = -1$

$P^2 = 1$ (5)

$\underline{P = \pm 1}$

$P = +1$ විට $Q = -1$ (5)

$P = -1$ විට $Q = 1$ (5)

$\therefore g(x) = (x^2+x-6)(x^2-x-6)$ (5)

$= \underline{(x+3)(x-2)(x-3)(x+2)}$ 30

$$\frac{1}{x^4-13x^2+36} = \frac{1}{(x+3)(x-2)(x-3)(x-2)}$$ (5)

$$= \frac{A}{x+3} + \frac{B}{x-3} + \frac{C}{x+2} + \frac{D}{x-2}$$ (5)

$1 = A(x-3)(x^2-4) + B(x+3)(x^2-4) + C(x-2)(x^2-9) + D(x+2)(x^2-9)$ (5)

15

x^3 සංගුණක $A+B+C+D = 0$ — (1)

x^2 " $-3A+3B-2C+2D = 0$ — (2)

$-3(A-B)-2(C-D) = 0$

x සංගුණක $-4(A+B)-9(C+D) = 0$ — (3)

x^0 සංගුණක $12(A-B)+18(C-D) = 1$ — (4)

(2) $\times 4 +$ (4) $10(C-D) = 1$

$C-D = 1/10$ — (5)

සමීකරණ (4) වල

(5)

(9)

$① \Rightarrow ③ \quad -4(-C-D) - 9(C+D) = 0$

$C + D = 0 \quad \text{—} ⑥$

$⑤ + ⑥ \quad 2C = \frac{1}{10} \qquad D = -C = -\frac{1}{20}$ 5

$C = \frac{1}{20}$ 5

$A + B = 0 \quad \text{—} ⑦$

$12(A-B) + \frac{18}{10} = 1$

$12(A-B) = 1 - 9/5$

$A - B = -\frac{1}{15} \quad \text{—} ⑧$

$⑦ + ⑧ \quad 2A = -\frac{1}{15} \qquad B = \frac{1}{30}$ 5

$A = -\frac{1}{30}$ 5

$$\frac{1}{x^4 - 13x^2 + 36} = \frac{-1}{30(x+3)} + \frac{1}{30(x-3)} + \frac{1}{20(x+2)} - \frac{1}{20(x}$$ 5

3

(II) සාධක ප්‍රමේයයේ විලෝමය.

$(x-a)$ යන්න ; $f(x)$ බහුපදයේ සාධකයක් නම් පමණි $f(a) = 0$ වේ. 5

සාධනය :- $x-a$; $f(x)$ බහුපදයේ සාධකයක් නිසා, $f(x) = (x-a)\phi(x)$ වන සේ $\phi(x)$ බහුපදයක් පවතී. 5

$x = a$ ආදේශ කළ විට

$f(x) = (a-a)\phi(a)$

$= 0$ 5

15

$f(x) = x^3 - x^2 + ax + b$;

$f(-1) = 0$ 5 $\therefore (x+1)$; $f(x)$ හි සාධකයක් බැවින් ;

$-1 - 1a + b = 0$

$b - a = 2$ —① 5

$x-1$ න් $f(x)$ බෙදූ විට ශේෂය $f(1) = -8$ 5

$1 - 1 + a \cdot 1 + b = -8$ 5

$a + b = -8$ —② 5

①+② $2b = -6$

$\underline{\underline{b = -3}}$ 5 එවිට 5

$\underline{\underline{a = -5}}$ 5 $\underline{\underline{f(x) = x^3 - x^2 - 5x - 3}}$

$\boxed{40}$

$f(x) = 0$ විට

$(x+1)^2(x-3) = 0$

$(x+1)^2 = 0$ හෝ $x - 3 = 0$

$x = -1$ හෝ $x = 3$ 10

$\therefore \underline{\underline{x = \{-1, 3\}}}$

$f(x) = x^3 - x^2 - 5x - 3$

$= (x+1)(x^2 - 2x - 3)$ 5

$= (x+1)(x+1)(x-3)$

$\underline{\underline{= (x+1)^2(x-3)}}$ 5

$\boxed{20}$

(13) (a) (i) $ax^2 + bx + c = 0$ හි මූල α හා β නම්,

$\alpha + \beta = -\frac{b}{a}$ 5, $\alpha\beta = \frac{c}{a}$ වේ 5

$$\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3 + \beta^3}{\alpha\beta} \;\; 5 = \frac{(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)}{\alpha\beta} \;\; 5$$

$$= \frac{-\frac{b^3}{a^3} - \frac{3c}{a}\left(-\frac{b}{a}\right)}{\frac{c}{a}} \;\; 5$$

$$= \frac{(-b^3 + 3abc)}{a^{\cancel{3}\,2}} \times \frac{\cancel{a}}{c}$$

$\boxed{30}$

$$\underline{\underline{= \frac{3abc - b^3}{a^2 c}}} \;\; 5$$

II). $\frac{\alpha^2}{\beta}$ හා, β^2/α මූල වන x හි වර්ගජ සමීකරණය

$(x-\alpha^2/\beta)(x-\beta^2/\alpha) = 0$ 5

$x^2 - \left(\alpha^2/\beta + \beta^2/\alpha\right)x + \alpha\beta = 0$ 5

$x^2 - \left(\frac{3abc-b^3}{a^2c}\right)x + c/a = 0$ 5

$a^2c\,x^2 - (3abc-b^3)x + ac^2 = 0$ 5 $\boxed{20}$

III). $\lambda = \frac{\alpha^2}{\beta}+1$ සහ $\mu = \frac{\beta^2}{\alpha}+1$ යැයි ගනිමු.

$y = x+1$ — ② 10 ලෙස සලකමු.

$x = \frac{\alpha^2}{\beta}$ විට $y = \frac{\alpha^2}{\beta}+1 = \lambda$ ද 5

$x = \beta^2/\alpha$ විට $y = \frac{\beta^2}{\alpha}+1 = \mu$ ද ලැබේ. 5

② $\Rightarrow$ $x = y-1$ — ① 10

① හි x මගින් ද $\frac{\alpha^2}{\beta}$ හා β^2/α නිරූපණයවේ.

$a^2c(y-1)^2 - (3abc-b^3)(y-1) + ac^2 = 0$ 5

$a^2cy^2 - (3abc-b^3+2a^2c)y + 2a^2c + 3abc - b^3 = 0$

සාධාරණ ලෙස, 5

$a^2c\,x^2 - (3abc-b^3+2a^2c)x + 2a^2c + 3abc - b^3 = 0$

$\boxed{40}$

(b) $f(x)$ බහුපදය $(x-a)(x-b)$ න් බෙදූවිට ලබ්ධිය $g(x)$ යැයිද, ශේෂය $px+q$ ලෙසද ගනිමු.

$$f(x) = (x-a)(x-b)\,g(x) + px + q \quad 5$$

$x=a$ ආදේශයෙන්,

$$f(a) = pa + q \quad \text{—}① \quad 5$$

$x=b$ ආදේශයෙන්, $f(b) = pb + q$ —② 5

① − ② $\quad f(a) - f(b) = p(a-b)$

$$p = \frac{f(a)-f(b)}{(a-b)} \quad 5$$

①×b − ②×a

$$\boxed{30}$$

$$b\,f(a) - a\,f(b) = q(b-a)$$

$$q = \frac{a f(b) - b f(a)}{(a-b)} \quad 5$$

∴ ශේෂය $= px + q$ 5

$$= \left\{\frac{f(a)-f(b)}{a-b}\right\}x + \left\{\frac{a f(b) - b f(a)}{a-b}\right\}$$ වේ

$$f(x) = x^6 + px + q$$

$f(x)$ බහුපදය $(x-1)(x+2)$ න් බෙදූවිට ශේෂය $2x+1$ නිසා,

$$\boxed{30}$$

$$2 = \frac{f(1) - f(-2)}{1-(-2)} \quad 5$$ සහ $$1 = \frac{1\cdot f(-2) + f(1)\times 2}{1-(-2)} \quad 5$$

$6 = 1^6 + p + q - [(-2)^6 - 2p + q]$ 5 $\qquad 3 = (-2)^6 - 2p + q + [1+p+q]$ 5

$6 = 1 + p + \cancel{q} - 64 + 2p - \cancel{q}$ $\qquad 3 = 64 + 3q + 2$ 5

$\frac{69}{3} = p$

$\underline{p = 23}$ 5 $\qquad \underline{q = -21}$ 5

(14) (a) $\tan(\theta - \pi/4)$

$$= \frac{\tan\theta - \tan\pi/4}{1 + \tan\theta\tan\pi/4} = \frac{\tan\theta - 1}{1 - \tan\theta}$$ 5 5

$$= \frac{\frac{\sin\theta}{\cos\theta} - 1}{1 + \frac{\sin\theta}{\cos\theta}} = \frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta}$$ 5 5

$\boxed{20}$

$$\tan(\pi/3 - \pi/4) = \tan\pi/12$$ 5

$$\tan\pi/12 = \frac{\tan\pi/3 - \tan\pi/4}{1 + \tan\pi/3\tan\pi/4} = \frac{\sqrt{3}-1}{1+\sqrt{3}}$$ 5 5

$$= \frac{(\sqrt{3}-1)(-\sqrt{3}+1)}{(\sqrt{3}+1)(-\sqrt{3}+1)} = \frac{2\sqrt{3}-4}{1-3}$$ 5

$$= \frac{-\cancel{2}(2-\sqrt{3})}{-\cancel{2}} = 2-\sqrt{3}$$ 5

$\boxed{25}$

$\tan\alpha = 2+\sqrt{3}$ යැයි ගනිමු.

$$= 2+\sqrt{3} \times \frac{(2-\sqrt{3})}{(2-\sqrt{3})}$$ 5

$$= \frac{4-3}{2-\sqrt{3}} = \frac{1}{2-\sqrt{3}}$$

$$= \frac{1}{\tan 15^\circ}$$

$$= \cot 15^\circ$$ 5

$$= \cot(90-75)$$

$$\tan\alpha = \tan 75^\circ$$

$$\alpha = 75$$ 5

$\boxed{15}$

(b). $\sin 2A + \sin 2B + \sin 2C$ 5

$= 2\sin A\cos A + 2\sin(B+C)\cos(B-C)$

$= 2\sin A\cos A + 2\sin(\pi - A)\cos(B-C)$ 5

$= 2\sin A\cos A + 2\sin A\cos(B-C)$

$= 2\sin A\left[\cos A + \cos(B-C)\right]$ 5

$= 2\sin A\left(\cos(\pi-(B+C)) + \cos(B-C)\right]$

$= 2\sin A\left[\cos(B-C) - \cos(B+C)\right]$

$= 2\sin A \times - 2\sin B\sin(-C)$ 5

$= \underline{\underline{4\sin A\sin B\sin C}}$ 5

$\boxed{25}$

(c) $4\cos x\cos\left(\pi/3 - x\right)\cdot\cos\left(\pi/3 + x\right)$

$= 2\cos x\cdot 2\cos\left(\pi/3 + x\right)\cos\left(\pi/3 - x\right)$

$= 2\cos x\cdot\left[\cos 2\pi/3 + \cos(-2x)\right]$ 5

$= 2\cos x\left[-1/2 + \cos 2x\right]$ 5

$= -\cos x + 2\cos x\cos 2x$ 5

$= -\cos x + 2\cos x\left(2\cos^2 x - 1\right)$

$= -\cos x + 4\cos^3 x - 2\cos x$ 5

$= 4\cos^3 x - 3\cos x$ 5

$= \underline{\underline{\cos 3x}}$

$\boxed{25}$

$8\underbrace{\cos x\cdot\cos\left(\pi/3 - x\right)\cdot\cos\left(\pi/3 + x\right)} = 1$

$2\cos 3x = 1$ 5

$\cos 3x = 1/2$

$\cos 3x = \cos \pi/3$ 5

$3x = 2n\pi \pm \pi/3 \quad n\in\mathbb{Z}$

$x = \frac{2}{3}n\pi \pm \pi/9$ 5 $n\in\mathbb{Z}$

$\boxed{15}$

d) $\underbrace{\tan^{-1}\left(\frac{x-1}{x-2}\right)}_{A} + \underbrace{\tan^{-1}\left(\frac{x+1}{x+2}\right)}_{B} = \pi/4$

$$A + B = \pi/4 \quad 5$$

$$\tan(A+B) = \tan \pi/4$$

$$= \tan(A+B)$$

$$= \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B} = \frac{\frac{x-1}{x-2} + \frac{x+1}{x+2}}{1 - \frac{x-1}{x-2} \times \frac{x+1}{x+2}} \quad 5 \quad 5$$

$$1 = \frac{(x-1)(x+2) + (x+1)(x-2)}{(x^2-4) - (x^2-1)}$$

$$1 = \frac{x^2 + \cancel{2x} - \cancel{x} - 2 + x^2 - \cancel{2x} + \cancel{x} - 2}{\cancel{x^2} - 4 - \cancel{x^2} + 1} \quad 5$$

$$-3 = 2x^2 - 4$$

$$2x^2 - 1 = 0$$

$$2x^2 = 1$$

$$x^2 = 1/2$$

$$x = \pm \frac{1}{\sqrt{2}} \quad 5$$

$\boxed{25}$

(15) i) $\dfrac{\log_b y^m}{\log_b b^n} = \log_{b^n} y^m$ 5 5

$\log_{b^n} y^m = \dfrac{m \log_b y}{n \underbrace{\log_b b}_{1}}$ 5

$\log_{b^n} y^m = \dfrac{m}{n} \log_b y$ 5

$\boxed{20}$

ඉහත ප්‍රතිඵලයෙන්,

ii) $\log_{b^1} y^1 + \log_{b^2} y^2 + \log_{b^3} y^3 + \ldots\ldots + \log_{b^{2020}} y^{2020}$

$= \frac{1}{1}\log_b y + \frac{2}{2}\log_b y + \frac{3}{3}\log_b y + \ldots + \frac{2020}{2020}\log_b y$ 10

$= \log_b y + \log_b y + \log_b y + \ldots\ldots + \log_b y$ 5

$= 2020 \log_b y$ 5

$= \log_b y^{2020}$ 5

$\boxed{25}$

15 17

II) $\dfrac{2x}{x+3} \le \dfrac{1}{x+1}$

$$\frac{2x}{x+3} - \frac{1}{x+1} \le 0 \quad 5$$

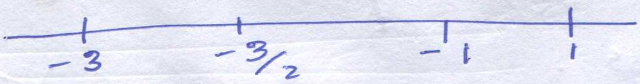
$$\frac{2x(x+1) - (x+3)}{(x+3)(x+1)} \le 0 \quad 5$$

$$\frac{2x^2 + 2x - x - 3}{(x+3)(x+1)} \le 0 \quad 5$$

$$\frac{2x^2 + x - 3}{(x+3)(x+1)} \le 0 \quad 5$$

$$\frac{(2x+3)(x-1)}{(x+3)(x+1)} \le 0 \quad 5$$

$\boxed{25}$



	$2x+3$	$x-1$	$x+3$	$x+1$	$\frac{(2x+3)(x-1)}{(x+3)(x+1)}$	
$x<-3$	−	−	−	−	+	5
$-3<x<-\frac{3}{2}$	−	−	+	−	−	5
$-\frac{3}{2}<x<-1$	+	−	+	−	+	5
$-1<x<1$	+	−	+	+	−	5
$x>1$	+	+	+	+	+	5

$\boxed{25}$

$$\left\{ \forall x \in \mathbb{R} \; ; \; (-3, -\tfrac{3}{2}] \cup (-1, 1] \right\}$$
5 5

හෝ

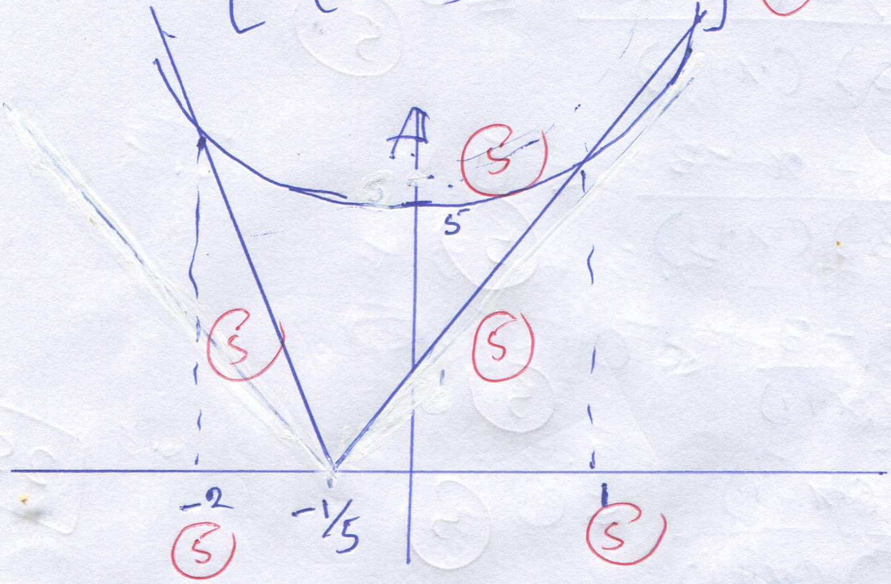
$\forall x \in \mathbb{R} \; ; \; -3 < x \le -\frac{3}{2} \cup -1 < x \le 1$ $\boxed{10}$

18

(III). $|5x+1| < x^2+5$

$y_1 = |5x+1|$, $y_2 = x^2+5$

$$y_1 = \begin{cases} 5x+1 & x \geq -\frac{1}{5} \\ -(5x+1) & x < -\frac{1}{5} \end{cases}$$ 5 5



$|5x+1| < x^2+5$ වන අගය පරාසය වන්නේ

$-2 < x < 1$ වේ.

10

$\boxed{45}$

16) $\dfrac{x^n-a^n}{x-a} = x^{n-1}+ax^{n-2}+a^2x^{n-3}+\dots+a^{n-1}$ 5

$$x-a \,\big)\, \overline{x^n-a^n} \quad x^{n-1}+ax^{n-2}+a^2x^{n-3}+\dots+a^n$$

$$\lim_{x\to a}\frac{x^n-a^n}{x-a} = \lim_{x\to a} x^{n-1} + a\lim_{x\to a} x^{n-2} + a^2\lim_{x\to a} x^{n-3} + \dots + a^n$$ 5 ~~15~~

$$= a^{n-1} + a.a^{n-2} + a^2.a^{n-3} + \dots + a^{n-1}$$ 5

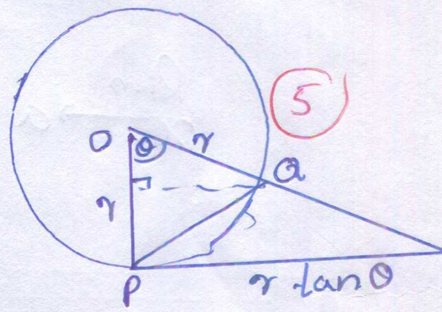
$$= a^{n-1} + a^{n-1} + a^{n-1} + \dots + a^{n-1}$$

$$= \underline{na^{n-1}}$$ 5

$\boxed{~~15~~}$ $\boxed{20}$

OPQ Δ ව.ඵ < OPQ කේන්ද්‍රික ඛ.ව.ඵ 5

< OPR Δ ව.ඵ 5



$\frac{1}{2}$ OP. LQ $< \frac{1}{2}r^2\theta < \frac{1}{2}$OP. PR

$$r.r\sin\theta < r^2\theta < r.r\tan\theta$$ 5

$$\sin\theta < \theta < \tan\theta$$

$\boxed{20}$

$\div\sin\theta$ $\quad 1 < \dfrac{\theta}{\sin\theta} < \dfrac{1}{\cos\theta}$

$$1 > \frac{\sin\theta}{\theta} > \cos\theta$$ 5

$$1 > \lim_{\theta\to 0}\frac{\sin\theta}{\theta} > \lim_{\theta\to 0}\cos\theta$$

$$1 > \lim_{\theta\to 0}\frac{\sin\theta}{\theta} > 1$$

$\therefore \lim_{\theta\to 0}\dfrac{\sin\theta}{\theta} \to \underline{=1}$ 5

$\boxed{10}$

(i) $\displaystyle\lim_{x\to a}\frac{\sin(x^5-a^5)}{x-a} = 80$

$$\lim_{x\to a}\frac{\sin(x^5-a^5)}{x^5-a^5} \times \lim_{x\to a}\frac{x^5-a^5}{x-a} = 80$$

$\underset{x\to a}{x^5-a^5} = \underset{\theta\to 0}{\theta}$ ගනිමු. 5

16) III) $\left[\lim_{\theta \to 0} \frac{\sin\theta}{\theta}\right] 5a^4 = 80$ (5)

$1 \times 5a^4 = 80$

$a^4 = 16$ (5)

$a^4 = \pm 2^4$

$\underline{\underline{a = \pm 2}}$ (5)

$\boxed{20}$

II). $\lim_{x \to a} \frac{\sin(x^7 - a^7)}{x - a}$.

$\lim_{x \to a} \frac{\sin(x^7 - a^7)}{x^7 - a^7} \times \lim_{x \to a} \frac{x^7 - a^7}{x - a}$ (5)

$x^7 - a^7 = \theta$.

$x \to a \quad \theta \to 0$.

$\Big(\underbrace{\lim_{\theta \to 0} \frac{\sin\theta}{\theta}}_{1}\Big] \times 7a^6$ (5)

(5) $1 \times 7a^6 = \underline{\underline{7a^6}}$ (5)

$\boxed{20}$

III) $\lim_{x \to 5} \frac{\sqrt{x-1} - 2}{x^2 - 25}$

$= \lim_{x \to 5} \frac{\sqrt{x-1} - 2}{x^2 - 25} \times \frac{\sqrt{x-1} + 2}{\sqrt{x-1} + 2}$ (5)

$= \lim_{x \to 5} \frac{(x - 1 - 4)}{(x+5)(x-5)} \times \frac{1}{\sqrt{x-1} + 2}$

$= \lim_{x \to 5} \frac{\cancel{(x-5)}}{(x+5)\cancel{(x-5)}} \times \frac{1}{\sqrt{x-1} + 2}$ $(x \neq 5)$ (5)

$$\lim_{x\to 5}\frac{1}{x+5}\ \lim_{x\to 5}\frac{1}{\sqrt{x-1}+2}$$

$$= \frac{1}{5+5}\cdot\frac{1}{\sqrt{5-1}+2} \quad 5$$

$$= \frac{1}{10}\times\frac{1}{4}$$

$$= \frac{1}{40} \quad 5 \qquad \boxed{20}$$

(iv) $$\lim_{\theta\to 0}\frac{1-\cos(a\sin\theta)}{\sin^2\theta} = 128 \qquad (a\in Z^+)$$

$$\lim_{\theta\to 0}\frac{2\sin^2(a\sin\theta)}{a^2\sin^2\theta}\times a^2 = 128 \quad 5$$

$$2\left[\lim_{\theta\to 0}\frac{\sin(a\sin\theta)}{a\sin\theta}\right]^2 a^2 = 128 \quad 5$$

$$2(1)^2a^2 = 128 \quad 5$$

$$2a^2 = 128$$

$$a^2 = 64$$

$a = \pm 8$ $\quad a\in Z^+$ බැවින්

$$a = 8 \quad 5 \qquad \boxed{20}$$

(v) $ax^2+bx+c=0$ හි මූල $\pi/3$ හා $\pi/4$ බැවින්

$ax^2+bx+c \equiv a(x-\pi/3)(x-\pi/4)$ 5 යැයි ගනිමු

$$\lim_{x\to\pi/3}\frac{\tan(ax^2+bx+c)}{x-\pi/3} = \pi/6$$

$$\lim_{x\to\pi/3}\frac{\tan a(x-\pi/3)(x-\pi/4)}{(x-\pi/3)} = \pi/6 \quad 5$$

$$= \lim_{x\to \pi/3} \frac{\sin a(x-\pi/3)(x-\pi/4)}{a(x-\pi/4)(x-\pi/3)} \times \frac{\cancel{a}\, a(x-\pi/4)}{\cos a(x-\pi/3)(x-\pi/4)} =$$

$$= \underbrace{\left(\lim_{x\to \pi/3} \frac{\sin a(x-\pi/3)(x-\pi/4)}{a(x-\pi/4)(x-\pi/3)}\right)}_{5} \frac{a \lim_{x\to \pi/3}(x-\pi/4)}{\lim_{x\to \pi/4} \cos a(x-\pi/3)(x-\pi/4)}$$

$$= \quad 1 \times \frac{a(\pi/3 - \pi/4)}{\cos 0} = \pi/6$$

$$a \, \frac{\cancel{\pi}}{\cancel{12}_{2}} = \frac{\cancel{\pi}}{\cancel{6}}$$

$$\underline{\underline{a = 2}} \quad 5$$

$\boxed{20}$

$\cancel{17}$

(17) a) ABC Δ ක් සඳහා සුපුරුදු අංකනයෙන් සාධනය කිරීමට ලකුණු (05) බැගින්

15

BC , CA , AB

a , b , c

$b-a = d \Longrightarrow b = a+d$ 5

$c-b = d \Longrightarrow c = a+2d$ 5

$$\cos C = \frac{a^2+b^2-c^2}{2ab}$$ 5

$$= \frac{a^2+(a+d)^2-(a+2d)^2}{2a(a+d)}$$ 5

$$= \frac{a^2+\cancel{a^2}+2ad+d^2-\cancel{a^2}-4ad-4d^2}{2a(a+d)}$$

$$= \frac{a^2-2ad-3d^2}{2a(a+d)}$$ 5

$$= \frac{a(a-3d)+d(a-3d)}{2a(a+d)}$$

$$= \frac{(a-3d)\cancel{(a+d)}}{2a\cancel{(a+d)}}$$ 5

$$\cos C = \frac{a-3d}{2a}$$ 5

$d>0$ විය යුතුය. 5

$2a\cos C = a-3d$

$$d = \frac{a-2a\cos C}{3}$$ 5

$$\frac{a-2a\cos C}{3} > 0 \quad 5$$

$$1 > 2\cos C$$

$$\frac{1}{2} > \cos C$$

$$\cos \frac{\pi}{3} > \cos C \quad 5$$

$$C > \frac{\pi}{3} \quad 5$$

$\boxed{60}$

b) $f(x) = \frac{\sin^2 2x}{16}\left(\cot^2 x - \tan^2 x\right) + \frac{3}{4}$

$$y = \frac{4\sin^2 x\cos^2 x}{16}\left(\frac{\cos^2 x}{\sin^2 x} - \frac{\sin^2 x}{\cos^2 x}\right) + \frac{3}{4} \quad 5$$

$$y = \frac{\sin^2 x\cos^2 x}{4}\left(\frac{\cos^4 x - \sin^4 x}{\sin^2 x\cos^2 x}\right) + \frac{3}{4} \quad 5$$

$$= \frac{1}{4}(\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x) + \frac{3}{4} \quad 5$$

$$y = \frac{1}{4}\cos 2x + \frac{3}{4} \quad 5$$

$$a = \frac{1}{4} \quad 5 \qquad b = \frac{3}{4} \quad 5$$

$$4\left(y - \frac{3}{4}\right) = \cos 2x$$

$$-1 \le \cos 2x \le 1 \quad 5$$

$$-1 \le 4\left(y - \frac{3}{4}\right) \le 1$$

$$-\frac{1}{4} \le \left(y - \frac{3}{4}\right) \le \frac{1}{4}$$

$$\frac{2}{4} \le y \le 1$$

$$\frac{1}{2} \le y \le 1 \quad 5$$

$y = \frac{1}{2}$

$\frac{1}{2} = \frac{1}{4}\cos 2x + \frac{3}{4}$

$-\frac{1}{4} = \frac{1}{4}\cos 2x$

$-1 = \cos 2x$

$2x = \pi$

$x = \pi/2$

$x = -\pi/2$

$y = \frac{1}{4}\cos(-\pi) + 3/4$

$y = 1/2$

$y = 1$

$1 = 1/4 \cos 2x + 3/4$

$\frac{1}{4} = \frac{1}{4}\cos 2x$

$\cos 2x = 1$

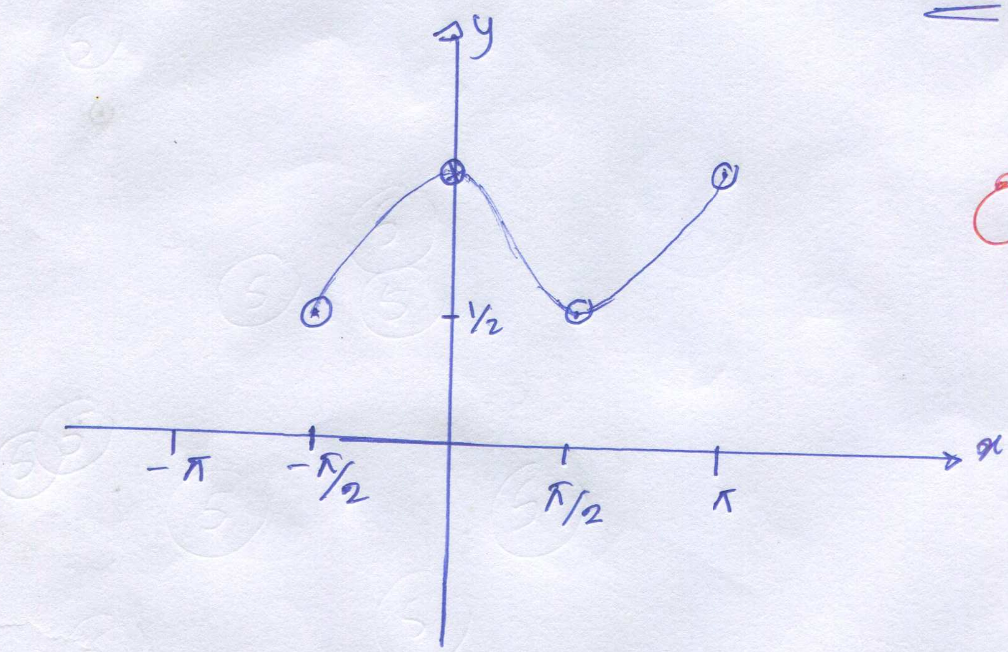
$x = 0$

$x = \pi$

$y = \frac{1}{4}\cos 2\pi) + 3/4$

$= \frac{1}{4} + 3/4$

$= \underline{1}$



උපරිම 10

අවම 10

(i) $k > 1$ හා $k < 1/2$ 5

(ii) $k = 1$ හා $k = 1/2$ 5

(iii) $1/2 < k < 1$ 5

$\boxed{75}$

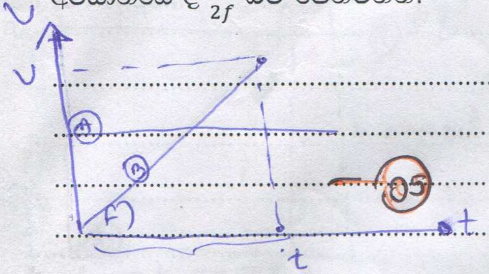
සංයුක්ත ගණිතය II 12 ශ්‍රේණිය

A කොටස 05 — $\omega b(c^2+a^2-b^2)$ විය යුතුයි.
$2abc \sin c$

A කොටස

ප්‍රශ්න සියල්ලටම පිළිතුරු සපයන්න

01) ලක්ෂ්‍යන් දෙකක් එකම සරල රේඛාවක් ඔස්සේ එහි එකම ලක්ෂයෙන් එකම මොහොතේ පිටත්ව චලනය වෙයි. පළමු වැනි ලක්ෂ්‍යය නියත u ප්‍රවේගයෙන් ද දෙවැන්න නියත f ත්වරණයෙන් ද චලනය වෙයි. දෙවැන්න පළමුවැන්න වෙතට පැමිණීමට පෙර අංශු දෙක අතර වැඩි ම දුර, ආරම්භයේ සිට $\frac{u}{f}$ කාලය අවසානයේ දී $\frac{u^2}{2f}$ බව පෙන්වන්න.



$f = v/t$

$v = ft$ — (1)

$\therefore K$ (උපරිම) $= \frac{u^2}{2f}$ 05

t කාලයේදී පරතරය K නම්

$K = ut - \frac{1}{2} \cdot ft \cdot t$ 05

$K = ut - \frac{ft^2}{2}$

$ft^2 - 2ut + 2K = 0$

t තාත්වික නිසා $\Delta \geq 0$

$4u^2 - 4f \cdot 2K \geq 0$ 05

$\frac{u^2}{2f} \geq K$

K උපරිම වූ විට

$ft^2 - 2ut + 2\frac{u^2}{2f} = 0$

$t^2 - 2\left(\frac{u}{f}\right)t + \left(\frac{u}{f}\right)^2 = 0$

$(t - u/f)^2 = 0$

$\therefore t = u/f$ 05

25

02) සුපුරුදු අංකනයෙන් $\underline{a} = 4\underline{i} + 3\underline{j}$, $\underline{b} = (1-k)\underline{i} + k\underline{j}$ වන අතර $k \in \mathbb{R}$ද $\underline{c} = 3\underline{i} + 4\underline{j}$ වේ.

i. $|\underline{a}|$, $|\underline{b}|$ හා $|\underline{c}|$ සොයන්න.

ii. k ඇසුරෙන් $\underline{a}.\underline{b}$ හා $\underline{b}.\underline{c}$ සොයන්න.

$\underline{a}$ හා $\underline{b}$ අතර කෝණය $\underline{b}$ හා $\underline{c}$ කෝණයට සමාන නම් $k = \frac{1}{2}$ බව පෙන්වන්න.

(i) $|a| = \sqrt{4^2+3^2} = 5$ $|b| = \sqrt{1-2k+2k^2}$ $|c| = \sqrt{3^2+4^2} = 5$ 05

$\underline{a}.\underline{b} = (4\underline{i} + 3\underline{j}).((1-k)\underline{i} + k\underline{j})$

$= 4(1-k) + 3k$

$= 4 - k$ 05

$\underline{b}.\underline{c} = ((1-k)\underline{i} + k\underline{j}).(3\underline{i} + 4\underline{j})$

$= 3(1-k) + 4k$

$= 3 + k$ 05

$\underline{a}.\underline{b} = |a||b|\cos\theta$

$\underline{b}.\underline{c} = |b||c|\cos\theta$

$\frac{a.b}{|a||b|} = \cos\theta = \frac{b.c}{|b||c|}$ ⇐ θ සමාන

05

$\frac{4-k}{|a||b|} = \frac{3+k}{|c||b|}$ $(|a| = |c| = 5)$

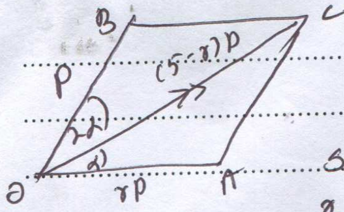
$1 = 2k$

$k = \frac{1}{2}$ 05

25

2

03) O නම් ලක්ෂයක් මත ක්‍රියා කරන γp , ($\gamma \epsilon Z^+$ හා $\gamma > 1$) සහ p නම් බල දෙකේ සම්ප්‍රයුක්ත බලය $(5 - \gamma)$p වන අතර එම සම්ප්‍රයුක්ත බලය මගින් γp සහ p අතර කෝණය 1 : 2 අනුපාතයට බෙදෙයි. $\gamma =$ 2 බව පෙන්වන්න.



$\frac{\sin(180-3\alpha)}{(5-\gamma)p} = \frac{\sin 2\alpha}{\gamma p} = \frac{\sin\alpha}{p}$ 05

$\sin 2\alpha = \gamma \sin\alpha \Rightarrow 2\sin\alpha\cos\alpha = \gamma\sin\alpha$ ($\sin\alpha \neq 0$)

$\cos\alpha = \gamma/2$ 05

$\cos^2\alpha = \frac{\gamma^2}{4}$

$\frac{\sin 3\alpha}{(5-\gamma)} = \sin\alpha$

$3\sin\alpha - 4\sin^3\alpha = \sin\alpha(5-\gamma)$ [$\sin\alpha \neq 0$] 05

$3 - 4(1-\cos^2\alpha) = (5-\gamma)$

$4\cos^2\alpha - \gamma - 6 = 0$

$\cancel{4} \times \frac{\gamma^2}{\cancel{4}} - \gamma - 6 = 0$

$\gamma^2 - \gamma - 6 = 0$ 05

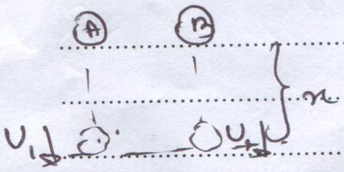
$(\gamma+3)(\gamma-2) = 0$

$\gamma = 2$, $\gamma = -3$ ($\gamma > 1$ නිසා)

$\gamma = 2$ 05

$\boxed{25}$

04) අංශුවක් g_1 ms^{-2} ත්වරණය යටතේ පහළට වැටේ.දෙවන අංශුවක් g_2 ms^{-2} ගුරුත්වජ ත්වරණය යටතේ එම දුරම පහළට වැටීමේදී අනිකට වඩා t (s) කාලයක් අඩුවෙන් ගත කරන අතර අංශු දෙකම එම දුරෙහි දී ලබා ගන්නා ප්‍රවේග අතර වෙනස v ms^{-1} වේ නම් $g_1 g_2 = \frac{v^2}{t^2}$ බව පෙන්වන්න.



(A) ට, $s = ut + \frac{1}{2}at^2$

$x = 0 \cdot t + \frac{1}{2}g_1 t_1^2$

$x = \frac{1}{2}g_1 t_1^2$

$t_1 = \sqrt{\frac{2x}{g_1}}$

(A) ට, $v^2 = u^2 + 2fs$

$v_1^2 = 2g_1 x$

$v_1 = \sqrt{2g_1 x}$ — (1)

(B) ට, $x = 0 \cdot t_2 + \frac{1}{2}g_2 t_2^2$

$t_2 = \sqrt{\frac{2x}{g_2}}$

(B) ට, $v_2^2 = 2g_2 x$

$v_2 = \sqrt{2g_2 x}$ — (2) 05

$t = \sqrt{2x}\left\{\frac{1}{\sqrt{g_2}} - \frac{1}{\sqrt{g_1}}\right\}$ 05

$|v_1 - v_2| = v$

$|\sqrt{2g_1 x} - \sqrt{2g_2 x}| = v$

$\sqrt{2x}|[\sqrt{g_1} - \sqrt{g_2}]| = v$ 05

$t = \frac{\sqrt{2x}}{\sqrt{g_1 g_1}}(\sqrt{g_1} - \sqrt{g_2}) = \frac{\cancel{\sqrt{2x}}}{\sqrt{g_1 g_2}} \cdot \frac{v}{\cancel{\sqrt{2x}}}$ 05

$t = \frac{v}{\sqrt{g_1 g_2}} \Rightarrow \frac{v^2}{t^2} = g_1 g_2$

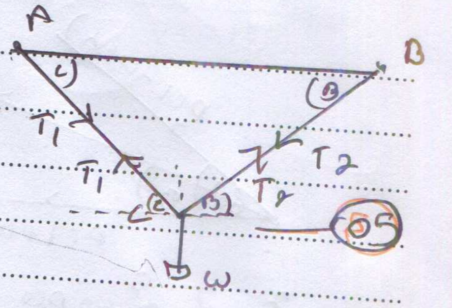
$g_1 g_2 = \frac{v^2}{t^2}$ 05

$\boxed{25}$

05) එකම තිරස් මට්ටමේ වූ A හා B ලක්ෂ්‍ය දෙකකට සවි කර ඇති ACB තන්තුවක C ලක්ෂ්‍යයකට W භාරයක් ගැට ගසා ඇත.

CA තන්තුවේ ආතතිය $\frac{Wb(c^2+a^2-b^2)}{2abc\sin C}$ බව පෙන්වන්න. a , b , c හා C ත්‍රිකෝණයක සම්මත අංකනයට වේ.

ලාමි ප්‍රමේය මගින්



$$\frac{T_1}{\sin(90+B)} = \frac{W}{\sin C} \quad 05$$

$$T_1 = \frac{W\cos B}{\sin C} \quad 05$$

05

$$\cos B = \frac{a^2+c^2-b^2}{2ac} \quad (\text{cos සූත්‍රයෙන්}) \quad 05$$

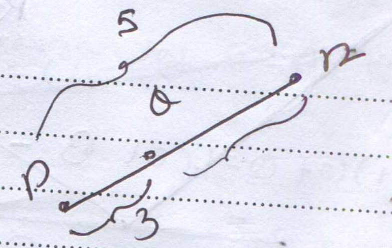
$$T_1 = \frac{W}{\sin C} \cdot \left(\frac{a^2+c^2-b^2}{2ac}\right)$$

$$T_1 = \frac{Wb(a^2+c^2-b^2)}{2abc\sin C} \quad 05$$

25

06) $-2\underline{a} + 5\underline{b}$, $7\underline{a}-\underline{b}$ හා $13\underline{a} - 5\underline{b}$ යනු පිළිවෙළින් P , Q හා R ලක්ෂ්‍ය තුනෙහි අචල O මූලයට අනුබද්ධව පිහිටුම් දෛශික යැයි ගනිමු. මෙහි $\underline{a}$ හා $\underline{b}$ සමාන්තර නොවන දෛශික දෙකකි. P , Q හා R ඒක රේඛීය බව පෙන්වා PQ : QR = 3 : 2 බව පෙන්වන්න.

$$\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ}$$
$$= \overrightarrow{OQ} - \overrightarrow{OP}$$
$$= 7\underline{a} - \underline{b} - (-2\underline{a} + 5\underline{b})$$
$$= 9\underline{a} - 6\underline{b}$$
$$= 3(3\underline{a} - 2\underline{b}) \quad 05$$



$\therefore PQ : QR = 3 : 2$ වේ. 05

$$\overrightarrow{PR} = \overrightarrow{PO} + \overrightarrow{OR}$$
$$= \overrightarrow{OR} - \overrightarrow{OP}$$
$$= 13\underline{a} - 5\underline{b} - (-2\underline{a} + 5\underline{b})$$
$$= 15\underline{a} - 10\underline{b}$$
$$= 5(3\underline{a} - 2\underline{b}) \quad 05$$

$$\overrightarrow{PR} = \frac{5\overrightarrow{PQ}}{3} \quad 05$$

එය

$PR = k\overrightarrow{PQ}$ ආකාර ගනී.

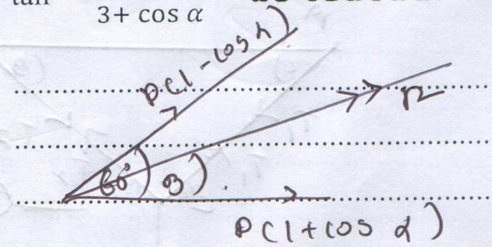
$\therefore P, Q, R$ ඒක රේඛීය වේ.

25

$$\therefore |\overrightarrow{PR}| = \frac{5}{3}|\overrightarrow{PQ}| \quad 05$$

4

07) අංශුවක් මත $P(1 + \cos\alpha)$ හා $P(1 - \cos\alpha)$ බල දෙකක් 60° ක් ආනතව ක්‍රියා කරයි. සම්ප්‍රයුක්ත බලය $\sqrt{(4 - \sin^2\alpha)}\,P$ බව පෙන්වන්න. පළමු බලය හා සම්ප්‍රයුක්ත බලය අතර කෝණය $\tan^{-1}\dfrac{\sqrt{3}(1-\cos\alpha)}{3+\cos\alpha}$ බව පෙන්වන්න.



$R^2 = P^2 + Q^2 + 2PQ\cos\theta$ සූත්‍රයට අනුව.

$= P^2(1+\cos\alpha)^2 + P^2(1-\cos\alpha)^2 + 2P^2(1+\cos\alpha)(1-\cos\alpha)\cos 60^\circ$ 05

$= P^2(1+\cancel{2\cos\alpha}+\cos^2\alpha+1-\cancel{2\cos\alpha}+\cos^2\alpha) + P^2 - P^2\cos^2\alpha$ 05

$= 3P^2 + P^2\cos^2\alpha$

$R = P\sqrt{3+\cos^2\alpha} = P\sqrt{3+1-\sin^2\alpha}$

$R = P\sqrt{4-\sin^2\alpha}$ 05

(ii) $\tan\beta = \dfrac{Q\sin\theta}{P+Q\cos\theta}$

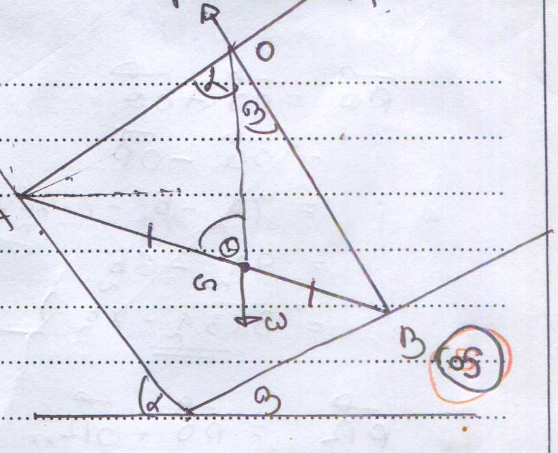
$= \dfrac{P(1-\cos\alpha)\sin 60^\circ}{P(1+\cos\alpha)+P(1-\cos\alpha)\cos 60^\circ}$ 05

$= \dfrac{(1-\cos\alpha)\sqrt{3}/2}{1+\cos\alpha+\frac{1}{2}-\frac{1}{2}\cos\alpha}$

$= \dfrac{\sqrt{3}(1-\cos\alpha)}{3+\cos\alpha}$ 05

25

08) බර ඒකාකාර දණ්ඩක් එහි දෙකෙලවර තිරස සමඟ α හා $\beta(<\alpha)$ කෝණවලින් ආනත වන සුමට ආනත තල දෙකක් හා ස්පර්ශව සිරසට θ සුළු කෝණයකින් ආනතව සමතුලිතාවයේ තබා ඇත්තේ දණ්ඩ හරහා යන සිරස් තලය, ආනත තල ඡේදනය වන රේඛාවට ලම්භකව පිහිටන පරිදි නම් $\cot\alpha + 2\cot\theta = \cot\beta$ බව පෙන්වන්න.



AOB Δයට cot ප්‍රමේයෙන්

$(1+1)\cot\theta = 1.\cot\beta - 1.\cot\alpha$ 10

$2\cot\theta = \cot\beta - \cot\alpha$

$2\cot\theta + \cot\alpha = \cot\beta$ 10

05

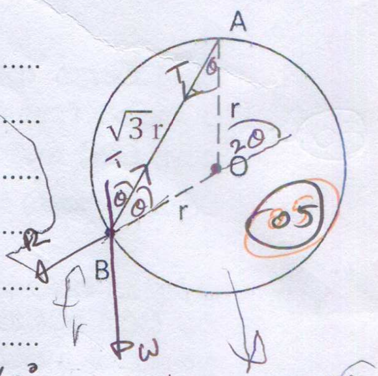
25

5

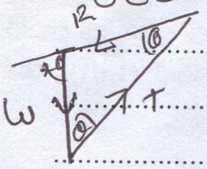
09) O කේන්ද්‍රය ද අරය r ද වන සුමට සිරස් දෘඩ වෘත්තාකාර කම්බියක් අචලව සවිකර ඇත. වෘත්තාකාර කම්බිය තුළින් යන බර $\sqrt{3}w$ වන සුමට B මුද්දකට වෘත්තාකාර සිරස් කම්බියේ චලිත වීමට නිදහස ඇත. $\sqrt{3}r$ දිග අවිතන්‍ය තන්තුවක එක් කෙළවරක් සිරස් වෘත්තාකාර කම්බියේ ඉහළම A ලක්ෂ්‍යයට ද අනෙක් කෙළවර මුද්දට ද සවිකර ඇත. රූපයේ දැක්වෙන පරිදි තන්තුව තදව මුද්ද සමතුලිත විට ඒ සඳහා බල ත්‍රිකෝණයක් ඇඳීමෙන් හෝ අන් අයුරකින් හෝ

i. තන්තුවේ ආතතිය 3w බව හා

ii. මුද්ද මත ප්‍රතික්‍රියාව $\sqrt{3}w$ බව පෙන්වන්න.



B මුද්ද w, R, T බල මගින් සමතුලිත වේ.



ලාමි ප්‍රමේයෙන්,

$\frac{T}{\sin(180-2\theta)} = \frac{\sqrt{3}w}{\sin\theta} = \frac{R}{\sin\theta}$ — 05

ABO ΔABO, sin ප්‍රමේයෙන්,

$\frac{\sqrt{3}r}{\sin(180-2\theta)} = \frac{r}{\sin\theta}$ — 05

$\frac{\sqrt{3}}{2\sin\theta\cos\theta} = \frac{1}{\sin\theta}$

$\cos\theta = \sqrt{3}/2 \Rightarrow \theta = 30°$

$T = \frac{\sqrt{3}w}{\sin 30°} \times \sin 60°$

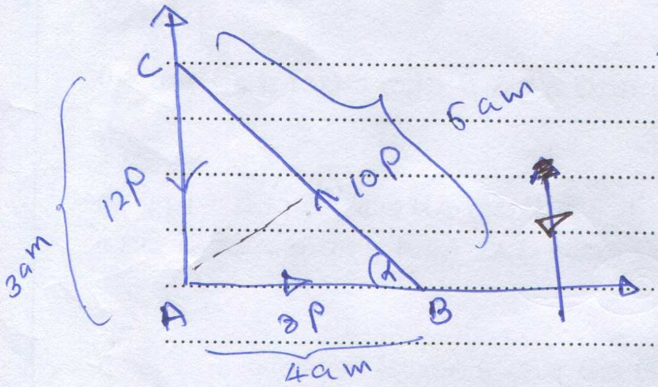
$= 3w$ — 05

$R = \sqrt{3}w$ — 05

05

25

10) A හිදී සෘජුකෝණී වූ සහ AB = 4a m , AC = 3a m වූ ABC ත්‍රිකෝණයක AB,BC හා CA පාද දිගේ පිළිවෙළින් නිවිටන 8p,10p,12p විශාලත්ව සහිත් බල ක්‍රියා කරයි. මෙම බල පද්ධතියේ සම්ප්‍රයුක්තයෙහි විශාලත්වය හා දිශාව ගණනය කරන්න. තව ද එහි ක්‍රියා රේඛාව AB හමුවන ලක්ෂ්‍යයද සොයන්න.



$\rightarrow X = 8P - 10P\cos\alpha = 8P - 10P \times 4/5$

$= 0$ — 05

$\uparrow Y = 10P\sin\alpha - 12P = 10P \times 3/5 - 12P$

$Y = -6P$ — 05

$\therefore Y \parallel CA$ හා $Y = 6P \downarrow$ වේ. — 05

$\cos\alpha = 4/5$

$\sin\alpha = 3/5$

A) $10P \cdot 4a \cdot \sin\alpha = 6P \times x_0$ — 05

$10 \times 2a \times \frac{3}{5} = 3x_0$

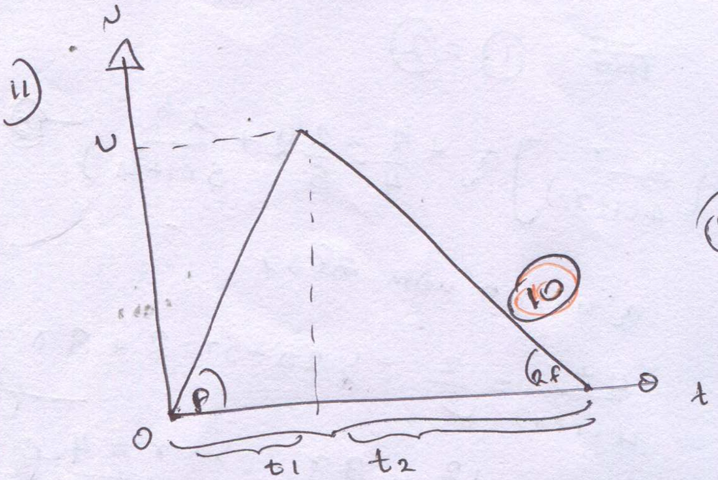
$x_0 = 4a$ — 05

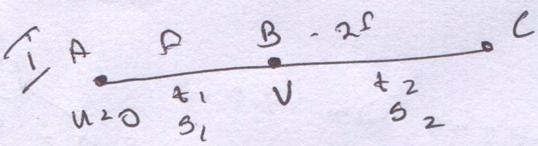
A සිට 4a දුරින් සම්ප්‍රයුක්තය ඡේදනය කරයි

25

6

11)





A → B චලිතය සලකා,

$v = u + at$

$v = ft_1$ —— ① 05

B → C චලිතය සලකා,

$v = u + at$

$0 = v - 2ft_2$

$v = 2ft_2$ —— ② 05

① = ②

$2ft_2 = ft_1 \Rightarrow t_1 = 2t_2$

$t_1 + t_2 = t \Rightarrow t_2 + 2t_2 = t$

$t_2 = \frac{t}{3}$ $t_1 = \frac{2t}{3}$ 10

A → B දක්වා විස්ථාපනය

$s = ut + \frac{1}{2}at^2$

$S_1 = \frac{1}{2} \cdot f \cdot t_1^2 = \frac{f}{2} \cdot \frac{4t^2}{9} = \frac{2ft^2}{9}$ 10

B → C දක්වා විස්ථාපනය,

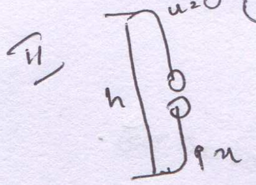
$v^2 = u^2 + 2as$

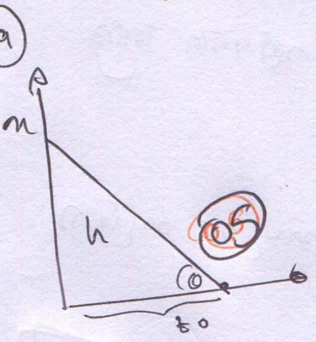
$0 = v^2 - 2 \cdot 2f s_2$

$s_2 = \frac{(ft_1)^2}{4f} = f^2 \cdot \frac{4t^2}{9} \cdot \frac{1}{4f}$ 10

$s_1 + s_2 = \frac{2ft^2}{9} + \frac{ft^2}{9} = \frac{ft^2}{3}$ 10

$\therefore h = \frac{ft^2}{3}$ 60

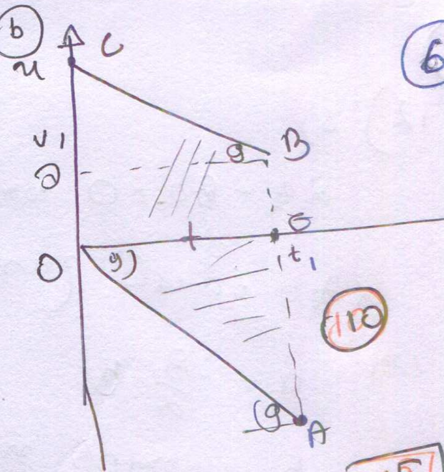




$\tan\theta = g$

a) $g = \frac{u}{t_0} \Rightarrow t_0 = \frac{u}{g}$ 05

$h = \frac{1}{2} \cdot u \cdot t_0 = \frac{1}{2} \cdot u \cdot \frac{u}{g} = \frac{u^2}{2g}$



15

b) [illegible]

[illegible]

[illegible] $= OABC$ ▱ ව.ඵ 10

$h = \frac{1}{2} \times 2u \times t_1 = \frac{u^2}{2g}$ $t_1 = \frac{u}{2g}$ 10

c) BCD Δයෙන්,

$g = \frac{u - v_1}{t_1} \Rightarrow v_1 = u - gt_1$ 10

$v_1 = u - g \cdot \frac{u}{2g}$ 10

$= \frac{u}{2}$

20

d) ගමන් උස h_1 නම්

$h_1 = OCBE$ ව.ඵ 05

$= \frac{1}{2}(u + v_1)t_1$

$= \frac{1}{2}(u + \frac{u}{2}) \cdot \frac{u}{2g}$ 05

$= \frac{3u^2}{8g} = \frac{3}{4}\left(\frac{u^2}{2g}\right)$ 05

$= \frac{3h}{4}$

20

7

12) a)

$\alpha \underset{\sim}{a} + \beta \underset{\sim}{b} = 0$ සත්‍ය වේ නම්

$\alpha \neq 0$ වේ යැයි උපකල්පනය කරමු

එවිට $\underset{\sim}{a} = -\dfrac{\beta}{\alpha} \underset{\sim}{b}$

$\underset{\sim}{a}$ හා $\underset{\sim}{b}$ ශුන්‍ය නොවන සමාන්තර නොවන දෛශික

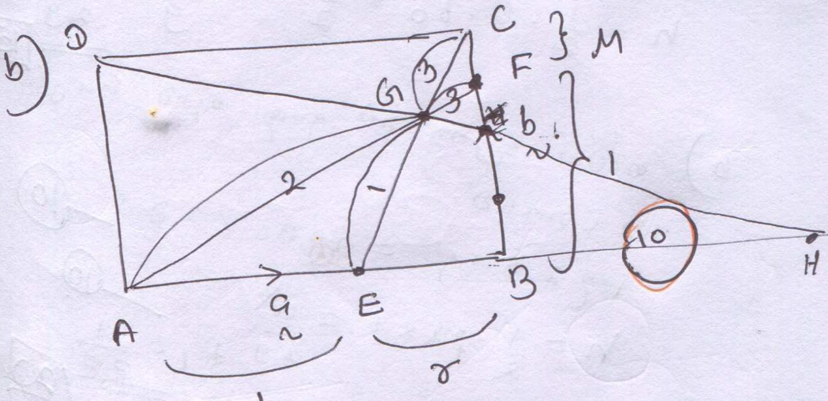
නිසා මෙය විරෝධාභාසයකි.

$\therefore \beta = 0$ වේ.

එලෙසම $\alpha = 0$ බව පෙන්විය හැක

$\boxed{25}$

b)



$AG : GC = 1 : 3$ $\quad$ $AE : EB = 1 : r$

$AG : GF = 2 : 3$ $\quad$ $BF : FC = 1 : M$

i/ $\overrightarrow{AG} = \overrightarrow{AE} + \overrightarrow{EG}$

$= \dfrac{1}{(1+r)} \underset{\sim}{a} + \dfrac{1}{4} \overrightarrow{EC}$ — 05

$= \dfrac{1}{(1+r)} \underset{\sim}{a} + \dfrac{1}{4} (\overrightarrow{EB} + \overrightarrow{BC})$ — 05

$= \dfrac{1}{(1+r)} \underset{\sim}{a} + \dfrac{1}{4} \left(\dfrac{r \underset{\sim}{a}}{(1+r)} + \underset{\sim}{b} \right)$

$= \left[\dfrac{4+r}{4(1+r)} \right] \underset{\sim}{a} + \dfrac{\underset{\sim}{b}}{4}$ — ① — 05

$\overrightarrow{AG} = \dfrac{2}{5} \overrightarrow{AF} = \dfrac{2}{5} (\overrightarrow{AB} + \overrightarrow{BF})$ — 05

$= \dfrac{2}{5} \left(\underset{\sim}{a} + \dfrac{1}{1+M} \underset{\sim}{b} \right)$

$= \dfrac{2}{5} \underset{\sim}{a} + \dfrac{2}{5(1+M)} \underset{\sim}{b}$ — ② — 10

$\overrightarrow{AG}$ ① = ②

$\left[\dfrac{4+r}{4(1+r)} \right] \underset{\sim}{a} + \dfrac{\underset{\sim}{b}}{4} = \dfrac{2}{5} \underset{\sim}{a} + \dfrac{2 \underset{\sim}{b}}{5(1+M)}$ — 05

$\underset{\sim}{a}$ සංගුණක සමාන කිරීමෙන්

$\dfrac{4+r}{4(1+r)} = \dfrac{2}{5} \Rightarrow 20 + 5r = 8 + 8r$ — 05

$12 = 3r \Rightarrow r = 4$ — 05

$\underset{\sim}{b}$ සංගුණක සමාන කිරීමෙන්

$\dfrac{1}{4} = \dfrac{2}{5(1+M)} \Rightarrow 5 + 5M = 8$ — 05

$M = \dfrac{3}{5}$ — 05

$\boxed{65}$

ii/ $\overrightarrow{AG} = \dfrac{2}{5} \underset{\sim}{a} + \dfrac{2 \underset{\sim}{b}}{5(1+\frac{3}{5})}$ — 05

$= \dfrac{2}{5} \underset{\sim}{a} + \dfrac{\cancel{2} \underset{\sim}{b}}{\cancel{8}\,4}$

$= \dfrac{8 \underset{\sim}{a} + 5 \underset{\sim}{b}}{20}$ — 05

$\boxed{10}$

iii/ $\overrightarrow{DH} = k \overrightarrow{DG}$

$\overrightarrow{AH} = \overrightarrow{AD} + \overrightarrow{DH}$

$= \underset{\sim}{b} + k \overrightarrow{DG}$ — 05

$= \underset{\sim}{b} + k (\overrightarrow{DA} + \overrightarrow{AG})$

$= \underset{\sim}{b} + k \left(-\underset{\sim}{b} + \dfrac{8 \underset{\sim}{a} + 5 \underset{\sim}{b}}{20} \right)$ — 05

$= \underset{\sim}{b} + k \left(\dfrac{-20 \underset{\sim}{b} + 5 \underset{\sim}{b} + 8 \underset{\sim}{a}}{20} \right)$

$= \dfrac{(20 - 15k) \underset{\sim}{b} + 8k \underset{\sim}{a}}{20}$ — 10

$\boxed{20}$

AH , AB එක රේඛීය නිසා — 05

$20 - 15k = 0$

$k = \dfrac{4}{3}$ — 05

8

12) iii) $7\overrightarrow{AH} = 8\overrightarrow{HB}$ බව

$$\overrightarrow{AH} = \frac{\cancel{8}^{2} k \underset{\sim}{a}}{5\,\cancel{20}} = \frac{2}{5} \times \frac{4}{3}\underset{\sim}{a}$$

$$\overrightarrow{AH} = \frac{8}{15}\underset{\sim}{a} \quad 05$$

$$\overrightarrow{AH} = \frac{8}{15} \times \overrightarrow{AB}$$

$$\overrightarrow{AH} = \frac{8}{15}(\overrightarrow{AH} + \overrightarrow{HB}) \quad 10$$

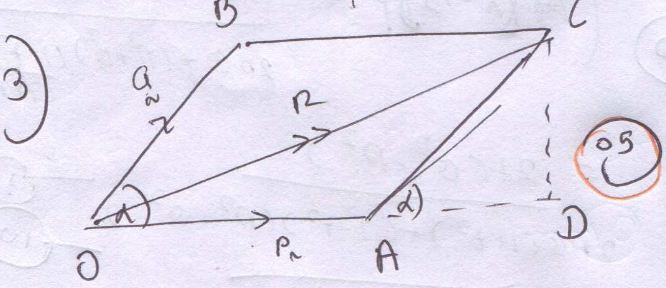
$$\overrightarrow{AH} - \frac{8}{15}\overrightarrow{AH} = \frac{8}{15}\overrightarrow{HB}$$

$$\frac{7\overrightarrow{AH}}{\cancel{15}} = \frac{8}{\cancel{15}}\overrightarrow{HB}$$

$$7\overrightarrow{AH} = 8\overrightarrow{HB} \quad 05$$

$\boxed{20}$

13)



$$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC}$$

$$\underset{\sim}{R} = \underset{\sim}{p} + \underset{\sim}{a}$$

$$OC = |\overrightarrow{OC}| = |\underset{\sim}{R}| = R$$

$$OC^2 = OD^2 + DC^2 \quad 05$$

$$= (OA + AD)^2 + AC^2 - AD^2 \quad 05$$

$$= OA^2 + 2.OA.AD + \cancel{AD^2} + AC^2 - \cancel{AD^2} \quad 05$$

$$= OA^2 + 2.OA.AC\cos\alpha \quad 05$$

$$= OA^2 + AC^2 + 2.OA.AC\cos\alpha$$

$$R^2 = P^2 + Q^2 + 2PQ\cos\alpha \quad 05$$

$$R = \sqrt{P^2 + Q_2 + 2PQ\cos\alpha}$$

$\boxed{30}$

$$R_{0°} = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha} \quad (\cos 0°) \quad 05$$

$$= \sqrt{P^2 + Q^2 + 2PQ \times 1} \quad 05$$

$$= \sqrt{(P+Q)^2}$$

$$r = P + Q \quad 05 \quad \left(\begin{array}{l}\cos\alpha = 1 \\ \alpha = 0\end{array}\right)$$

$\boxed{15}$

$$R_{\pi} = \sqrt{P^2 + Q^2 + 2PQ \times (-1)} \quad 05$$

$$= \sqrt{P^2 + Q^2 - 2PQ}$$

$$= \sqrt{(P-Q)^2} \quad 05$$

$$M = P - Q \quad 05 \quad \left(\begin{array}{l}\cos\alpha = -1 \\ \alpha = \pi\end{array}\right)$$

$\boxed{15}$

$$\frac{r + M}{2} = P \quad 05 \qquad \frac{r - M}{2} = Q \quad 05$$

$\boxed{10}$

(9)

1) $R^2 = P^2 + Q^2 + 2PQ\cos 2\theta$

$= (P+Q)^2 - 2PQ + 2PQ\cos 2\theta$

$= (P+Q)^2 - 2PQ(1-\cos 2\theta)$ — 05

$= (P+Q)^2 - 2PQ \cdot 2\sin^2\theta$

$= r^2 - 4\left(\frac{r+M}{2}\right)\left(\frac{r-M}{2}\right)\sin^2\theta$ — 05

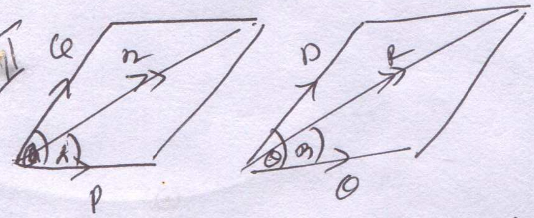
$= r^2 - 4\frac{(r^2-M^2)}{4}\sin^2\theta$ — 05

$= r^2(1-\sin^2\theta) + M^2\sin^2\theta$

$= r^2\cos^2\theta + M^2\sin^2\theta$ — 05

$\boxed{20}$

13) ii)



$\tan\alpha = \frac{Q\sin\theta}{P+Q\cos\theta}$ $\quad \tan\beta = \frac{P\sin\theta}{Q+P\cos\theta}$ — 10

R mob n A $|(\alpha-\beta)|$ 6d.

$\tan|\alpha-\beta| = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}$ — 05

$= \frac{\frac{Q\sin\theta}{(P+Q\cos\theta)} - \frac{P\sin\theta}{(Q+P\cos\theta)}}{1 + \frac{Q\sin\theta}{(P+Q\cos\theta)}\frac{P\sin\theta}{(Q+P\cos\theta)}}$ — 05

$= \frac{Q\sin\theta(Q+P\cos\theta) - P\sin\theta(P+Q\cos\theta)}{(P+Q\cos\theta)(Q+P\cos\theta) + PQ\sin^2\theta}$

$= \frac{Q^2\sin\theta + PQ\sin\theta\cos\theta - P^2\sin\theta - PQ\sin\theta\cos\theta}{PQ + P^2\cos\theta + Q^2\cos\theta + PQ(\underbrace{\cos^2\theta + \sin^2\theta}_{1})}$

$= \frac{\sin\theta(Q^2-P^2)}{2PQ + \cos\theta(P^2+Q^2)}$

$\tan\theta/2 = t$ and

~~$\sin\theta = 2\tan\alpha$~~

$\sin\theta = \frac{2t}{1-t^2}$ $\quad \cos\theta = \frac{1-t^2}{1+t^2}$

$(\tan(\alpha-\beta)) = \frac{(Q^2-P^2)\,2t/(1-t^2)}{2PQ + (P^2+Q^2)\frac{(1-t^2)}{(1+t^2)}}$

$= \frac{2t(Q^2-P^2)}{2PQ(1+t^2) + (1-t^2)(P^2+Q^2)}$ — 1 — 10

mob n A $(\alpha-\beta) = 2\tan^{-1}\underbrace{\left(\frac{P-Q}{P+Q}\right)\tan}_{r}$

$(\alpha-\beta) = 2r$

$\tan(\alpha-\beta) = \tan 2r$ $\quad \tan r = \left(\frac{P-Q}{P+Q}\right)$

$\tan 2r = \frac{2\tan r}{1-\tan^2 r}$ — 10

$= \frac{2\left(\frac{P-Q}{P+Q}\right)\tan\alpha/2}{1-\left(\frac{P-Q}{P+Q}\right)^2\tan^2\alpha/2}$

$= \frac{2(P-Q)(P+Q)\tan\alpha/2}{(P+Q)^2 - (P-Q)^2\tan^2\alpha/2}$

$= \frac{2(P-Q)(P+Q)\tan\alpha/2}{}$

13) II $= \dfrac{2(P^2-Q^2)\tan \alpha/2}{2PQ(1+t^2) + (P^2+Q^2)(1-\tan^2 \alpha/2)}$

$= \dfrac{2(P^2-Q^2)t}{2PQ(1+t^2)(1-t^2)(P^2+Q^2)}$ 2

05

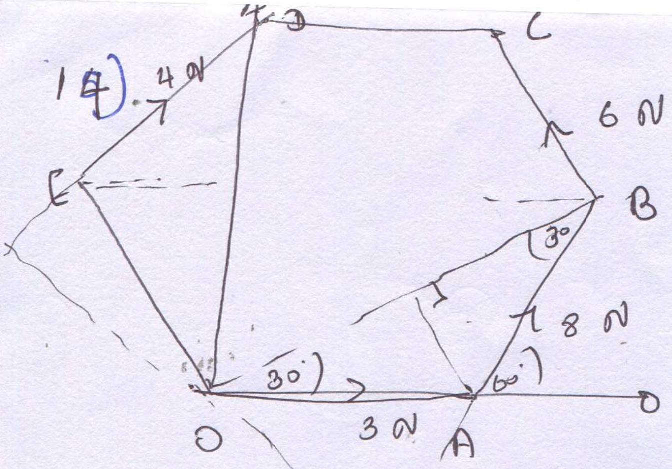
1 = 2 නිසා

[illegible] $\angle\ |\alpha - \beta| = 2\tan^{-1}\left(\dfrac{P-Q}{P+Q}\right)\tan \dfrac{\alpha}{2}$ වේ.

05

$\boxed{50}$

14)



(20)

$\rightarrow x = 3 + 8\cos 60^\circ - 6\cos 60^\circ + 4\cos 60^\circ$ — 10

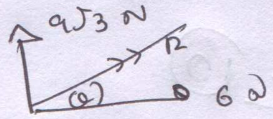
$= 3 + 4 - 3 + 2$ — 10

$= 6N$

$\uparrow Y = 18\sin 60^\circ$ — 10 $= 18\sqrt{3}/2 = 9\sqrt{3}\,N$ — 10

$R^2 = 6^2 + (9\sqrt{3})^2 = 36 + 81\times 3 = 279N$

$R = \sqrt{279}\,N$ — 10



$\tan\theta = \frac{9\sqrt{3}}{6} = 3\sqrt{3}/2$

$\theta = \tan^{-1}\left(\frac{3\sqrt{3}}{2}\right)$ — 10

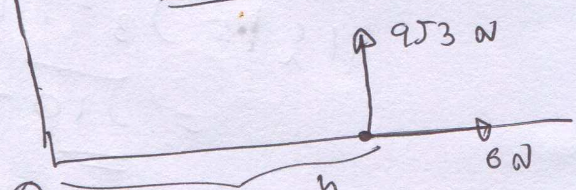
80

$O \circlearrowleft \; 6 \times 2 \cdot 2\cos 30^\circ - 4 \cdot 2\sin 60^\circ + 8 \cdot 2\sin 60^\circ$ — 10

$= 12\sqrt{3} - 4\sqrt{3} + 8\sqrt{3}$

$= 16\sqrt{3}\,Nm$ — 10

20

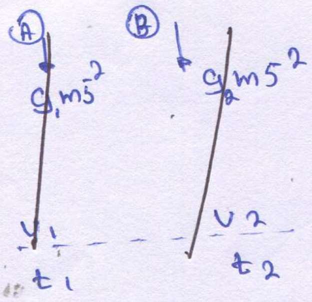


$O \circlearrowleft \; 16\sqrt{3} = 9\sqrt{3}\,h$ — 10

$h = 16/9$ — 10

20

(12) 4)



ක්‍රියාරේඛාවේ සමීකරණය

$y - y_1 = m(x - x_1)$ — 10

$y - 0 = \frac{3\sqrt{3}}{2}\left(x - \frac{16}{9}\right)$ — 10

$y = \frac{\cancel{3}\sqrt{3}}{2} \frac{(9x - 16)}{\cancel{9}\,3}$

$6y = 9\sqrt{3}x - 16\sqrt{3}$ — 10

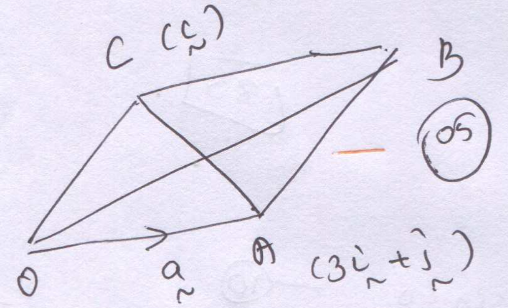
$\boxed{30}$

(15) a) $\underset{\sim}{a} \cdot \underset{\sim}{b} = |\underset{\sim}{a}|\,|\underset{\sim}{b}| \cos\alpha$

α යනු a හා b අතර කෝණය. $(0 < \alpha < \pi)$ වේ. — 10

$\boxed{10}$ 10

b)



— 05

$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$

$= \underset{\sim}{a} + \overrightarrow{OC}$

$= \underset{\sim}{a} + \underset{\sim}{c}$ — 10 $\quad (\overrightarrow{OA} = \underset{\sim}{a})$

$\overrightarrow{CA} = \overrightarrow{CO} + \overrightarrow{OA}$

$= -\underset{\sim}{c} + \underset{\sim}{a}$

$= \underset{\sim}{a} - \underset{\sim}{c}$ — 10

$\boxed{20}$

රොම්බසයේ විකර්ණ ⊥ නිසා,

$\overrightarrow{OB} \cdot \overrightarrow{CA} = 0$ — 05

$(\underset{\sim}{a} + \underset{\sim}{c}) \cdot (\underset{\sim}{a} - \underset{\sim}{c}) = 0$

$\underset{\sim}{a} \cdot \underset{\sim}{a} - \underset{\sim}{c} \cdot \underset{\sim}{c} = 0$ — 05

$|\underset{\sim}{a}|^2 - |\underset{\sim}{c}|^2 = 0$ — 05

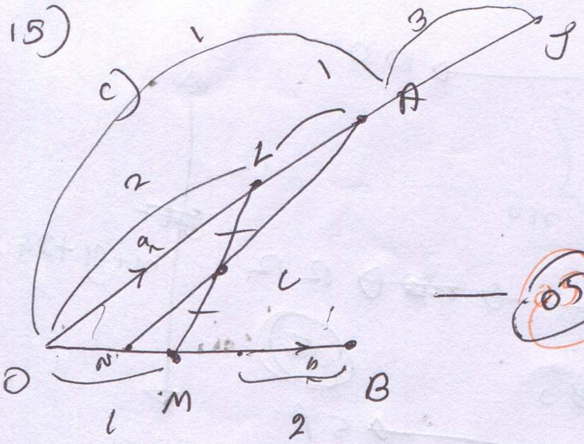
$|\underset{\sim}{a}| = |\underset{\sim}{c}|$ — 05

$|\underset{\sim}{c}| = \sqrt{3^2 + 1^2}$

$= \sqrt{10}$ — 05

$\boxed{25}$

15)



(05)

$\overrightarrow{OA} = \underset{\sim}{a}$, $\overrightarrow{OB} = \underset{\sim}{b}$

i/ $\overrightarrow{OL} = \overrightarrow{OM} + \overrightarrow{ML}$

$= \frac{1}{3}\overrightarrow{OB} + \frac{1}{2}\overrightarrow{ML}$ (05)

$= \frac{1}{3}\overrightarrow{OB} + \frac{1}{2}(\overrightarrow{MO} + \overrightarrow{OL})$

$= \frac{1}{3}\overrightarrow{OB} + \frac{1}{2}\left(\frac{1}{3}\overrightarrow{BO} + \frac{2}{3}\overrightarrow{OA}\right)$ (05)

$= \left(\frac{1}{3} - \frac{1}{6}\right)\underset{\sim}{b} + \frac{1}{3}\underset{\sim}{a}$

$= \frac{\underset{\sim}{a}}{3} + \frac{1}{6}\underset{\sim}{b}$ (05)

$\boxed{20}$

ii/ $\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC} = \cancel{\overrightarrow{AO}} +$ (05)

$= -\underset{\sim}{a} + \frac{\underset{\sim}{a}}{3} + \frac{\underset{\sim}{b}}{6}$ (05)

$= \frac{1}{6}\underset{\sim}{b} - \frac{2}{3}\underset{\sim}{a}$ (05)

$\boxed{15}$

iii/ $\overrightarrow{AC} = r\overrightarrow{AN}$, $\overrightarrow{ON} = \mu\overrightarrow{OB}$

$\frac{1}{6}\underset{\sim}{b} - \frac{2}{3}\underset{\sim}{a} = r(\overrightarrow{AO} + \overrightarrow{ON})$ (05)

$\frac{1}{6}\underset{\sim}{b} - \frac{4}{6}\underset{\sim}{a} = r(-\underset{\sim}{a} + \mu\overrightarrow{OB})$ (05)

$\underset{\sim}{b} - 4\underset{\sim}{a} = 6r(\mu\underset{\sim}{b} - \underset{\sim}{a})$

$(3r-2)\underset{\sim}{a} + (1-6r\mu)\underset{\sim}{b} = 0$ (05)

$\boxed{15}$

$\underset{\sim}{a}$ හා $\underset{\sim}{b}$ ඒකරේඛීය නොවන නිසා

$3r - 2 = 0$

$r = \frac{2}{3}$ (05)

$\frac{1}{6} - \cancel{r}\mu = 0$

$\frac{2}{3}\mu = \frac{1}{6}$

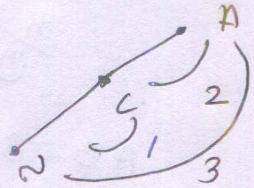
$\mu = \frac{1}{4}$ (05)

$\frac{1}{6}r\mu = 1$

$\frac{1}{6} \times \frac{3}{2}$ $\mu = 1$

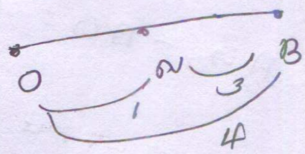
$\mu = 2$

$\frac{\overrightarrow{AC}}{\overrightarrow{AN}} = \frac{2}{3}$



$\therefore \frac{AL}{CN} = \frac{2}{1}$ (05)

$\frac{\overrightarrow{ON}}{\overrightarrow{OB}} = \frac{1}{4}$



$\frac{ON}{NB} = \frac{1}{3}$

$\frac{ON}{NB} = \frac{1}{3}$ (05)

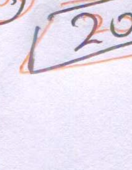
$\boxed{20}$

iv/ $\overrightarrow{JB} = \overrightarrow{JO} + \overrightarrow{OB}$

$= \underset{\sim}{b} - \overrightarrow{OJ}$ (05)

$= \underset{\sim}{b} - 4\overrightarrow{OA}$

$= \underset{\sim}{b} - 4\underset{\sim}{a}$ (05)



$\frac{OA}{OJ} = \frac{1}{4}$

$= \frac{6(\underset{\sim}{b} - 4\underset{\sim}{a})}{6}$ (05)

$= 6\left(\frac{\underset{\sim}{b}}{6} - \frac{4\underset{\sim}{a}}{6}\right)$

$= 6\left(\frac{1}{6}\underset{\sim}{b} - \frac{2\underset{\sim}{a}}{3}\right)$

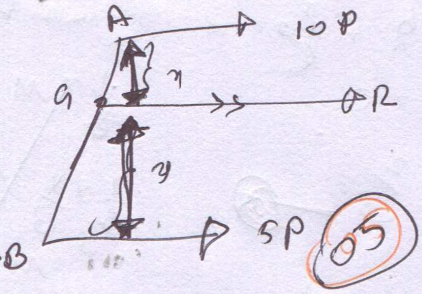
$\overrightarrow{JB} = 6 \cdot \overrightarrow{AC}$ (10) $\boxed{25}$

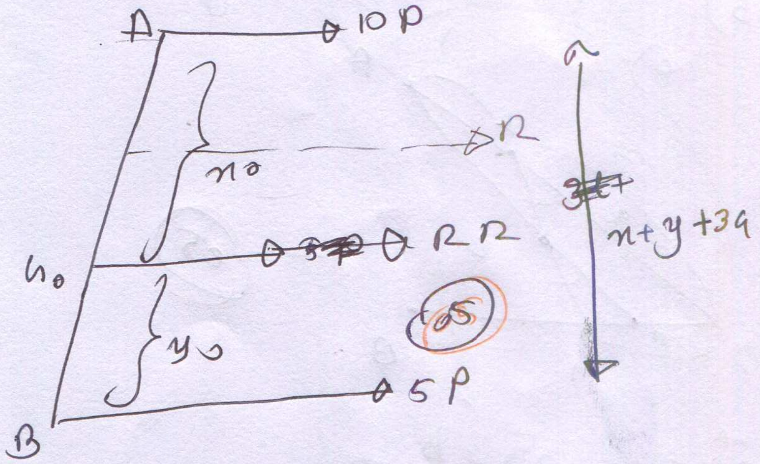
1

(14)

16)

(I)





G වට $10P \times x - 5Py = 0$

$\frac{x}{y} = \frac{1}{2}$ —— (1) 05

$AB = 3l$ නම්.

$AG = l$, $GB = 2l$ වේ.

—— 10

G_0 වට $= 10P x_0 - 5P y_0 = 0$

$\frac{x_0}{y_0} = \frac{1}{2}$

$x + y + 3a = 3(a + l)$

$AG_0 = \frac{1}{3} 3(a + l)$ 10

$= a + l$

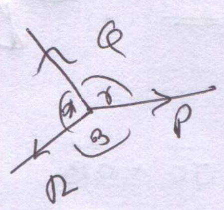
$AG_0 - AG = a + l - l = a$ 05

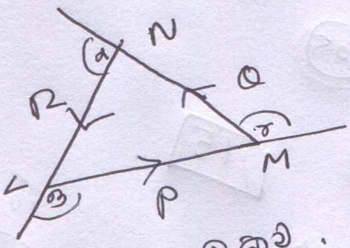
30

සම්ප්‍රයුක්තය a දුරකින් චලනය විය. 05

එනම් සම්ප්‍රයුක්තය ... වේ නම්, එම

(II) ලෙකාමිගේ ප්‍රමේයය කරන බල ඉතා සමතුලිතතාවයේ ... බල එක් එක් බලයක් අනෙක් බල දෙක අතර කෝණයේ සයිනයට සමානුපාතිකවේ. 10





05

සමතුලිත බාහිර බල ත්‍රිකෝණය සලකමු.

$\frac{LM}{\sin(\pi - \alpha)} = \frac{MN}{\sin(\pi - \beta)} = \frac{NL}{\sin(\pi - \gamma)} = k$ —— 05

$LM = k \sin\alpha$, $MN = k \sin\beta$, $NL = k \sin\gamma$

$P = k \sin\alpha$ $Q = k \sin\beta$ $R = k \sin\gamma$

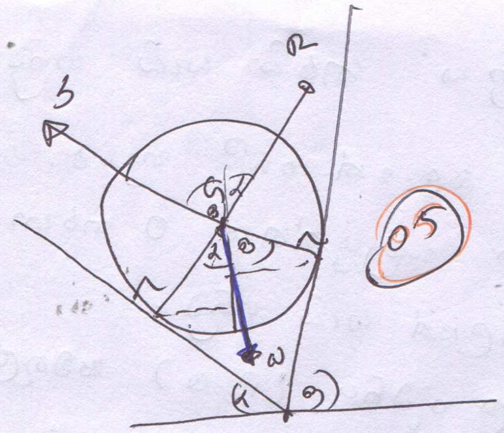
$R \propto \sin\gamma$ වේ. —— 05

$\therefore P \propto \sin\alpha$ $Q \propto \sin\beta$,

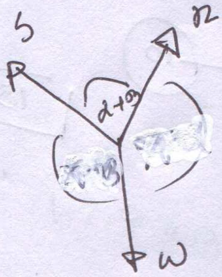
බලයක් අනෙක් බල 2 අතර කෝණයේ සයිනයට සමානුපාතික වේ. —— 05

30

16) (ii)



05



$$\frac{R}{\sin(\pi-\theta)} = \frac{W}{\sin(\alpha+\theta)} = \frac{S}{\sin(\pi-\alpha)}$$ 05

$$\frac{R}{\sin\theta} = \frac{W}{\sin(\alpha+\theta)} = \frac{S}{\sin\alpha}$$ 05

$$R = \frac{W\sin\theta}{\sin(\alpha+\theta)}$$ 05

$$S = \frac{W\sin\alpha}{\sin(\alpha+\theta)}$$

20

$R = \frac{W}{2}$ ගත් 05

$$\frac{W}{2} = \frac{W\sin\theta}{\sin(\alpha+\theta)}$$ 10

$\sin\alpha\cos\theta + \cos\alpha\sin\theta = 2\sin\theta$ 05

$\sin\alpha\cos\theta = \sin\alpha\cos\theta$ 05

$(2-\cos\alpha)\sin\theta = \sin\alpha$ 05

$$\tan\theta = \frac{\sin\alpha}{2-\cos\alpha}$$

30

$\tan\theta = \frac{\sin\alpha}{2-\cos\alpha} = y$ නම්

$2y - y\cos\alpha = \sin\alpha$

$y\cos\alpha + \sin\alpha = 2y$ 05

α තාත්වික වීම 05

$\left(\frac{4y^2}{1+y^2}\right)^2 \leq 1$ විය යුතුයි. 05

$4y^2 \leq 1+y^2$

$3y^2 \leq 1$ 05

$3y^2 - 1 \leq 0$

$(\sqrt{3}y-1)\underbrace{(\sqrt{3}y+1)}_{(+)} \leq 0$ 05

$\sqrt{3}y - 1 \leq 0$

$y \leq \frac{1}{\sqrt{3}}$ 05

$\tan\theta \leq \frac{1}{\sqrt{3}}$

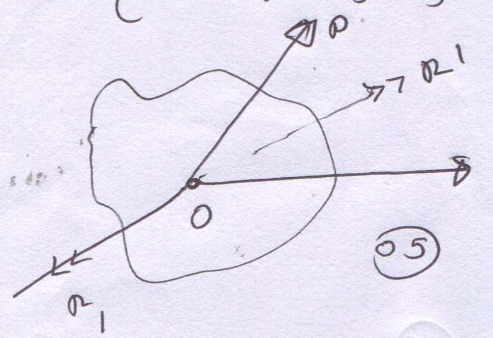
$\tan\theta \leq \tan\frac{\pi}{6}$ 05

$\theta \leq \frac{\pi}{6}$ 05

40

(16)

17) දෘඪ වස්තුවක් මත P, Q, හා R බල ක්‍රියා කරයි යැයි ගනිමු.



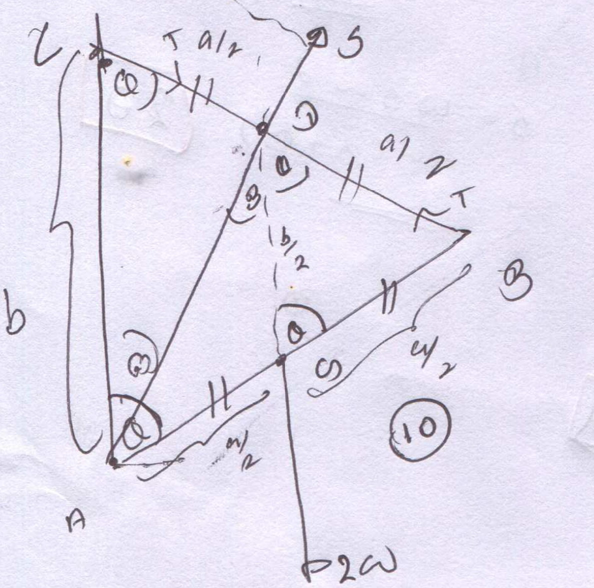
(05)

P හා Q බල දෙක O හිදී හමුවේ. එම බල දෙකෙහි සම්ප්‍රයුක්තය O හරහා යන R' බලයක් යැයි ගනිමු.

ඉහත බල පද්ධතිය (P, Q) සමතුලිත වීම

R' බලය O හරහා ක්‍රියාකරන අතර විශාලත්වයෙන් R'ට සමාන ද දිශාවෙන් ප්‍රතිවිරුද්ධ ද විය යුතුය. — (10)

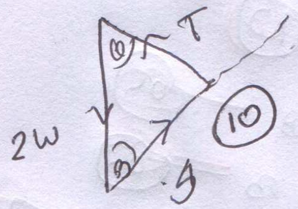
15



(10)

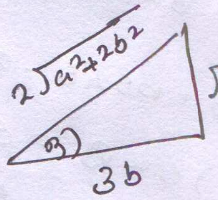
සමතුලිතතාව

බල 3 යටතේ සමතුලිත



(10)

$\cot\beta = 3\cot\theta = \frac{3 \cdot b}{\sqrt{4a^2-b^2}}$



— (10)

ABD Δ මට cot සූත්‍රයෙන්,

$2\cot\theta = \cot\beta - \cot\theta$ — (10)

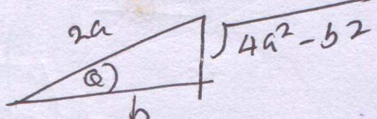
$3\cot\theta = \cot\beta$ — (1)

ABC Δ මට sin සූත්‍රය යෙදීමෙන්,

$\frac{a}{\sin\theta} = \frac{b}{\sin(\pi - 2\theta)}$

$\frac{a}{\sin\theta} = \frac{b}{2\sin\theta\cos\theta}$ $(\sin\theta \neq 0)$ — (10)

$\cos\theta = \frac{b}{2a}$ — (2)



$\frac{2w}{\sin(\pi-(\theta+\beta))} = \frac{T}{\sin\beta} = \frac{S}{\sin\theta}$

$T = \frac{2w\sin\beta}{\sin(\theta+\beta)}$

$= \frac{2w\sin\beta}{\sin\theta\cos\beta + \cos\theta\sin\beta}$

$= \frac{2w}{\sin\theta\cot\beta + \cos\theta}$

$= \frac{2w}{\frac{\cancel{\sqrt{4a^2-b^2}}}{2a}\cdot\frac{3b}{\cancel{\sqrt{4a^2-b^2}}} + \frac{b}{2a}}$

$T = \frac{4aw}{4b} = \underline{\underline{\frac{aw}{b}}}$ — (10)