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மேல் மாகாணக் கல்வித் திணைக்களம்
Department of Education - Western Province

සංයුක්ත ගණිතය I
இணைந்த கணிதம் I
Combined Mathematics I

10 S I

පැය තුනයි
மூன்று மணித்தியாலம்
Three hours

අමතර කියවීම් කාලය - මිනිත්තු 10 යි
மேலதிக வாசிப்பு நேரம் - 10 நிமிடங்கள்
Additional Reading Time - 10 minutes

Use additional reading time to go through the question paper, select the questions you will answer and decide which of them you will prioritise.

Index number : ……………………………………………………..

Instructions :

* This paper consists two parts: Part A (questions 1-10) and Part B (questions 11-17)
* **Part A :**
  Answer all questions. Write your answers to each questions in the space provided. You may use additional sheets if more space is needed.
* **Part B :**
* Answer **five** questions only. Write your answers on the sheets provided.
* At the end of the time allocated, tie the answer script of the two parts together so that **part A** is on the top of **part B** and hand them over to the supervisor.
* You are permitted to remove **only part B** of the question paper from the examination hall.

Examiner's use only

(10) CombinedMathematicsI		
Part	Question No	Marks
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	

Total

In Number	
In words	

Code numbers

Marking examiner		
Checked by	1	
	2	
Supevised by		

Part A

1) Using the **principle of mathematical induction** prove that, $(2x+1)^n - (x-3)^n$ is divisible by $(x+4)\ for\ n\epsilon\mathbb{Z}^+, x\epsilon\mathbb{R}$.

2) Sketch the graph of $y = 3 - 2|x-1|$, and $y = |2x+1|$ in the same diagram. **Hence or otherwise** for all real values of x satisfying the inequality $|4x+1| \leq 3 - 2|2x-1|$.

3) Sketch on the same Argond diagram, the loci of the points representing complex numbers Z satisfying $|Z - 2i| = 4, and \;\; arg(Z + 2\sqrt{3}) = \frac{\pi}{6}$ and find the complex number represented by the point of intersection of these loci in the form *x*+*iy*.

4) Find the number of words that can be made by using all letters of the word **"NEEDLE".** Find how many of these words have all three **E**'s together and only two **E**'s together.

5) If $\lim\limits_{x \to \alpha} \left(\frac{\sqrt{x+\alpha} - \sqrt{2\alpha}}{\sin(x-\alpha)} \right) = \frac{1}{4}$ find the value of α

6) The region enclosed by the curves $y = \sqrt{\frac{\cos x - 2\sin 2x}{\sin x + \cos 2x}}$, x axis, y axis, and $x = \frac{\pi}{3}$ is rotated about x axis through 2π radians. Show that the volume of the solid thus generated is $\pi \ln\left(\frac{\sqrt{3}-1}{2}\right)$.

7) The curve C is given by the parametric equation $y = sin^2\theta, x = 2\cos\theta$. Show that the equation of the tangent drawn to the curve C at $\theta = \frac{\pi}{3}$ is $4y + 2x - 5 = 0$ and find the equation of the normal at that point.

8) Find the coordinate of the point which lies on the perpendicular line drawn from $A \equiv (2,1)$ to the straight line $l \equiv x + y - 1 = 0$ and whose distance from A is twice the distance from A to the line l.

9) Show that the circle whose diameter lies on the straight line $2x + 3y = 7$ and passes through the points of intersection of the circles represented by $x^2 + y^2 - 4x - 6y + 11 = 0 \, , and \;\; x^2 + y^2 - 10x - 4y + 21 = 0$ is given $x^2 + y^2 - 4x - 2y + 3 = 0$.

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10) In a triangle ABC with usual notation if $acos^2\frac{C}{2} + ccos^2\frac{A}{2} = \frac{ab}{2}$, Show that the length of the sides of the triangle are in geometric progression.

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Part B

* **Answer only 5 questions**.

11) (a) Show that the quadratic equation $x^2+px+p=0$ has real roots, when $p\geq 4$ only where $p\in\mathbb{R}$. If the roots of the above equation are α, β write the values of $(\alpha+\beta)$, αβ and show that $\left(\alpha+\frac{1}{\beta}\right)\left(\beta+\frac{1}{\alpha}\right)=\frac{(p+1)^2}{p}$ .Show that the quadratic equation whose roots $\left(\alpha+\frac{1}{\beta}\right),\left(\beta+\frac{1}{\alpha}\right)$ is given by $px^2+p(p+1)x+(p+1)^2=0$ and deduce the quadratic equation whose roots are $\left(\alpha+\frac{1}{\beta}\right)^2,\left(\beta+\frac{1}{\alpha}\right)^2$.

(b) Let $f(x)=x^5+80x^2+240x+192$ Using remainder theorem show that $(x+2)^3$ is a factor of $f(x)$ .Show also that $f(x)=0$ has only one real root.

12) (a) Given that $(bx+2a)^8=C_0+C_1x+C_2x^2+\cdots+C_ix^i+\cdots+C_8x^8$ where a and b are constants. The coefficient of x_i is C_i where $i=0,1,2,\dots,8$. If $7aC_1=2C_2$ show that $b=2a^2$ .Hence or otherwise show that $\sum_{i=1}^{8}C_i=(2a)^8[(a+1)^8-1]$.

(b) Write rth term T_r of the sequence $\frac{8}{2.5}.\left(\frac{1}{2}\right)+\frac{11}{5.8}.\left(\frac{1}{2}\right)^2+\frac{14}{8.11}.\left(\frac{1}{2}\right)^3+\cdots$ and show that

$$T_r=f(r)-f(r+1) \text{ where } f(r)=\frac{1}{(3r-1)}\left(\frac{1}{2}\right)^{r-1}$$

Hence show that $\sum_{r=1}^{n}T_r=\frac{1}{2}-\frac{1}{(3n+1)}\left(\frac{1}{2}\right)^n$ and the sequence is convergent. Further show that $\frac{2}{5}\leq\sum_{r=1}^{\infty}T_r\leq\frac{1}{2}$.

13) (a) Given that $A=\begin{pmatrix}2 & 1\\ 0 & -1\end{pmatrix}, B=\begin{pmatrix}1 & 1\\ 0 & -2\end{pmatrix}$ Find the values of the constants λ, μ such that $A^2+\lambda A+\mu I=0$ I is a 2×2 unit matrix. Show that $B^{-1}=\frac{1}{2}A$ and find A^{-1} .

(b) Let $Z=x+iy$ where $x,y\in\mathbb{R}$. Write $\bar{Z}$ the conjugate of Z and $|Z|$. Show that

I. $Z.\bar{Z}=|Z|^2$
II. $Z-\bar{Z}=2i\,Im(Z)$

Also show that, when , $|Z|=1$ $\frac{Z-1}{Z+1}$ is a pure imaginary number.

(c) Find the square roots P and Q of the complex number $12 + 5i$ in the form $a + ib,\ (a, b \in \mathbb{R})$ and represent P and Q on an Argond diagram. If Q' is the conjugate complex of Q, find the comples number R such that $OPRQ'$ is a rhombus.

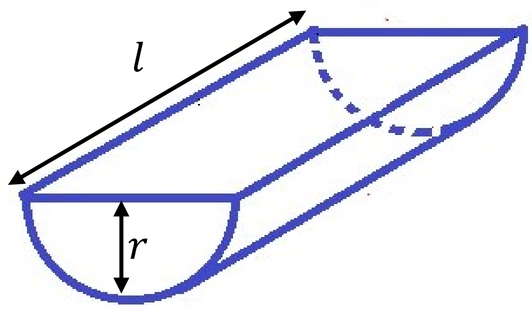
14) (a) Let $f(x) = \frac{x^2+4}{(x-2)^2}$,for $x \neq 2$.show that $f'(x)$ the derivative of $f(x)$ is given by $f'(x) = \frac{-4(x+2)}{(x-2)^3}$ for $x \neq 2$.. Find the coordinates of the turning point of $f(x)$.

Hence find the interval of which $y = f(x)$ is increasing and the interval of which $y = f(x)$ is decreasing.

Given that $f''(x) = \frac{8(x+4)}{(x-2)^4}$, $x \neq 2$ find the coordinates of the point of inflection of the graph $y = f(x)$

Sketch the graph of $y = f(x)$ indicating the asymptotes, the turning point and the point of inflection.

(b) The diagrm shown below is a semi cylindrical container of radius $r\ cm$ and length $l\ cm$. If the volume of the container is $512\pi\ cm^3$ and the surface area is $S\ cm^2$ show that S is given by $S = \pi r\left(r + \frac{1024}{r^2}\right)$ Also find the value of r when S is minimum and find the minimum surface area.



15) (a) Find $\int \frac{3}{(x-1)(x^2+x-2)}dx$ using partial fractions.

If $I = \int_0^{\frac{\pi}{6}} \frac{\cos x}{(\sin x-1)(\sin x+2)}dx$ show that, $I = \frac{1}{3}\ln\left(\frac{5}{2}\right)$

(b) Integration by parts and suitable substitution, show that,

$\int_0^{\frac{\pi}{3}} \tan^{-1}(\sin x).\cos x\, dx = \frac{2\pi-3}{2\sqrt{3}}$

(c) Show that $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$ where a is a constant

Let $I = \int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x+\cos x}dx$. Using the above results show that,

$I = \frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{1}{\sin x+\cos x}dx$. Also show that $I = \frac{1}{\sqrt{2}}\, ln(\sqrt{2}+1)$.

16) In a parallelogram $ABCD$, $AD \equiv x + y = 12$, $AB \equiv y = 2x$, $B \equiv (0, \lambda)$ and $C \equiv (-2\sqrt{10}, \mu)$. Find λ, and μ. Show that any point on AC can be written in the form $(4 + t, 8 + (\sqrt{10} - 3)t)$ where t is a parameter. Show that the circle which touches BC and the center lies on AC is given by $x^2 + y^2 - 2(t + 4)x - 2(8 + (\sqrt{10} - 3)t)y + c = 0$ Where $c \in \mathbb{R}$. When $c = t^2(12 - 4\sqrt{10}) + t(4\sqrt{10} - 16) + 8$ if the circle touches AB show that $t = -(\sqrt{10} + 2)$. Find the coordinate of the center of this circle and show that the center is the point of intersection of the diagonals of the parallelogram. Show also that this circle touches CD and find the equations of the circles touch internally and externally at B.

17) (a) Express $\sin x + \sqrt{3} \cos x$ in the form $R \sin(x + \alpha)$ where) $R > 0$, $0 < \alpha < \frac{\pi}{2}$.

If $\sec x + \sqrt{3} cosec\, x = 4$, for $0 \leq x \leq \pi$, show that

$\sin 2x - \sin\left(x + \frac{\pi}{3}\right) = 0$. Hence or otherwise find the general solution of

$\sec x + \sqrt{3} cosec\, x = 4$

(b) In a triangle ABC angle $B = \frac{\pi}{2}$. O is a point inside the triangle which makes an angle $\frac{2\pi}{3}$ with one side of the triangle. If $C\hat{B}O = \theta$ show that, $\tan\theta = \frac{c + a\sqrt{3}}{a + c\sqrt{3}}$.

(c) Solve the equation $\tan^{-1}\left(\frac{x+1}{x+2}\right) + \tan^{-1}\left(\frac{x-1}{x-2}\right) = \frac{\pi}{4}$.